Negative Dimensionality – Cartesian Spaces or Space-Times

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Abstract

We will consider spaces or space-times of cartesian character and with negative dimensionality.

Negative Dimensionality

Just take a metric with only diagonal elements of the following kind:

$$g_{\alpha\beta}=\begin{pmatrix}\frac{1}{x_0^4} & 0 & \cdots\\ 0 & \frac{1}{x_1^4} & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}\quad\text{for}\quad\forall\quad x_i\,. \tag{1}$$

Now evaluate the volume for such a space assuming a cube of side-length a for all x_i and you end with the formula $V=a^{-n}$, with n giving the dimensionality of the space. We might just see this as a space with negative dimensionality and the funny thing is that this apparently negative dimensionality comes from a completely ordinary cartesian space under the simple transformation:

$$f=f\left[x_i\right]=\pm\frac{1}{x_i}+\text{const}\quad\text{for}\quad\forall\quad x_i\,. \tag{2}$$

Following the usual tensor transformation rule:

$$g_{\alpha\beta}\left[x_i\right]=\frac{\partial X^a}{\partial x^\alpha}\frac{\partial X^b}{\partial x^\beta}g_{ab}\left[X_j\right]=\frac{\partial X^a}{\partial x^\alpha}\frac{\partial X^b}{\partial x^\beta}\begin{pmatrix}1 & 0 & \cdots\\ 0 & 1 & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}$$

$$=\frac{\partial f\left[x^a\right]}{\partial x^\alpha}\frac{\partial f\left[x^b\right]}{\partial x^\beta}\begin{pmatrix}1 & 0 & \cdots\\ 0 & 1 & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}=\frac{\partial\left(\pm\frac{1}{x^a}+\text{const}\right)}{\partial x^\alpha}\frac{\partial\left(\pm\frac{1}{x^b}+\text{const}\right)}{\partial x^\beta}\begin{pmatrix}1 & 0 & \cdots\\ 0 & 1 & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}. \tag{3}$$

$$=\delta_\alpha^a\delta_\beta^b\begin{pmatrix}\frac{1}{x_0^4} & 0 & \cdots\\ 0 & \frac{1}{x_1^4} & \cdots\\ \vdots & \vdots & \ddots\end{pmatrix}_{ab}$$

We see that this directly gives us (1), hence, a negative dimensional space or space-time.

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