Quantum Gravity Waves – Part 3: Towards Newton's Gravity

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Abstract

One of the standard text-book derivations of Newton's gravity from Einstein's General Theory of Relativity uses the Einstein-linearization of his field equations. It is the same linearization which also results in solutions of gravitational waves, but it requires a few more approximations in order to come to Newton.

We have recently shown that by directly applying the classical approaches for the simplest linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we not only obtained wave-like field equations in vacuum for the original Einstein equations, but also for the corresponding quantum gravity field equations. These quantum field equations already contain matter or matter-like terms without the need of adding them artificially (as Hilbert and Einstein did) via a matter Lagrange density (Hilbert) or directly (Einstein) via an energy-momentum tensor.

In this paper we demonstrate how our "matter" from the quantum field equations connects with Newton's mass density in the classical gravity theory from Newton.

Introduction

Gravity waves are an outcome of Einstein's great general theory of relativity and they are a standard topic in the corresponding textbook literature (e.g. [1], pp 325). Usually, the derivations are restricted to the linearized Einstein Field Equations and – if even mentioned – the problem of quantization, respectively quantized wave solutions is not discussed.

In this paper, we will present a simple extension of the classical linear gravity waves, showing how this might lead to a Quantum Gravity approach. We will demonstrate that it requires only a small adaptation of the classical Hilbert and Einstein achievements [2, 3] to come up with such a Quantum Gravity Theory or – as it is sometimes also been called - a "Theory of Everything".

In a variety of previous publications [4 – 15], this author has shown how a simple scaling factor to the metric tensor already leads to a quantum gravity field equational outcome. The "problem" with this finding of course is, that Hilbert should get all the credit for already having found a - or THE - "Theory of Everything" over 100 years ago. There wasn't much to be done, unless one counts adding a scaling factor as "much", which this author definitively does not. He sees such a "work" as a mere finding in another's paper, namely, Hilbert's "Die Grundlagen der Physik" from 1915 [1]. However, with tens of thousands of jobs at stake because they all depend on the fact that – apparently - there is no Theory of Everything yet, people obviously have problems or – to use a psychological term in order to give - at least - some of these scientists an excuse for their blindness – face an undeniable cognitive dissonant barrier to recognize that Hilbert has already done almost all the work. Worse still, it was also shown [16] that – in principle – variational kernels in the Einstein-Hilbert action of the type f[R], as they are necessary to produce the endless output of new field equations some of the more creative and paper-productive researcher are permanently proposing, and as they are also necessary for so many other Trans-Planckian approaches, are not of need, because those could always be substituted by a suitable metric scaling factor [16] without changing the total variational (Hilbert!!!) integral, which – after all – is a scalar. With the potential wall of recognition too high to be overcome by most of the string, loop gravity, trans Planckians and whatever else researchers, perhaps the quantum mechanical tunneling will help them to just – one day – miraculously diffuse through this barrier. Until then, the circus will probably produce tens of thousands more of completely useless "scientific" "contributions", while the simple fact that the job was already done over 100 years ago continues to be hushed up.

De facto, one may even say that this author also just "produces papers". Even worse, he always repeats the same things. This, however, is more for convenience in order to give the essentials before adding the new aspect, the author intends to consider. We see no need to hide the redundancy. Those who are already familiar with the basics, can easily skip the introduction and first theory sections.

In this paper, we will apply the rather straight forward and very rich Hilbert approach… just a tiny bit expanded, respectively, generalized and then try to extract Newton's gravity from there. Thereby it should be noted that due to the inconsistencies resulting from the classical approximation process, we have a variety of options. It should also be pointed out that – in principle – we already have a very simple transition from the Quantum Gravity field equations to Newton's gravity (thereby finding Newton's gravity as a pure quantum or fuzziness effect) [15], being based on the fact that any perturbational factor in the Hilbert variational kernel could be absorbed by the metric as a volume scaling [16].

A Quantum Gravity Theory

We start with the following scaled metric tensor and force it into the Einstein-Hilbert action variational problem [2] as follows:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \quad \rightarrow \quad \delta W = 0 = \delta \int_V d^n x \sqrt{-G} \cdot \left(R^* - 2 \cdot \Lambda \right) \tag{1}$$

Here Λ is the cosmological constant, G denotes the determinant of the metric tensor from (1) and R^* gives the corresponding Ricci scalar. Performing the variation with respect to the metric $G_{\alpha\beta}$ results in:

$$0 = \left(\begin{array}{c} \boxed{R_{\alpha\beta} - R\frac{g_{\alpha\beta}}{2} + \Lambda \cdot g_{\alpha\beta}} \\ -\frac{1}{2F}\left(\begin{array}{c} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b}\right) - \\ F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd}\left(\begin{array}{c} g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ +\frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{array} \right) \end{array} \right) \\ +\frac{1}{4F^2}\left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n) \right) \\ +(n-1)\left(\frac{1}{2F}\left(\begin{array}{c} 2\Delta F - 2F_{,d}g^{cd}{}_{,c} \\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) + \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2}(n-6) \right) \cdot \frac{g_{\alpha\beta}}{2} \end{array} \right) \delta G^{\alpha\beta} \tag{2}$$

and shows us that we have not only obtained the classical Einstein Theory of General Relativity [3] (see boxed terms exactly giving the Einstein Field Equations in vacuum plus the cosmological constant term), but also a set of quantum field equations for the scaling function F, clearly playing the role of the wave-function. It was shown in our previous publications [4, 5, 6, 7, 8] that these additional terms are quantum equations, fully covering the main aspects of relativistic classical quantum theory. Everything else can be obtained by a few generalizations, structural shaping and the introduction of the variation with respect to the degrees of freedom or number of dimensions [4, 5, 6, 7, 8]. So, we conclude, that we indeed have a Quantum Gravity Theory or Theory of Everything, as one also calls it, at hand, whereby it should be pointed out that (2) has to be considered the simplest possible – and still general (see [4, 5, 6, 7, 8]) - form for the corresponding quantum gravity field equations.

"Weak Gravity" and Linearity – The Transition to the Classical Quantum Theory

It was shown in [5, 6, 7, 8] that the so-called "weak gravity" condition:

$$\delta G^{\alpha\beta} = G^{\alpha\beta}\cdot\delta_0 + \overbrace{G^{ab}\delta^{\alpha\beta}_{ab}}^{\text{Gravity}} \xrightarrow{\forall\,\delta^{\alpha\beta}_{ab}\ll\delta_0} = \frac{g^{\alpha\beta}}{F}\cdot\delta_0, \tag{3}$$

together with a setting for the scaling function F[f] as follows:

$$F[f] = \begin{cases} C_F\cdot\left(f + C_f\right)^{\frac{4}{n-2}} & n \neq 2 \\ C_F\cdot e^{f\cdot C_f} & n = 2 \end{cases}. \tag{4}$$

leads to a significant simplification and scalarization of the quantum gravity field equations (2), namely:

$$0 = R - \frac{F'}{2F}\left((n-1)\left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}\right) - nf_{,d}g^{cd}g^{ab}g_{ac,b}\right). \tag{5}$$

This equation is completely linear in f, which not only has the characteristics of a quantum function, but – for a change – gives us the opportunity to metrically see what QUANTUM actually means, namely, a volume jitter to the metric of the system in question… at least this is one quantum option, because we have already seen others, like the perturbated kernel (e.g. see [8]).

Interestingly, for metrics without shear elements:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \tag{6}$$

and applying the solution for F[f] from (4) the derivative terms in (5), which is to say:

$$(n-1)\left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}\right) - nf_{,d}g^{cd}g^{ab}g_{ac,b}. \tag{7}$$

converge to the ordinary Laplace operator, namely:

$$\begin{aligned} R^* = 0 \quad &\rightarrow \quad 0 = F\cdot R + F'\cdot(1-n)\cdot\Delta f \\ \Rightarrow \quad 0 = &\begin{cases} \left(f - C_f\right)^{\frac{4}{n-2}}\cdot C_F\left(R + \frac{4}{n-2}\cdot\frac{(1-n)}{\left(f-C_f\right)}\cdot\Delta f\right) & n > 2 \\ e^{C_f\cdot f}\cdot C_F\left(R + C_f\cdot(1-n)\cdot\Delta f\right) & n = 2 \end{cases}. \end{aligned} \tag{8}$$

We recognize the relativistic Klein-Gordon equation.

Thus, in the case of n>2 we always also have the option for a constant (broken symmetry) solution of the kind:

$$0=\mathrm{f}-C_{\mathrm{f}0}\quad\Rightarrow\quad\mathrm{f}=C_{\mathrm{f}0}\,. \tag{9}$$

In all other cases, meaning where $\mathrm{f}\neq C_{\mathrm{f}0}$, we have the simple equations:

$$0=\begin{cases}\left(\mathrm{f}-C_{\mathrm{f}0}\right)\cdot\mathrm{R}+\left(1-\mathrm{n}\right)\cdot\dfrac{4}{\mathrm{n}-2}\cdot\Delta\mathrm{f} & \mathrm{n}>2\\ \mathrm{R}+C_{\mathrm{f}0}\cdot\left(1-\mathrm{n}\right)\cdot\Delta\mathrm{f} & \mathrm{n}=2\end{cases}. \tag{10}$$

A critical argument should now be that this equation is not truly of Klein-Gordon character as it does contain neither potential nor mass, but this author has already shown that this problem is easily solved by adding additional dimensions carrying the right properties to produce masses and potentials due to entanglement, being provided by the right scaling function F[f] (e.g. [4 – 8]).

Using these results we were able to develop a quantum gravity statistics [9, 10], formulate a Heisenberg uncertainty principle containing gravity [12] and even suggesting a path for answering the riddle of the 3 generations of elementary particles [13].

Finding Matter

Observing our variational result (2) and comparing with the classical equations from [3]:

$$\mathrm{R}_{\alpha\beta}-\frac{1}{2}\mathrm{R}\cdot\mathrm{g}_{\alpha\beta}+\Lambda\cdot\mathrm{g}_{\alpha\beta}=-\kappa\cdot\mathrm{T}_{\alpha\beta}, \tag{11}$$

where we have: $R_{\alpha\beta}$, $T_{\alpha\beta}$ the Ricci- and the energy momentum tensor, respectively, while the parameters Λ and κ are constants (usually called cosmological and coupling constant, respectively), we realize that the following terms of (2) are just the most natural energy momentum tensor elements, yielding the following identity:

$$\kappa\cdot\mathrm{T}_{\alpha\beta}=\left(\begin{array}{c}-\dfrac{1}{2\mathrm{F}}\left(\begin{array}{c}\mathrm{F}_{,\alpha\beta}\left(\mathrm{n}-2\right)+\mathrm{F}_{,ab}\mathrm{g}_{\alpha\beta}\mathrm{g}^{ab}+\mathrm{F}_{,a}\mathrm{g}^{ab}\left(\mathrm{g}_{\beta b,\alpha}-\mathrm{g}_{\beta\alpha,b}\right)-\\ \mathrm{F}_{,\alpha}\mathrm{g}^{ab}\mathrm{g}_{\beta b,a}-\mathrm{F}_{,\beta}\mathrm{g}^{ab}\mathrm{g}_{\alpha b,a}+\mathrm{F}_{,d}\mathrm{g}^{cd}\left(\begin{array}{c}\mathrm{g}_{\alpha c,\beta}-\dfrac{1}{2}\mathrm{n}\mathrm{g}_{\alpha c,\beta}-\dfrac{1}{2}\mathrm{n}\mathrm{g}_{\beta c,\alpha}\\ +\dfrac{1}{2}\mathrm{n}\mathrm{g}_{\alpha\beta,c}+\dfrac{1}{2}\mathrm{g}_{\alpha\beta}\mathrm{g}_{ab,c}\mathrm{g}^{ab}\end{array}\right)\end{array}\right)\\ +\dfrac{1}{4\mathrm{F}^{2}}\left(\mathrm{F}_{,\alpha}\cdot\mathrm{F}_{,\beta}\left(3\mathrm{n}-6\right)+\mathrm{g}_{\alpha\beta}\mathrm{F}_{,c}\mathrm{F}_{,d}\mathrm{g}^{cd}\left(4-\mathrm{n}\right)\right)\\ +\left(\mathrm{n}-1\right)\left(\dfrac{1}{2\mathrm{F}}\left(\begin{array}{c}2\Delta\mathrm{F}-2\mathrm{F}_{,d}{\mathrm{g}^{cd}}_{,c}\\ -\dfrac{\mathrm{n}}{\left(\mathrm{n}-1\right)}\mathrm{F}_{,d}\mathrm{g}^{cd}\mathrm{g}^{ab}\mathrm{g}_{ac,b}\end{array}\right)+\dfrac{\mathrm{g}^{ab}\mathrm{F}_{,a}\cdot\mathrm{F}_{,b}}{4\mathrm{F}^{2}}\left(\mathrm{n}-6\right)\right)\cdot\dfrac{\mathrm{g}_{\alpha\beta}}{2}\end{array}\right). \tag{12}$$

Einstein's Linearization and Newton's Gravity Potential

Directly applying the results from [1], thereby using the classical linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we obtain the following field equations in vacuum:

$$R_{\alpha\beta}-\frac{1}{2}R\cdot g_{\alpha\beta}=0\quad\xrightarrow{\text{weak gravity}}\quad\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha\partial x^\beta}\Psi_{\mu\nu}=\Delta_{MG}\Psi_{\mu\nu}=\Delta_\eta\Psi_{\mu\nu}=0\,. \tag{13}$$

Thereby we have:

$$\begin{gathered}
\Psi_{\mu\nu}=h_{\mu\nu}-h\cdot\eta_{\mu\nu}=h_{\mu\nu}-\frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu}\\
h=\frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}=\frac{1}{2}\cdot\eta_{\alpha\beta}h^{\alpha\beta}\quad\Rightarrow\quad h_{\alpha\beta}\eta^{\alpha\beta}=2h\\
\eta^{\mu\nu}\Psi_{\mu\nu}=\eta^{\mu\nu}h_{\mu\nu}-h\cdot\eta^{\mu\nu}\eta_{\mu\nu}=2h-h\cdot n=(2-n)\cdot h\\
g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}+\varepsilon\left[h^2\right]\cong\eta_{\mu\nu}+h_{\mu\nu}\\
\Rightarrow g_{\mu\nu}\cong\eta_{\mu\nu}+\Psi_{\mu\nu}-\frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu}=\eta_{\mu\nu}+\Psi_{\mu\nu}-h\cdot\eta_{\mu\nu}=\eta_{\mu\nu}+\Psi_{\mu\nu}+\frac{\eta^{\alpha\beta}\Psi_{\alpha\beta}}{n-2}\cdot\eta_{\mu\nu}
\end{gathered}\;, \tag{14}$$

with the gauge condition:

$$\Psi_{\mu\nu,}{}^{\nu}=0\,, \tag{15}$$

and $\eta_{\mu\nu}$ denoting the metric tensor in Minkowski or Cartesian space-times.

The wave solution to (13) is given in [1, 13], but as it is not of interest here, we skip this part and move directly on to the corresponding quantized linearized field equations, thereby interpreting all quantum aspects (F-terms) as potential matter. Using (12), combining with (11) in the linearized version of (13) this gives us:

$$R_{\alpha\beta}-\frac{1}{2}R\cdot g_{\alpha\beta}=-\kappa\cdot T_{\alpha\beta}\quad\xrightarrow{\text{weak gravity}}\quad\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha\partial x^\beta}\Psi_{\mu\nu}=\Delta_\eta\Psi_{\mu\nu}=-\kappa\cdot T_{\alpha\beta}$$

$$\Rightarrow$$

$$\Delta_\eta\Psi_{\mu\nu}=-\left(\begin{array}{c}
-\dfrac{1}{2F}\left(\begin{array}{c}
F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)-\\
F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}
g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\
+\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}
\end{array}\right)
\end{array}\right)\\
+\dfrac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\\
+(n-1)\left(\dfrac{1}{2F}\left(\begin{array}{c}
2\Delta F-2F_{,d}g^{cd}{}_{,c}\\
-\dfrac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}
\end{array}\right)+\dfrac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6)\right)\cdot\dfrac{g_{\alpha\beta}}{2}
\end{array}\right). \tag{16}$$

The corresponding energy momentum tensor from above simplifies dramatically in the current set of approximations:

$$
\kappa\cdot T_{\alpha\beta}=\left(\begin{array}{c}
-\frac{1}{2F}\left(\begin{array}{c} F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)- \\ F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c} g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\ +\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\end{array}\right)\end{array}\right)\\
+\frac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\\
+(n-1)\left(\frac{1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6)\right)\cdot\frac{g_{\alpha\beta}}{2}
\end{array}\right)
$$

$$
\simeq\frac{1}{2F}\left((n-1)\Delta_\eta F\cdot\psi_{\alpha\beta}-F_{,\alpha\beta}(n-2)\right)+\frac{1}{4F^2}F_{,\alpha}\cdot F_{,\beta}(3n-6)
$$

$$
=\frac{1}{2F}\left((n-1)\Delta_\eta F\cdot\psi_{\alpha\beta}-F_{,\alpha\beta}(n-2)+\frac{3n-6}{2F}F_{,\alpha}\cdot F_{,\beta}\right)
$$

$$
\xrightarrow{F=F[f]=C_F\left(f+C_f\right)^{-2}}
$$

$$
=\frac{1}{2F}\left((n-1)F'\Delta_\eta f\cdot\psi_{\alpha\beta}-F'f_{,\alpha\beta}(n-2)+(n-2)\left(\frac{3}{2F}(F')^2-F''\right)f_{,\alpha}\cdot f_{,\beta}\right)
$$

$$
=-\frac{1}{f+C_f}\left((n-1)\Delta_\eta f\cdot\psi_{\alpha\beta}-f_{,\alpha\beta}(n-2)\right) \quad . \qquad (17)
$$

$$
\xrightarrow{\Psi=f+C_f}
$$

$$
=-\frac{1}{\Psi}\left((n-1)\Delta_\eta\Psi\cdot\psi_{\alpha\beta}-\Psi_{,\alpha\beta}(n-2)\right)
$$

So, the final result from (16) should read:

$$
\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha\partial x^\beta}\psi_{\mu\nu}=\Delta_\eta\psi_{\mu\nu}=\frac{1}{\Psi}\left((n-1)\Delta_\eta\Psi\cdot\psi_{\mu\nu}-\Psi_{,\mu\nu}(n-2)\right). \qquad (18)
$$

However, it should be pointed out that the linearization leads to (see corresponding discussing in the textbook literature, e.g. [1]) some ambiguous consequences and that by placing the volume or F-scaling at different positions within the linearization, we can obtain different results. So, for instance, from [13, 14] we could also attract the following equations under certain conditions (c.f. [14]):

$$
F[f]=C_F\cdot\sqrt{f+C_f}
$$

$$
\Rightarrow
$$

$$
\Delta_g h=\frac{F'}{F}\cdot\left(f_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g f\right)=\frac{1}{2\left(f+C_f\right)}\cdot\left(f_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g f\right), \qquad (19)
$$

$$
\xrightarrow{\Psi=f+C_f}
$$

$$
\Delta_g h=\frac{1}{2\Psi}\cdot\left(\Psi_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g\Psi\right)
$$

$$\Rightarrow$$

$$\frac{1}{f+C_f}\psi_{\mu\nu}\cdot\Delta_\eta f-4\cdot\Delta_\eta\psi_{\mu\nu}=0 \qquad (20)$$

$$\xrightarrow{\Psi=f+C_f}$$

$$4\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_\eta\Psi=0$$

$$F[f]=C_F\cdot\left(f+C_f\right)^2$$

$$\Rightarrow$$

$$F\cdot\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot F'\Delta_\eta f$$

$$=\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot\frac{\Delta_\eta f}{f+C_f}=0 \qquad (21)$$

$$\xrightarrow{\Psi=f+C_f}$$

$$\Psi\cdot\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot\Delta_\eta\Psi=0$$

We will not discuss these other options here, but only concentrate on (18) and investigate the transition to Newton's gravity. From [1] we extract the rather extreme necessity for further simplifications in order to finally end up with the Newton gravity potential given as:

$$4\cdot\pi\cdot G\cdot\rho=\Delta_{G-\text{spatial}}\Phi_{\text{Newton}} \qquad (22)$$

with G here denoting the Newton constant and ρ giving the mass density.

Taking (18) and comparing with (22), we see that we have to assume one component of our effective matter term to be dominant, namely the 00-component, leading us to:

$$\begin{gathered}\Delta_{\text{spatial}}\psi_{\mu\nu}=\frac{1}{\Psi}\left((n-1)\Psi_{,00}\cdot\psi_{\mu\nu}-\Psi_{,00}(n-2)\delta_\mu^0\delta_\nu^0\right)\\=\frac{\Psi_{,00}}{\Psi}\left((n-1)\cdot\psi_{\mu\nu}-(n-2)\delta_\mu^0\delta_\nu^0\right)\\\Rightarrow\\\left\{\begin{matrix}\Delta_{\text{spatial}}\psi_{00}=\frac{\Psi_{,00}}{\Psi}(2-n)\\\Delta_{\text{spatial}}\psi_{\mu\nu}=\frac{\Psi_{,00}}{\Psi}(n-1)\cdot\psi_{\mu\nu};\quad\mu,\nu=1,2,3,\ldots\end{matrix}\right\}\end{gathered}. \qquad (23)$$

We also already incorporated the classical simplification with (c.f. [1]):

$$\psi_{0\nu}=0\quad\text{for}\quad\nu\neq0. \qquad (24)$$

While the first equation (second last line in (23)) mirrors the Newton potential via:

$$\Delta_{G-\text{spatial}}\Phi_{\text{Newton}}=\Delta_{\text{spatial}}\psi_{00}=\frac{\Psi_{,00}}{\Psi}(2-n)=4\cdot\pi\cdot G\cdot\rho, \qquad (25)$$

we find no direct equivalent for the eigenvalue equation in the last line, reading:

$$\Delta_{\text{spatial}}\psi_{\mu\nu}=\frac{\Psi_{,00}}{\Psi}(n-1)\Psi_{,00}\cdot\psi_{\mu\nu};\quad\mu,\nu=1,2,3,\ldots, \qquad (26)$$

in the Newton theory. Hence, just as it is usually been done, we set all non-zero components to zero.

The interesting fact is now that, what classically is the gravity-producing matter, respectively the mass density in our approximation appears as something dynamic, because – apparently - it requires a second order derivation of the "Führungsfeld" Ψ with respect to the time-coordinate in order to obtain mass. Establishing the connection is straight forward. We simply use the second half of (25) and integrate:

$$\frac{\Psi_{,00}}{\Psi}(2-n)=4\cdot\pi\cdot G\cdot\rho \quad\Rightarrow\quad \Psi=C_{\Psi}\cdot e^{\pm i\cdot t\cdot 2\cdot\sqrt{\frac{\pi\cdot G\cdot\rho}{n-2}}}. \tag{27}$$

We recognize the similarity to the Dirac particle at rest [17] with:

$$f[t]=e^{\pm i\cdot\frac{m_R\cdot c^2}{\hbar}\cdot t}\cdot C_f \tag{28}$$

and – temporarily ignoring the fact that too many approximations are in our current derivation to consider any result of this paper as fundamental - might be tempted to conclude that by setting Ψ=f:

$$C_{\Psi}\cdot e^{\pm i\cdot t\cdot 2\cdot\sqrt{\frac{\pi\cdot G\cdot\rho}{n-2}}}=e^{\pm i\cdot\frac{m_R\cdot c^2}{\hbar}\cdot t}\cdot C_f \xrightarrow{C_\Psi=C_f} \frac{m_R\cdot c^2}{\hbar}=2\cdot\sqrt{\frac{\pi\cdot G\cdot\rho}{n-2}} \tag{29}$$

we have found a "rule" for the connection of Planck's constant, speed of light and Newton's constant. Meaning that with something like:

$$\begin{aligned}(n-2)^2\cdot\frac{m_R^2\cdot c^4}{4\cdot\pi\cdot G\cdot\rho\cdot\hbar^2}&=(n-2)^2\cdot V\cdot\frac{m_R^2\cdot c^4}{4\cdot\pi\cdot G\cdot m_R\cdot\hbar^2}=(n-2)^2\cdot V\cdot\frac{m_R\cdot c^4}{4\cdot\pi\cdot G\cdot\hbar^2}\\ &\xrightarrow{r_S=\frac{2m_R G}{c^2}}\\ &=(n-2)^2\cdot V\cdot\frac{r_S\cdot c^6}{8\cdot\pi\cdot G^2\cdot\hbar^2}=(n-2)^2\cdot\frac{V\cdot r_S}{8\cdot\pi\cdot\ell_P^4}=1\end{aligned}, \tag{30}$$

where the symbol ℓ_P stands for the Planck length, we found something which puts these constants in some relation to each other, thereby compromising their current independence. However, as already hinted above, with so many approximations in place, we better refrain from drawing such conclusions and rather consider the result (30) as an artefact of the recipe applied in here (and the corresponding classical text-book literature).

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