

Derivation of a Metric-Dimensional Based Dirac Equation

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Abstract

In the classical derivation of the Dirac equation, there is always the problem of justifying the introduction of the spinor or vector wave function. The usual justification is based on the fact that the operational root extraction of the Klein-Gordon equation leads to a rank discrepancy which requires the introduction of a wave vector function instead of a scalar.

In this brief paper, we will show a different path.

The Dimensional Dirac

We start with the Klein-Gordon equation:

$$0 = f_{,\alpha\beta} g^{\alpha\beta} \quad (1)$$

and expand it as follows:

$$0 = f_{,\alpha\beta} g^{\alpha\beta} \rightarrow 0 = \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta}. \quad (2)$$

This allows us to derive:

$$\begin{aligned} 0 &= \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta} = f_{,\alpha\beta} \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} + \gamma_k^{j\beta} \gamma_j^{i\alpha}}{2} \\ &= \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} + \gamma_k^{j\beta} \gamma_j^{i\alpha}}{2} \partial_\beta \partial_\alpha f = \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} \partial_\alpha \partial_\beta f + \gamma_k^{j\beta} \gamma_j^{i\alpha} \partial_\beta \partial_\alpha f}{2} \\ &= \frac{\gamma_j^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_j^{i\alpha} \partial_\alpha f}{2} \end{aligned} \quad (3)$$

Now the next step usually is to just perform a split-up as follows [1]:

$$0 = \frac{\gamma_j^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_j^{i\alpha} \partial_\alpha f}{2}, \quad (4)$$

claim that, in order to obtain the zero outcome, it would suffice to just satisfy the inner derivatives, which is to say:

$$\begin{aligned} 0 &= \gamma_k^{j\beta} \partial_\beta f \\ 0 &= \gamma_j^{i\alpha} \partial_\alpha f \end{aligned} \quad (5)$$

and that in order to do so, the function f needs to become a wave vector.

Going back to the last line of equation (3), however, we see there is something fundamentally wrong with this approach. We see, namely, that there is a summation we cannot simply tear apart:

$$0 = \frac{\gamma_{[j]}^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f + \gamma_k^{[j]\beta} \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f}{2}. \quad (6)$$

In classically doing so, one would have to assume something like:

$$\begin{aligned} \gamma_{[j]}^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f &= \Gamma^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f_{[j]}; \quad \gamma_k^{[j]\beta} \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f = \Gamma_k^\beta \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f^{[j]} \\ &\Rightarrow \\ 0 &= \frac{\gamma_{[j]}^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f + \gamma_k^{[j]\beta} \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f}{2} = \frac{\Gamma^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f_{[j]} + \Gamma_k^\beta \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f^{[j]}}{2}, \end{aligned} \quad (7)$$

and the question arises how can this be justified.

So, here is our reasoning:

$$\begin{aligned} 0 &= \gamma_{[j]}^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f + \gamma_k^{[j]\beta} \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f = \cancel{\partial_{[j]}^{i\alpha}} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f + \cancel{\partial_k^{[j]\beta}} \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f, \\ &= \Gamma^{i\alpha} \partial_\alpha \gamma_k^{[j]\beta} \partial_\beta f_{[j]} + \Gamma_k^\beta \partial_\beta \gamma_{[j]}^{i\alpha} \partial_\alpha f^{[j]}, \end{aligned} \quad (8)$$

where we understand the j-contraction of the gamma matrices in combination with the partial derivatives $\partial_\alpha, \partial_\beta$ with respect to the ordinary coordinates as some kind of special derivative acting on f. Of course, in this sense we would result in a gradient $f_{[j]}, f^{[j]}$ and as with every gradient, it is a vector.

Now we can perform the split-up and obtain the usual mass-free Dirac equation:

$$\begin{aligned} 0 &= \gamma_k^{[j]\beta} \partial_\beta f_{[j]}, \\ 0 &= \gamma_{[j]}^{i\alpha} \partial_\alpha f^{[j]}. \end{aligned} \quad (9)$$

How to add mass via dimensional mass creation has been shown many times in our previous publications (e.g. [2 - 5]).

References

- [1] P. A. M. Dirac, "The Quantum Theory of the Electron", Published 1 February 1928. DOI: 10.1098/rspa.1928.0023
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