

The Dimensional Dirac Equation with Potential

By Dr. rer. nat. habil. Norbert Schwarzer

Abstract

In our most recent paper [A1] we have shown how the Dirac-spinor can come out of the Klein-Gordon equation without the usual rank discussion [A2] but via a derivative-like process similar to the evaluation of a gradient. In here we will now derive the Dirac equation with potential in the metric picture. Thereby the potential is based on additional dimensions with certain properties.

[A1] N. Schwarzer, "Derivation of a Metric-Dimensional Based Dirac Equation", 2026, a SIO publication, www.siomec.com

[A2] P. A. M. Dirac, "The Quantum Theory of the Electron", Published 1 February 1928.DOI: 10.1098/rspa.1928.0023

The Dimensional Dirac

In [1] our starting point was the Klein-Gordon equation:

$$0 = f_{,\alpha\beta} g^{\alpha\beta}, \quad (1)$$

which we expanded as follows:

$$0 = f_{,\alpha\beta} g^{\alpha\beta} \rightarrow 0 = \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta}. \quad (2)$$

This allows us to derive:

$$\begin{aligned} 0 &= \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta} = f_{,\alpha\beta} \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} + \gamma_k^{j\beta} \gamma_j^{i\alpha}}{2} \\ &= \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} + \gamma_k^{j\beta} \gamma_j^{i\alpha}}{2} \partial_\beta \partial_\alpha f = \frac{\gamma_j^{i\alpha} \gamma_k^{j\beta} \partial_\alpha \partial_\beta f + \gamma_k^{j\beta} \gamma_j^{i\alpha} \partial_\beta \partial_\alpha f}{2} \\ &= \frac{\gamma_j^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_j^{i\alpha} \partial_\alpha f}{2}. \end{aligned} \quad (3)$$

In the last line we see that there is a summation we cannot simply tear apart:

$$0 = \frac{\gamma_{[j}^{i\alpha} \partial_\alpha \gamma_{k]}^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_{[j}^{i\alpha} \partial_\alpha f}{2}. \quad (4)$$

Instead of just classically doing so (e.g. [2]), and then introducing or assuming something like:

$$\begin{aligned} \gamma_{[j}^{i\alpha} \partial_\alpha \gamma_{k]}^{j\beta} \partial_\beta f &= \Gamma^{i\alpha} \partial_\alpha \gamma_{[j}^{j\beta} \partial_\beta f_{k]}; \quad \gamma_k^{j\beta} \partial_\beta \gamma_{[j}^{i\alpha} \partial_\alpha f = \Gamma_k^\beta \partial_\beta \gamma_{[j}^{i\alpha} \partial_\alpha f_{j]} \\ &\Rightarrow \\ 0 &= \frac{\gamma_{[j}^{i\alpha} \partial_\alpha \gamma_{k]}^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_{[j}^{i\alpha} \partial_\alpha f}{2} = \frac{\Gamma^{i\alpha} \partial_\alpha \gamma_{[j}^{j\beta} \partial_\beta f_{k]} + \Gamma_k^\beta \partial_\beta \gamma_{[j}^{i\alpha} \partial_\alpha f_{j]}}{2} \end{aligned} \quad (5)$$

we aim for a slightly different reasoning, namely:

$$\begin{aligned} 0 &= \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} \gamma_k^{j\beta} \partial_{\beta} f + \gamma_k^{j\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f = \cancel{\partial_{[\underline{j}}^{i\alpha} \partial_{\alpha} \gamma_k^{j\beta} \partial_{\beta} f} + \cancel{\partial_k^{j\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f} \\ &= \Gamma^{i\alpha} \partial_{\alpha} \gamma_k^{j\beta} \partial_{\beta} f_{[\underline{j}} + \Gamma_k^{j\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f^{i]} \end{aligned} \quad (6)$$

where we understand the j-contraction of the gamma matrices in combination with the partial derivatives $\partial_{\alpha}, \partial_{\beta}$ with respect to the ordinary coordinates as some kind of special derivative acting on f. Of course, in this sense we would result in a gradient $f_{[\underline{j}}, f^{i]}$ and as with every gradient, it is a vector.

Now we can perform the split-up and obtain the usual mass-free Dirac equation:

$$\begin{aligned} 0 &= \gamma_k^{j\beta} \partial_{\beta} f_{[\underline{j}} \\ 0 &= \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f^{i]} \end{aligned} \quad (7)$$

How to add mass via dimensional mass creation has been shown many times in our previous publications (e.g. [4 - 7]).

Adding the Potential

We know from basic quantum theory (e.g. [3]) that many fields and potentials couple into the quantum equations via the so-called weak coupling. Picking the example of the electromagnetic field with the potential A_{μ} , this would be done as follows:

$$f_{,\alpha} = \partial_{\alpha} f \rightarrow (\partial_{\alpha} + A_{\alpha}) f. \quad (8)$$

This could be interpreted as a coordinate transformation $x \rightarrow X, Y$ with:

$$\begin{aligned} f &= f[x^{\sigma}] \rightarrow f[X^i[x^{\sigma}], Y^j[x^{\sigma}]] = f[x^{\sigma}, Y^j[x^{\sigma}]] = f_X[x^{\sigma}] \cdot f_Y[Y^j[x^{\sigma}]] \\ &\rightarrow \\ f_{,\alpha} &= \frac{\partial f}{\partial(X^i)} \frac{\partial X^i}{\partial x^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial(x^{\sigma})} \frac{\partial x^{\sigma}}{\partial x^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial(x^{\sigma})} \delta_{\alpha}^{\sigma} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}}, \quad (9) \\ &= \frac{\partial f}{\partial(X^{\sigma} = x^{\sigma})} \delta_{\alpha}^{\sigma} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial X^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = f_Y \frac{\partial f_X}{\partial X^{\alpha}} + f_X \frac{\partial f_Y}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} \\ &\xrightarrow{\frac{\partial f_Y}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = f_Y A_{\alpha}} = f_Y \frac{\partial f_X}{\partial X^{\alpha}} + f_X f_Y A_{\alpha} = \left(\frac{\partial}{\partial X^{\alpha}} + A_{\alpha} \right) f \end{aligned}$$

with an additional coordinate Y, which agrees with our earlier findings about dimensional mass creation [4 – 7] and the construction of various potentials via additional dimensions [8, 9].

Combining the Above

Inserting (8) into (7) gives us the classical form:

$$\begin{aligned} 0 &= \left(\gamma_{\mathbf{k}}^{\mathbf{j}\beta} \partial_{\beta} + A_{\alpha} \right) f_{\mathbf{j}} \\ 0 &= \left(\gamma_{\mathbf{j}}^{i\alpha} \partial_{\alpha} + A_{\alpha} \right) f_{\mathbf{j}} \end{aligned} \quad (10)$$

However, the derivation itself shows us that there is more to win (and to understand) when finding the metric origin of the potential and this we already did in variety of cases [10 - 13]. This we will demonstrate in our next paper [14].

References

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