

The Dimensional Dirac Equation with Potential and Mass

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Abstract

In our most recent paper [A1] we have shown how the Dirac-spinor can come out of the Klein-Gordon equation without the usual rank discussion [A2] but via a derivative-like process similar to the evaluation of a gradient. In [A3] we added the potential to the metric picture. Thereby the potential is based on additional dimensions with certain properties. In this paper we now add mass.

[A1] N. Schwarzer, "Derivation of a Metric-Dimensional Based Dirac Equation", 2026, a SIO publication, www.siomec.com

[A2] P. A. M. Dirac, "The Quantum Theory of the Electron", Published 1 February 1928. DOI: 10.1098/rspa.1928.0023

[A3] N. Schwarzer, "The Dimensional Dirac Equation with Potential", 2026, a SIO publication, www.siomec.com

The Dimensional Dirac

In [1] our starting point was the Klein-Gordon equation:

$$0 = f_{,\alpha\beta} g^{\alpha\beta}, \quad (1)$$

which we expanded as follows:

$$0 = f_{,\alpha\beta} g^{\alpha\beta} \rightarrow 0 = \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta}. \quad (2)$$

This allows us to derive:

$$0 = \delta_j^i \delta_k^j f_{,\alpha\beta} g^{\alpha\beta} = \frac{\gamma_j^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_j^{i\alpha} \partial_\alpha f}{2}. \quad (3)$$

In the last line we see that there is a summation we cannot simply tear apart:

$$0 = \frac{\gamma_{[j}^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_{j]}^{i\alpha} \partial_\alpha f}{2}. \quad (4)$$

Instead of just classically doing so (e.g. [2]), and then introducing or assuming something like:

$$\begin{aligned} \gamma_{[j}^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f &= \Gamma^{i\alpha} \partial_\alpha \gamma_k^{[j\beta} \partial_{\beta]} f_{j]}; & \gamma_k^{j\beta} \partial_\beta \gamma_{j]}^{i\alpha} \partial_\alpha f &= \Gamma_k^\beta \partial_\beta \gamma_{[j}^{i\alpha} \partial_{\alpha]} f_{j]} \\ &\Rightarrow & & \\ 0 &= \frac{\gamma_{[j}^{i\alpha} \partial_\alpha \gamma_k^{j\beta} \partial_\beta f + \gamma_k^{j\beta} \partial_\beta \gamma_{j]}^{i\alpha} \partial_\alpha f}{2} = \frac{\Gamma^{i\alpha} \partial_\alpha \gamma_k^{[j\beta} \partial_{\beta]} f_{j]} + \Gamma_k^\beta \partial_\beta \gamma_{[j}^{i\alpha} \partial_{\alpha]} f_{j]}}{2} \end{aligned} \quad (5)$$

We aim for a slightly different reasoning, namely:

$$\begin{aligned}
0 &= \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} \gamma_{\underline{k}}^{j\beta} \partial_{\beta} f + \gamma_{\underline{k}}^{j\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f = \cancel{\partial_{[\underline{j}}^{i\alpha} \partial_{\alpha} \gamma_{\underline{k}}^{j\beta} \partial_{\beta} f} + \cancel{\partial_{\underline{k}}^{j\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f} , \\
&= \Gamma^{i\alpha} \partial_{\alpha} \gamma_{\underline{k}}^{j\beta} \partial_{\beta} f_{[\underline{j}} + \Gamma_{\underline{k}}^{\beta} \partial_{\beta} \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f_{\underline{j}]} ,
\end{aligned} \tag{6}$$

where we understand the j-contraction of the gamma matrices in combination with the partial derivatives $\partial_{\alpha}, \partial_{\beta}$ with respect to the ordinary coordinates as some kind of special derivative acting on f. Of course, in this sense we would result in a gradient $f_{[\underline{j}], f^{[\underline{j}]}$ and as with every gradient, it is a vector.

Now we can perform the split-up and obtain the usual mass-free Dirac equation:

$$\begin{aligned}
0 &= \gamma_{\underline{k}}^{j\beta} \partial_{\beta} f_{[\underline{j}} \\
0 &= \gamma_{[\underline{j}}^{i\alpha} \partial_{\alpha} f_{\underline{j}]} .
\end{aligned} \tag{7}$$

How to add mass via dimensional mass creation has been shown many times in our previous publications (e.g. [4 - 7]).

Adding the Potential

We know from basic quantum theory (e.g. [3]) that many fields and potentials couple into the quantum equations via the so-called weak coupling. Picking the example of the electromagnetic field with the potential A_{μ} , this would be done as follows:

$$f_{,\alpha} = \partial_{\alpha} f \rightarrow (\partial_{\alpha} + A_{\alpha}) f . \tag{8}$$

This could be interpreted as a coordinate transformation $x \rightarrow X, Y$ with:

$$\begin{aligned}
f &= f[x^{\sigma}] \rightarrow f[X^i[x^{\sigma}], Y^j[x^{\sigma}]] = f[x^{\sigma}, Y^j[x^{\sigma}]] = f_X[x^{\sigma}] \cdot f_Y[Y^j[x^{\sigma}]] \\
&\rightarrow \\
f_{,\alpha} &= \frac{\partial f}{\partial(X^i)} \frac{\partial X^i}{\partial x^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial(x^{\sigma})} \frac{\partial x^{\sigma}}{\partial x^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial(x^{\sigma})} \delta_{\alpha}^{\sigma} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} , (9) \\
&= \frac{\partial f}{\partial(X^{\sigma} = x^{\sigma})} \delta_{\alpha}^{\sigma} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = \frac{\partial f}{\partial X^{\alpha}} + \frac{\partial f}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = f_Y \frac{\partial f_X}{\partial X^{\alpha}} + f_X \frac{\partial f_Y}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} \\
&\xrightarrow{\frac{\partial f_Y}{\partial(Y^j)} \frac{\partial Y^j}{\partial x^{\alpha}} = f_Y A_{\alpha}} = f_Y \frac{\partial f_X}{\partial X^{\alpha}} + f_X f_Y A_{\alpha} = \left(\frac{\partial}{\partial X^{\alpha}} + A_{\alpha} \right) f
\end{aligned}$$

with an additional coordinate Y, which agrees with our earlier findings about dimensional mass creation [4 – 7] and the construction of various potentials via additional dimensions [8, 9].

Combining the Above

Inserting (8) into (7) gives us the classical form:

$$\begin{aligned} 0 &= \left(\gamma_{\mathbf{j}}^{\mathbf{i}\beta} \partial_{\beta} + A_{\alpha} \right) f_{\mathbf{j}} \\ 0 &= \left(\gamma_{\mathbf{j}}^{i\alpha} \partial_{\alpha} + A_{\alpha} \right) f_{\mathbf{j}} \end{aligned} \quad (10)$$

However, the derivation itself shows us that there is more to win (and to understand) when finding the metric origin of the potential and this we already did in variety of cases [10 - 13].

Adding Mass

In the classical “weak coupling” strategy, the mass would just be added via:

$$f_{,\alpha} = \partial_{\alpha} f \rightarrow (\partial_{\alpha} + A_{\alpha} + m) f, \quad (11)$$

but when insisting to perform the dimensional mass creation, we require higher dimensional Dirac matrices. Using the technique presented in [14, 15] (see also https://en.wikipedia.org/wiki/Higher-dimensional_gamma_matrices), we can create Dirac matrices for dimensions higher than four using Kronecker products of Pauli matrices. For dimension n , there are n Dirac matrices, and the dimension of the matrices is $2^{\frac{n}{2}} \cdot 2^{\frac{n}{2}}$, whereby in the cases of odd dimensions, the matrices have the same physical size than the matrices in the previous even case but there is always one matrix more¹. Just as an example we here give the corresponding matrices in five dimensions, reading:

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} & & 1 \\ & 1 & \\ & & \\ 1 & & \end{pmatrix}; \quad \gamma^1 = \begin{pmatrix} & & -i \\ & i & \\ -i & & \\ i & & \end{pmatrix}; \quad \gamma^2 = \begin{pmatrix} & 1 & \\ & & -1 \\ 1 & & \\ & -1 & \end{pmatrix} \\ \gamma^3 &= \begin{pmatrix} & -i & \\ & & -i \\ i & & \\ & i & \end{pmatrix}; \quad \gamma^5 = \gamma^4 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \\ & & & -1 \end{pmatrix}; \quad I = \delta_{ij} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \end{aligned} \quad (12)$$

while in four dimensions we have the well-known Dirac originals with:

¹ From https://en.wikipedia.org/wiki/Gamma_matrices:

“The Clifford algebra in odd dimensions behaves like two copies of the Clifford algebra of one less dimension, a left copy and a right copy. Thus, one can employ a bit of a trick to repurpose $i^* \gamma^5$ as one of the generators of the Clifford algebra in five dimensions. In this case, the set $\{\gamma^0, \gamma^1, \gamma^2, \gamma^3, i^* \gamma^5\}$ therefore, by the last two properties (keeping in mind that $i^2 \equiv -1$) and those of the ‘old’ gammas, forms the basis of the Clifford algebra in 5 space-time dimensions for the metric signature (1,4). In metric signature (4,1), the set $\{\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\}$ is used, where the γ^{μ} are the appropriate ones for the (3,1) signature. This pattern is repeated for space-time dimension $2n$ even and the next odd dimension $2n + 1$ for all $n \geq 1$.”

$$\gamma^0 = \begin{pmatrix} & & 1 \\ & 1 & \\ 1 & & \end{pmatrix}; \quad \gamma^1 = \begin{pmatrix} & & -i \\ & i & \\ i & -i & \end{pmatrix}; \quad \gamma^2 = \begin{pmatrix} & 1 & \\ & & -1 \\ 1 & -1 & \end{pmatrix}.$$

$$\gamma^3 = \begin{pmatrix} & -i & \\ & & -i \\ i & & \\ & i & \end{pmatrix}; \quad I = \delta_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$
(13)

Please note that the latter version of Dirac matrices can be obtain from:

$$\gamma_D^0 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}; \quad \gamma_D^1 = \begin{pmatrix} & & 1 \\ & 1 & \\ -1 & -1 & \end{pmatrix},$$

$$\gamma_D^2 = \begin{pmatrix} & & -i \\ & i & \\ i & & \\ -i & & \end{pmatrix}; \quad \gamma_D^3 = \begin{pmatrix} & 1 & \\ & & -1 \\ -1 & & \\ & 1 & \end{pmatrix}; \quad I = \delta_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$
(14)

via matrix multiplication with $-\gamma_D^0 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$, respectively just γ_D^0 . The evaluation is

performed as follows:

$$-\gamma_D^1 \gamma_D^0 = \gamma^0; \quad -\gamma_D^2 \gamma_D^0 = \gamma^1; \quad -\gamma_D^3 \gamma_D^0 = \gamma^2; \quad ; \quad -\gamma_D^3 \gamma_D^0 = \gamma^3; \quad I = \delta_{ij} = \gamma_D^0 \gamma_D^0. \quad (15)$$

It should be noted that only in two and in four dimensions the number of elements in line and column of the matrices is equal to the dimension of the corresponding space-time.

By applying the five-dimensional Dirac matrices (12), we can just use (13) within the resulting final Dirac equations, because one of the matrices would produce the mass term, resulting in:

$$0 = \left(\gamma_k^{\underline{j}\underline{\beta}} \partial_{\beta} + A_{\alpha} + m \right) f_{\underline{j}}.$$

$$0 = \left(\gamma_{\underline{j}}^{i\alpha} \partial_{\alpha} + A_{\alpha} + m \right) f^{\underline{j}}.$$
(16)

Another, and more metric-orientated path was used in [12,13]. Simply by starting with the following metric tensor:

$$\begin{aligned}
g_{\alpha\beta}^8 &= \begin{pmatrix} -c^2 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & r^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2 \varphi_1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{77} \end{pmatrix} \\
&\times F[f[t, r, \vartheta, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]]; \quad F[f] = f^{\frac{2}{3}} \\
f[t, r, \vartheta, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5] &= f_t[t] \cdot f_r[r] \cdot f_\vartheta[\vartheta] \cdot \prod_{i=1}^5 f_{\varphi_i}[\varphi_i]; \\
g_{44} &= -\left(g_{\varphi_2}[\varphi_2]'\right)^2 \cdot r^1; \\
g_{55} &= \left(g_{\varphi_3}[\varphi_3]'\right)^2 \cdot r^2; \\
g_{66} &= \left(g_{\varphi_4}[\varphi_4]'\right)^2 \cdot r^0; \\
g_{77} &= \left(g_{\varphi_5}[\varphi_5]'\right)^2 \cdot r^0;
\end{aligned} \tag{17}$$

and a partially “fixing” of some of the functions as follows (for $i=3, 4, 5$):

$$\begin{aligned}
f_{\varphi_i}[\varphi_i] &= f_{\varphi_i}[g_{\varphi_i}[\varphi_i]] \equiv f_{\varphi_i}[g_{\varphi_i}]; \\
f_{\varphi_i}[g_{\varphi_i}] &= C_{-i} \cdot e^{-i \cdot A_i \cdot g_{\varphi_i}} + C_{+i} \cdot e^{+i \cdot A_i \cdot g_{\varphi_i}} = C_{ci} \cdot \cos[A_i \cdot g_{\varphi_i}] + C_{si} \cdot \sin[A_i \cdot g_{\varphi_i}] \\
&\text{especially (classically):} \\
f_{\varphi_l}[g_{\varphi_l}] &= C_{-l} \cdot e^{-i \cdot A_l \cdot g_{\varphi_l}} + C_{+l} \cdot e^{+i \cdot A_l \cdot g_{\varphi_l}} = C_{cl} \cdot \cos\left[\overset{\triangle m}{A_l \cdot g_{\varphi_l}}\right] + C_{sl} \cdot \sin\left[\overset{\triangle m}{A_l \cdot g_{\varphi_l}}\right]
\end{aligned} \tag{18}$$

$$f_t[t] = C_{t1} \cdot \cos[c \cdot E \cdot t] + C_{t2} \cdot \sin[c \cdot E \cdot t], \tag{19}$$

$$f_\vartheta[\vartheta = \varphi_0] = C_{p\vartheta} \cdot P_L^{A_1}[\cos[\vartheta = \varphi_0]] + C_{Q\vartheta} \cdot Q_L^{A_1}[\cos[\vartheta = \varphi_0]], \tag{20}$$

with the associated Legendre polynomials $P_L^{A_1}, Q_L^{A_1}$, gives us the following differential equation for the r -dependency of $f[\dots]$, when evaluating the Ricci scalar R^* of that very metric:

² If not used as index “ i ” stands for the imaginary number $i = \sqrt{-1}$.

$$R^* = 0 = \left(\frac{28 \cdot L \cdot (L+1) - 39}{28 \cdot r^2} + \frac{A_3^2}{r^2} - \frac{A_2^2}{r} + A_4^2 + A_5^2 - \frac{3}{2} \frac{\partial}{r \cdot \partial r} - \overbrace{\Delta_{3D-sphere} [r]}^{2 \frac{\partial}{r \cdot \partial r} + \frac{\partial^2}{\partial r^2}} - E^2 \right) \Psi$$

$$\Psi = \Psi[r, \vartheta = \varphi_0, \varphi = \varphi_1] = f_r[r] \cdot f_\vartheta[\vartheta = \varphi_0] \cdot f_\varphi[\varphi = \varphi_1]$$

$\Rightarrow$

$$0 = f_\vartheta[\vartheta] \cdot f[\varphi] \cdot \left(\frac{28 \cdot L \cdot (L+1) - 39}{28 \cdot r^2} + \frac{A_3^2}{r^2} - \frac{A_2^2}{r} + A_4^2 + A_5^2 - \frac{7}{2} \frac{\partial}{r \cdot \partial r} - \frac{\partial^2}{\partial r^2} - E^2 \right) f_r[r]$$

Now we can also solve (21) via:

$$f_r[r] = e^{-r \cdot \mu} \cdot r^{\ell - \frac{3}{4}} \cdot \left(C_U U[-nn, 2 \cdot (1 + \ell), 2 \cdot r \cdot \mu] + C_L \cdot L_{nn}^{1+2 \cdot \ell} [2 \cdot r \cdot \mu] \right)$$

$$\mu = \sqrt{A_4^2 + A_5^2 - E^2}; \quad \ell = \frac{1}{28} \left(\sqrt{133 + 784 A_3^2 + 784 L(1 + L)} - 14 \right) \cdot$$

$$nn = \frac{A_2^2}{2 \cdot \mu} - 1 - \ell$$

With the hypergeometric function $U[a, b, z]$ and the Laguerre polynomials $L_{nn}^{1+2 \cdot L}$ in the solutions above, we recognize the similarity to the Schrödinger hydrogen solution (e.g. [12]).

Now we see, why we had to make the coordinate φ_2 time-like in the metric tensor (17). With a space-like coordinate, namely, we would not have been able to obtain a stable potential setting for our $1/r$ -Potential, at least not when being interested in allowing for bonded solutions and only oscillating solutions for the φ_2 -function.

In the same way we may also be interested in obtaining a “confining interaction potential” with the following metric:

$$g_{\alpha\beta}^8 = \begin{pmatrix} -c^2 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & r^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2 \varphi_1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{77} \end{pmatrix}$$

$$\times F[f[t, r, \vartheta, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]]; \quad F[f] = f^{\frac{2}{3}}$$

$$f[t, r, \vartheta, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5] = f_t[t] \cdot f_r[r] \cdot f_\vartheta[\vartheta] \cdot \prod_{i=1}^5 f_{\varphi_i}[\varphi_i];$$

$$g_{44} = -\left(g_{\varphi_2}[\varphi_2]\right)' \cdot r^1;$$

$$g_{55} = \left(g_{\varphi_3}[\varphi_3]\right)' \cdot r^2;$$

$$g_{66} = \left(g_{\varphi_4}[\varphi_4]\right)' \cdot r^{-1};$$

$$g_{77} = \left(g_{\varphi_5}[\varphi_5]\right)' \cdot r^0;$$

With the settings from (18) to (20) we obtain:

$$R^* = 0 = \left(\frac{28 \cdot L \cdot (L+1) - 27}{28 \cdot r^2} + \frac{A_3^2}{r^2} - \frac{A_2^2}{r} + r \cdot A_4^2 + A_5^2 - \frac{\partial}{r \cdot \partial r} - \overbrace{\Delta_{3D\text{-sphere}}[r]}^{2 \frac{\partial}{r \cdot \partial r} + \frac{\partial^2}{\partial r^2}} - E^2 \right) \Psi$$

$$\Psi = \Psi[r, \vartheta = \varphi_0, \varphi = \varphi_1] = f_r[r] \cdot f_\vartheta[\vartheta = \varphi_0] \cdot f_\varphi[\varphi = \varphi_1]$$

$$\Rightarrow$$

$$0 = f_\vartheta[\vartheta] \cdot f[\varphi] \cdot \left(\frac{28 \cdot L \cdot (L+1) - 27}{28 \cdot r^2} + \frac{A_3^2}{r^2} - \frac{A_2^2}{r} + r \cdot A_4^2 + A_5^2 - 3 \frac{\partial}{r \cdot \partial r} - \frac{\partial^2}{\partial r^2} - E^2 \right) f_r[r]$$

The “confining potential” would be the term $r \cdot A_4^2$. Unfortunately, we only find a closed form solution for vanishing A_4 , which reads:

$$f_r[r] = e^{-r \cdot \mu} \cdot r^{\ell - \frac{1}{2}} \cdot \left(C_U U[-nn, 2 \cdot (1 + \ell), 2 \cdot r \cdot \mu] + C_L \cdot L_{nn}^{1+2 \cdot \ell} [2 \cdot r \cdot \mu] \right)$$

$$\mu = \sqrt{A_5^2 - E^2}; \quad \ell = \frac{1}{2} \left(\sqrt{\frac{1}{7} + 3A_3^2 + 4L(1+L)} - 1 \right); \quad nn = \frac{A_2^2}{2 \cdot \mu} - 1 - \ell$$

Choosing:

$$g_{\alpha\beta}^{10} = \begin{pmatrix} -c^2 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & r^2 & 0 & \dots & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2 \varphi_1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{99} \end{pmatrix}$$

$$\times F[f[t, r, \vartheta, \varphi, \tau, \rho, \theta, \phi]]; \quad F[f] = f^{\frac{1}{2}}$$

$$f[t, r, \vartheta, \varphi, \tau, \rho, \theta, \phi, x, y] = \begin{pmatrix} f_t[t] \cdot f_r[r] \cdot f_\vartheta[\vartheta] \cdot f_\varphi[\varphi] \cdot f_\tau[\tau] \\ f_\rho[\rho] \cdot f_\theta[\theta] \cdot f_\phi[\phi] \cdot f_x[x] \cdot f_y[y] \end{pmatrix};$$

$$g_{44} = -c_4^2 \cdot r; \quad g_{55} = r; \quad g_{66} = \rho^2; \quad g_{77} = \rho^2 \cdot \sin[\theta]^2; \quad g_{88} = 1; \quad g_{99} = 1$$

$$f_x[x] = C_{-x} \cdot e^{-A_x \cdot x} + C_{+x} \cdot e^{+A_x \cdot x};$$

$$f_y[y] = C_{-y} \cdot e^{-A_y \cdot y} + C_{+y} \cdot e^{+A_y \cdot y}$$

$$f_{\varphi_i}[g_{\varphi_i}] = C_{-i} \cdot e^{-A_i \cdot g_{\varphi_i}} + C_{+i} \cdot e^{+A_i \cdot g_{\varphi_i}}$$

and taking the solutions from (18) to (20), the equation (21) is clearly giving us the connection to the classical picture with potential, mass and so on. Thereby the angular part is completely sucked into the L-term, which – as shown above – vanishes in the setting $L=1/2$.

$$0 = \left(\frac{?}{r \cdot \partial r} + \frac{\overbrace{A_3^2 + A_4^2}^{V[r]}}{r} + \overbrace{A_5^2 + A_6^2}^M + \frac{\overbrace{2 \cdot \partial}{\Delta_{3D\text{-sphere}}} + \frac{\partial^2}{\partial r^2}}{r \cdot \partial r} - \overbrace{E^2}^{\text{energy}} \right) f_r[r].$$

We see how we can allocate $V[r]$, M and energy to the terms in the metric above and it becomes clear how mass, potential and energy are metrically just get manifest as dimensions. The

corresponding multidimensional Dirac equation is then obtained via the classical root-extraction process.

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