

Quantum Gravity Waves – Part 1: The Math of Linear Tensor Waves

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Abstract

Directly applying the classical approaches for the simplest linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we not only obtain clearly wave-like field equations in vacuum for the original Einstein equations, but also for the corresponding quantum gravity field equations.

We even find similar solutions for gravitational waves, though with more flexibility (degrees of freedom), no need for a postulation of matter and – of course – the right limiting behavior in the case of vanishing quantum effects. The convergence to the classical case concerns both, the field equations and its wave-like solutions, but with respect to the field equations, we realize that the transition allows us a glimpse into the nature of quantum attributes.

Introduction

Gravity waves are an outcome of Einstein's great general theory of relativity and they are a standard topic in the corresponding textbook literature (e.g. [1], pp 325). Usually, the derivations are restricted to the linearized Einstein Field Equations and – if even mentioned – the problem of quantization, respectively quantized wave solutions is not discussed... simply, so it is said, there is no quantum gravity theory.

In this paper, we will present a simple extension of the classical linear gravity waves, showing how this might lead to a linearized Quantum Gravity approach. Thereby, one can show that we are able to obtain such equations via the quantization of the classical recipe and a direct linearization of our own quantum gravity starting point. In this paper, however, we will only demonstrate the first approach. We will demonstrate that it requires only a small adaptation of the classical Hilbert and Einstein achievements [2, 3] to come up with such a Quantum Gravity Theory or – as it is sometimes also been called - a "Theory of Everything".

In a variety of previous publications [4 – 13], this author has shown how a simple scaling factor to the metric tensor already leads to a quantum gravity field equational outcome. The "problem" with this finding of course is, that Hilbert should get all the credit for already having found a - or THE - "Theory of Everything" over 100 years ago, namely in 1915 [1]. There wasn't much to be done, unless one counts adding a scaling factor [4 - 13] or a set of such [6, 7, 8, 14, 15] as "much", which this author definitively does not do. He sees such a "work" as a mere finding in another's paper, namely, Hilbert's "Die Grundlagen der Physik" from 1915 [1]. However, with tens of thousands of jobs at stake because they all depend on the fact that – apparently - there is no Theory of Everything yet, people obviously have problems or – to use a psychological term in order to give - at least - some of these scientists an excuse for their blindness – face an undeniable cognitive dissonant barrier to recognize that Hilbert has already done almost all the work. Worse still, it was also shown [19] that – in principle – variational kernels in the Einstein-Hilbert action of the type $f[R]$, as they are necessary to produce the endless output of new field equations, some of the more creative and paper-productive researcher are permanently proposing, and as they are also necessary for so many other Trans-Planckian

approaches (e.g. [16, 17, 18]), are not of need, because those could always be substituted by a suitable metric scaling factor [8, 19] without changing the total variational (Hilbert!!!) integral, which – after all – is a scalar. With the potential wall of recognition too high to be overcome by most of the string, loop-gravity, trans Planckians and whatever else researchers, perhaps the quantum mechanical tunneling will help them to just – one day – miraculously diffuse through this barrier. Until then, the circus will probably produce tens of thousands more of completely useless “scientific” “contributions”, while the simple fact that the job was already done over 100 years ago continues to be ignored or hushed up.

Nevertheless, in here, we will apply the rather straight forward and very rich Hilbert approach... just a tiny bit expanded, respectively, generalized.

A Quantum Gravity Theory

We start with the following scaled metric tensor and force it into the Einstein-Hilbert action variational problem [2] as follows:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \rightarrow \delta W = 0 = \delta \int_V d^n x \sqrt{-G} \cdot (R^* - 2 \cdot \Lambda) \quad (1)$$

Here Λ is the cosmological constant, G denotes the determinant of the metric tensor from (1) and R^* gives the corresponding Ricci scalar. Performing the variation with respect to the metric $G_{\alpha\beta}$ results in:

$$0 = \left(\begin{array}{c} \boxed{R_{\alpha\beta} - R \frac{g_{\alpha\beta}}{2} + \Lambda \cdot g_{\alpha\beta}} \\ -\frac{1}{2F} \left(\begin{array}{c} F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} + F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - \\ F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} + F_{,d} g^{cd} \left(\begin{array}{c} g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{array} \right) \end{array} \right) \\ + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \\ + (n-1) \left(\frac{1}{2F} \left(\begin{array}{c} 2\Delta F - 2F_{,d} g^{cd}{}_{,c} \\ - \frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac, b} \end{array} \right) + \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \frac{g_{\alpha\beta}}{2} \end{array} \right) \delta G^{\alpha\beta} \quad (2)$$

and shows us that we have not only obtained the classical Einstein Theory of General Relativity [3] (see boxed terms exactly giving the Einstein Field Equations in vacuum plus the cosmological constant term), but also a set of quantum field equations for the scaling function F , clearly playing the role of the wave-function. It was shown in our previous publications [4, 5, 6, 7, 8] that these additional terms are quantum equations, fully covering the main aspects of relativistic classical quantum theory. Everything else can be obtained by a few generalizations, structural shaping and the introduction of the variation with respect to the degrees of freedom or number of dimensions [4, 5, 6, 7, 8]. So, we conclude, that we indeed have a Quantum Gravity Theory or Theory of Everything, as one also calls it, at hand, whereby it should be pointed out that (2) has to be considered the simplest possible – and still general (see [4, 5, 6, 7, 8]) - form for the corresponding quantum gravity field equations. More complex versions are containing many scaling factors (e.g. [6, 7, 8, 14, 15]).

“Weak Gravity” and Linearity – The Transition to the Classical Quantum Theory

It was shown in [5, 6, 7, 8] that the so-called “weak gravity” condition:

$$\delta G^{\alpha\beta} = G^{\alpha\beta} \cdot \delta_0 + \overbrace{G^{ab} \delta_{ab}^{\alpha\beta}}^{\text{Gravity}} \xrightarrow{\forall \delta_{ab}^{\alpha\beta} \ll \delta_0} = \frac{g^{\alpha\beta}}{F} \cdot \delta_0, \quad (3)$$

together with a setting for the scaling function $F[f]$ as follows:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases}. \quad (4)$$

leads to a significant simplification and scalarization of the quantum gravity field equations (2), namely:

$$0 = R - \frac{F'}{2F} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right). \quad (5)$$

This equation is completely linear in f , which not only has the characteristics of a quantum function, but – for a change – gives us the opportunity to metrically see what QUANTUM actually means, namely, a volume jitter to the metric of the system in question... at least this is one quantum option, because we have already seen others, like the perturbed kernel (e.g. see [8]).

Interestingly, for metrics without shear elements:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \quad (6)$$

and applying the solution for $F[f]$ from (4) the derivative terms in (5), which is to say:

$$(n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b}. \quad (7)$$

converge to the ordinary Laplace operator, namely:

$$\begin{aligned} R^* = 0 &\rightarrow 0 = F \cdot R + F' \cdot (1-n) \cdot \Delta f \\ \Rightarrow 0 &= \begin{cases} (f - C_f)^{\frac{4}{n-2}} \cdot C_F \left(R + \frac{4}{n-2} \cdot \frac{(1-n)}{(f - C_f)} \cdot \Delta f \right) & n > 2 \\ e^{C_f \cdot f} \cdot C_F \left(R + C_f \cdot (1-n) \cdot \Delta f \right) & n = 2 \end{cases} \end{aligned} \quad (8)$$

We recognize the relativistic Klein-Gordon equation.

Thus, in the case of $n > 2$ we always also have the option for a constant (broken symmetry) solution of the kind:

$$0 = f - C_{f0} \Rightarrow f = C_{f0}. \quad (9)$$

In all other cases, meaning where $f \neq C_{f0}$, we have the simple equations:

$$0 = \begin{cases} (f - C_{f0}) \cdot R + (1-n) \cdot \frac{4}{n-2} \cdot \Delta f & n > 2 \\ R + C_{f0} \cdot (1-n) \cdot \Delta f & n = 2 \end{cases}. \quad (10)$$

A critical argument should now be that this equation is not truly of Klein-Gordon character as it does contain neither potential nor mass, but this author has already shown that this problem is easily solved by adding additional dimensions carrying the right properties to produce masses and potentials due to entanglement, being provided by the right scaling function $F[f]$ (e.g. [4 – 8]).

Using these results we were able to develop a quantum gravity statistics [9, 10], formulate a Heisenberg uncertainty principle containing gravity [12] and even suggesting a path for answering the riddle of the 3 generations of elementary particles [13].

Finding Matter

Observing our variational result (2) and comparing with the classical equations from [3]:

$$R_{\alpha\beta} - \frac{1}{2} R \cdot g_{\alpha\beta} + \Lambda \cdot g_{\alpha\beta} = -\kappa \cdot T_{\alpha\beta}, \quad (11)$$

where we have: $R_{\alpha\beta}$, $T_{\alpha\beta}$ the Ricci- and the energy momentum tensor, respectively, while the parameters Λ and κ are constants (usually called cosmological and coupling constant, respectively), we realize that the following terms of (2) are just the most natural energy momentum tensor elements, yielding the following identity:

$$\kappa \cdot T_{\alpha\beta} = \left(\begin{array}{c} F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} + F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - \\ - \frac{1}{2F} \left(F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} + F_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \right) \right. \\ \left. + \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha \beta} g_{ab, c} g^{ab} \right) \\ \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \\ (n-1) \left(\frac{1}{2F} \left(-\frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac, b} \right) + \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \frac{g_{\alpha\beta}}{2} \end{array} \right). \quad (12)$$

That being said, however, it should be pointed out that still Hilbert's introduction of the matter Lagrange density into his variational integral or Einstein's incorporation of the mass energy tensor into his field equations makes still sense as it accounts for a potential non-perfect extremal condition [8, 12, 20, 21], a universal cellular structure [14] and a few other things [20].

Towards Quantum Gravity Wave Solutions

Starting With the Classical Linearized Field Equations

Directly applying the results from [1], thereby using the classical linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we obtain the following field equations in vacuum:

$$\begin{aligned}
& R_{\alpha\beta} - \frac{1}{2} R \cdot g_{\alpha\beta} = 0 \\
& \xrightarrow{\text{weak gravity}} \\
& R_{\alpha\beta} - \frac{1}{2} R \cdot g_{\alpha\beta} = \frac{1}{2} \left(\left(\psi^\sigma_{\beta,\sigma} \right)_{,\alpha} + \left(\psi^\sigma_{\alpha,\sigma} \right)_{,\beta} - \left(\psi^{\mu\nu}_{,\mu} \right)_{,\nu} \eta_{\alpha\beta} - \Delta_\eta \psi_{\alpha\beta} \right) \cdot \\
& \xrightarrow{\text{gauging} \rightarrow \psi^{\mu\nu}_{,\mu} = 0} \\
& \eta^{\alpha\beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} \psi_{\mu\nu} = \Delta_{MG} \psi_{\mu\nu} = \Delta_\eta \psi_{\mu\nu} = 0
\end{aligned} \tag{13}$$

Thereby we have:

$$\begin{aligned}
\psi_{\mu\nu} &= h_{\mu\nu} - h \cdot \eta_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \cdot h_{\alpha\beta} \eta^{\alpha\beta} \cdot \eta_{\mu\nu} \\
h &= \frac{1}{2} \cdot h_{\alpha\beta} \eta^{\alpha\beta} = \frac{1}{2} \cdot \eta_{\alpha\beta} h^{\alpha\beta} \Rightarrow h_{\alpha\beta} \eta^{\alpha\beta} = 2h \\
\eta^{\mu\nu} \psi_{\mu\nu} &= \eta^{\mu\nu} h_{\mu\nu} - h \cdot \eta^{\mu\nu} \eta_{\mu\nu} = 2h - h \cdot n = (2-n) \cdot h \\
g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} + \varepsilon[h^2] \cong \eta_{\mu\nu} + h_{\mu\nu} \\
\Rightarrow g_{\mu\nu} &\cong \eta_{\mu\nu} + \psi_{\mu\nu} - \frac{1}{2} \cdot h_{\alpha\beta} \eta^{\alpha\beta} \cdot \eta_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} - h \cdot \eta_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} + \frac{\eta^{\alpha\beta} \psi_{\alpha\beta}}{n-2} \cdot \eta_{\mu\nu}
\end{aligned} \tag{14}$$

with the gauge condition:

$$\psi_{\mu\nu,}{}^\nu = 0, \tag{15}$$

and $\eta_{\mu\nu}$ denoting the metric tensor in Minkowski or Cartesian space-times.

The solution to (13) is known to be:

$$\psi_{\mu\nu} = e_{\mu\nu} \cdot u[k_\sigma x^\sigma]; \quad \eta_{\mu\nu} k^\mu k^\nu = 0; \quad e_{\mu\nu} k^\nu = 0, \tag{16}$$

Where we have the wave vector k_σ and the amplitude matrix of constants $e_{\mu\nu}$.

Now we remember that we can always extend the metric tensor in accordance with (1) and that then (13) can be expanded as follows:

$$\begin{aligned}
& R^*_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} = 0 \\
& \xrightarrow{\text{weak gravity}} , \\
& = \frac{1}{2} \cdot F \left(\left(F \cdot \psi^\sigma_{\beta,\sigma} \right)_{,\alpha} + \left(F \cdot \psi^\sigma_{\alpha,\sigma} \right)_{,\beta} - \left(F \cdot \psi^{\mu\nu}_{,\mu} \right)_{,\nu} \eta_{\alpha\beta} - \Delta_\eta (F \cdot \psi_{\alpha\beta}) \right)
\end{aligned} \tag{17}$$

because instead of (14), with a scaled metric tensor we have:

$$\begin{aligned}
G_{\mu\nu} &= F \cdot g_{\mu\nu} \cong F \cdot \left(\eta_{\mu\nu} + \psi_{\mu\nu} + \frac{1}{2} \cdot h_{\alpha\beta} \eta^{\alpha\beta} \cdot \eta_{\mu\nu} \right) \\
F \cdot \psi_{\mu\nu} &= F \cdot h_{\mu\nu} - F \cdot h \cdot \eta_{\mu\nu} = F \cdot h_{\mu\nu} - \frac{F}{2} \cdot h_{\alpha\beta} \eta^{\alpha\beta} \cdot \eta_{\mu\nu} \\
F \cdot g_{\mu\nu} &= F \cdot \left(\eta_{\mu\nu} + h_{\mu\nu} + \varepsilon[h^2] \right) \cong F \cdot \left(\eta_{\mu\nu} + h_{\mu\nu} \right) = F \cdot \left(\eta_{\mu\nu} + \psi_{\mu\nu} + \frac{\eta^{\alpha\beta} \psi_{\alpha\beta}}{n-2} \cdot \eta_{\mu\nu} \right)
\end{aligned} \tag{18}$$

The Laplace operation in (17) can be expanded using the following pattern:

$$\begin{aligned}
\Delta_G f \cdot \Phi &= \frac{1}{\sqrt{G}} \partial_\alpha \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot \partial_\beta f \cdot \Phi \right) = \frac{1}{\sqrt{G}} \partial_\alpha \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot (\Phi \cdot \partial_\beta f + f \cdot \partial_\beta \Phi) \right) \\
&= \frac{1}{\sqrt{G}} \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot \partial_\alpha (\Phi \cdot \partial_\beta f + f \cdot \partial_\beta \Phi) + (\Phi \cdot \partial_\beta f + f \cdot \partial_\beta \Phi) \partial_\alpha (\sqrt{G} \cdot G^{\alpha\beta}) \right) \\
&= \frac{1}{\sqrt{G}} \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot (\partial_\alpha \Phi \cdot \partial_\beta f + \partial_\alpha f \cdot \partial_\beta \Phi) + (\Phi \cdot \partial_\beta f + f \cdot \partial_\beta \Phi) \partial_\alpha (\sqrt{G} \cdot G^{\alpha\beta}) \right) \quad (19) \\
&= f \cdot \Delta_G \Phi + \Phi \cdot \Delta_G f + \frac{1}{\sqrt{G}} \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot (\partial_\alpha \Phi \cdot \partial_\beta f + \partial_\alpha f \cdot \partial_\beta \Phi) \right) \\
&= f \cdot \Delta_G \Phi + \Phi \cdot \Delta_G f + \frac{2}{\sqrt{G}} \left(\sqrt{G} \cdot G^{\alpha\beta} \cdot \partial_\alpha \Phi \cdot \partial_\beta f \right)
\end{aligned}$$

and results in:

$$\begin{aligned}
&\mathbf{R}^*_{\alpha\beta} - \frac{1}{2} \mathbf{R}^* \cdot \mathbf{G}_{\alpha\beta} = 0 \\
&\xrightarrow{\text{weak gravity}} \\
&\xrightarrow{\text{gauging} \rightarrow (\mathbf{F} \cdot \boldsymbol{\psi}^{\mu\nu})_{,\mu} = 0} \quad . \quad (20)
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2 \cdot \mathbf{F}} \eta^{\alpha\beta} \frac{\partial^2}{\partial \mathbf{x}^\alpha \partial \mathbf{x}^\beta} (\mathbf{F} \cdot \boldsymbol{\psi}_{\mu\nu}) &= \frac{1}{2 \cdot \mathbf{F}} \Delta_\eta (\mathbf{F} \cdot \boldsymbol{\psi}_{\mu\nu}) = 0 \Rightarrow \Delta_\eta (\mathbf{F} \cdot \boldsymbol{\psi}_{\mu\nu}) = 0 \\
\Delta_\eta (\mathbf{F} \cdot \boldsymbol{\psi}_{\mu\nu}) &= \mathbf{F} \cdot \Delta_\eta \boldsymbol{\psi}_{\mu\nu} + \boldsymbol{\psi}_{\mu\nu} \cdot \Delta_\eta \mathbf{F} + 2 \cdot \eta^{\alpha\beta} \cdot \partial_\alpha \boldsymbol{\psi}_{\mu\nu} \cdot \partial_\beta \mathbf{F} = 0. \quad (21)
\end{aligned}$$

Usually, the expansion is also necessary for the operator itself, whereby it generally holds:

$$\begin{aligned}
\Delta_G \Phi &= \frac{1}{\sqrt{\mathbf{F}^n \cdot \mathbf{g}}} \partial_\alpha \left(\sqrt{\mathbf{F}^n \cdot \mathbf{g}} \cdot \mathbf{F}^{-1} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi \right) \\
&= \frac{1}{\mathbf{F}^{\frac{n}{2}} \cdot \sqrt{\mathbf{g}}} \partial_\alpha \left(\sqrt{\mathbf{g}} \cdot \mathbf{F}^{\frac{n-1}{2}} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi \right) \\
&= \frac{1}{\mathbf{F}^{\frac{n}{2}} \cdot \sqrt{\mathbf{g}}} \left(\sqrt{\mathbf{g}} \cdot \mathbf{F}^{\frac{n-2}{2}} \cdot \mathbf{F}_{,\alpha} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi + \mathbf{F}^{\frac{n-1}{2}} \cdot \partial_\alpha \left(\sqrt{\mathbf{g}} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi \right) \right) \\
&= \frac{1}{\mathbf{F}^2} \cdot \mathbf{F}_{,\alpha} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi + \frac{1}{\mathbf{F}^1 \cdot \sqrt{\mathbf{g}}} \cdot \partial_\alpha \left(\sqrt{\mathbf{g}} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi \right) \quad . \quad (22) \\
&= \frac{1}{\mathbf{F}^2} \cdot \mathbf{F}_{,\alpha} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi + \frac{\Delta_{\mathbf{g}} \Phi}{\mathbf{F}^1} = 0 \\
&\Rightarrow \\
\frac{\mathbf{F}_{,\alpha}}{\mathbf{F}} \cdot \mathbf{g}^{\alpha\beta} \cdot \partial_\beta \Phi + \Delta_{\mathbf{g}} \Phi &= 0 \Rightarrow \mathbf{F}_{,\alpha} \cdot \mathbf{g}^{\alpha\beta} \cdot \Phi_{,\beta} + \mathbf{F} \cdot \Delta_{\mathbf{g}} \Phi = 0
\end{aligned}$$

Applying this on (21) would generally yield:

$$\begin{aligned}
0 &= F \cdot \Delta_{\text{MG}} \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \Delta_{\text{MG}} F + 2 \cdot G^{\alpha\beta} \cdot \partial_\alpha \psi_{\mu\nu} \cdot \partial_\beta F \\
&= F \cdot \left(\frac{F_{,\alpha}}{F^2} \cdot g^{\alpha\beta} \cdot \partial_\beta \psi_{\mu\nu} + \frac{\Delta_g \psi_{\mu\nu}}{F} \right) + \psi_{\mu\nu} \cdot \left(\frac{F_{,\alpha}}{F^2} \cdot g^{\alpha\beta} \cdot \partial_\beta F + \frac{\Delta_g F}{F} \right) + 2 \cdot G^{\alpha\beta} \cdot \partial_\alpha \psi_{\mu\nu} \cdot \partial_\beta F \\
&= \frac{F_{,\alpha}}{F} \cdot \eta^{\alpha\beta} \cdot \partial_\beta \psi_{\mu\nu} + \Delta_\eta \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \left(\frac{F_{,\alpha}}{F^2} \cdot \eta^{\alpha\beta} \cdot \partial_\beta F + \frac{\Delta_\eta F}{F} \right) + \frac{2}{F} \cdot \eta^{\alpha\beta} \cdot \partial_\alpha \psi_{\mu\nu} \cdot \partial_\beta F \\
&= \frac{F_{,\alpha}}{F} \cdot \eta^{\alpha\beta} \cdot \psi_{\mu\nu,\beta} + \Delta_\eta \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \left(\frac{F_{,\alpha}}{F^2} \cdot \eta^{\alpha\beta} \cdot F_{,\beta} + \frac{\Delta_\eta F}{F} \right) + \frac{2}{F} \cdot \eta^{\alpha\beta} \cdot \psi_{\mu\nu,\alpha} \cdot F_{,\beta} \quad , \quad (23) \\
&= \Delta_\eta \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \left(\frac{F_{,\alpha}}{F^2} \cdot \eta^{\alpha\beta} \cdot F_{,\beta} + \frac{\Delta_\eta F}{F} \right) + \frac{3}{F} \cdot \eta^{\alpha\beta} \cdot \psi_{\mu\nu,\alpha} \cdot F_{,\beta} \\
&\Rightarrow \\
F \cdot \Delta_\eta \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \left(\frac{F_{,\alpha}}{F} \cdot \eta^{\alpha\beta} \cdot F_{,\beta} + \Delta_\eta F \right) + 3 \cdot \eta^{\alpha\beta} \cdot F_{,\beta} \cdot \psi_{\mu\nu,\alpha} &= 0
\end{aligned}$$

but in the case of the classical approximation we only have the operator as given in the last line of (13). Of course, one could always think about a superposition of plane wave solutions to effectively form more complex geometries, but this will not be covered in here. Hence, our final equation here would be (21), which, when compared with the classical equation (13), offers a few more options and might account for matter without the need to artificially introduce it. However, we find that with the setting (16) and:

$$F = f[k_\sigma x^\sigma]; \quad \eta_{\mu\nu} k^\mu k^\nu = 0, \quad (24)$$

both equations, (21) and (23), would be satisfied. So, the last line of (23) then gives:

$$\begin{aligned}
&\overbrace{F \cdot \Delta_\eta \psi_{\mu\nu}}^{=0} + \psi_{\mu\nu} \cdot \left((F')^2 \frac{k_\alpha}{F} \cdot \eta^{\alpha\beta} \cdot k_\beta + F'' \eta^{\alpha\beta} k_\alpha k_\beta \right) + 3 \cdot \eta^{\alpha\beta} \cdot F' k_\beta \cdot \psi'_{\mu\nu} k_\alpha \\
&= \left(\psi_{\mu\nu} \cdot \left(\frac{(F')^2}{F} + F'' \right) + 3 \cdot F' \cdot \psi'_{\mu\nu} \right) \overbrace{\eta^{\alpha\beta} k_\alpha k_\beta}^{=0} = 0
\end{aligned} \quad (25)$$

and we would still have the classical gravitational wave solution, only that, this time, we also have an additional scaling factor F , which shows the same wave character as the metric tensor. So, we already have reproduced Einstein's gravity waves and the basis of our quantum gravity field equations by applying Einstein's approximation for the linearization. When setting $F=\text{const}=1$ we even obtain the classical original. It should be noted that we now even have the principle option of the metric being a metric of constants and only F depending on the coordinates, then, only considering (21), satisfying the resulting field equation:

$$\begin{aligned}
F \cdot \Delta_\eta \psi_{\mu\nu} + \psi_{\mu\nu} \cdot \Delta_\eta F + 2 \cdot \eta^{\alpha\beta} \cdot \partial_\alpha \psi_{\mu\nu} \cdot \partial_\beta F &= 0 \\
\begin{array}{c} \xrightarrow{\psi_{\mu\nu} = \text{const}_{\mu\nu}} \\ = \text{const}_{\mu\nu} \cdot \Delta_\eta F = 0 \end{array} &, \quad (26)
\end{aligned}$$

which – as before – with (24) gives:

$$\text{const}_{\mu\nu} \cdot \Delta_\eta F = \text{const}_{\mu\nu} \cdot (F'') \overbrace{\eta^{\alpha\beta} k_\alpha k_\beta}^{=0} = 0. \quad (27)$$

We were able to not only extract the classical gravity waves from our quantum gravity field equations (2), but also to achieve extended (“quantum gravity”) versions of the latter. The approach with the scaling factor (1) may appear trivial, even primitive, because – after all - the total metric solution for our case here reads:

$$\begin{aligned}
 G_{\mu\nu} &= f[k_\sigma x^\sigma] \cdot g_{\mu\nu} \cong f[k_\sigma x^\sigma] \cdot \left(\psi_{\mu\nu} + \left(1 + \frac{\eta^{\alpha\beta} \psi_{\alpha\beta}}{n-2} \right) \cdot \eta_{\mu\nu} \right) \\
 &= f[k_\sigma x^\sigma] \cdot \left(e_{\mu\nu} \cdot u[k_\sigma x^\sigma] + \left(1 + \frac{\eta^{\alpha\beta} e_{\alpha\beta} \cdot u[k_\sigma x^\sigma] \cdot f[k_\sigma x^\sigma]}{(n-2) \cdot f[k_\sigma x^\sigma]} \right) \cdot \eta_{\mu\nu} \right), \quad (28) \\
 &= f[k_\sigma x^\sigma] \cdot \left(e_{\mu\nu} \cdot u[k_\sigma x^\sigma] + \left(1 + \frac{\eta^{\alpha\beta} e_{\alpha\beta} \cdot u[k_\sigma x^\sigma]}{(n-2)} \right) \cdot \eta_{\mu\nu} \right) \\
 &\quad \eta_{\mu\nu} k^\mu k^\nu = 0; \quad e_{\mu\nu} k^\nu = 0
 \end{aligned}$$

but even when only considering the wave option, we are off with something richer than the classical, non-scaled case.

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