Quantum Gravity Waves – Part 2: A By-Product and the 4 "Big" Dimensions

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Abstract

We have recently shown that by directly applying the classical approaches for the simplest linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we not only obtained wave-like field equations in vacuum for the original Einstein equations, but also for the corresponding quantum gravity field equations.

As a by-product we also found a path to explain the special appearance of the 4 big dimensions.

Introduction

Gravity waves are an outcome of Einstein's great general theory of relativity and they are a standard topic in the corresponding textbook literature (e.g. [1], pp 325). Usually, the derivations are restricted to the linearized Einstein Field Equations and – if even mentioned – the problem of quantization, respectively quantized wave solutions is not discussed.

In this paper, we will present a simple extension of the classical linear equations, showing how this might lead to a Quantum Gravity approach. Yes, we know that – in principle – we already have a Quantum Gravity theory, but the linearized version, achieved with Einstein's approximation for the derivation of his famous quadrupole equation (e.g. [1], p. 340, which we here give because Einstein made a trivial mistake in his original work), leads to some interesting connections with important physical aspects like the question of the metric origin of the Klein-Gordon equation and the dominance of our global 4 dimensions. These two aspects will be considered here. Thereby, we will demonstrate that it requires only a small adaptation of the classical Hilbert and Einstein achievements [2, 3] to come up with internally consistent and non-approximated Quantum Gravity Theory or – as it is sometimes also been called - a "Theory of Everything".

In a variety of previous publications [4 – 13], this author has shown how a simple scaling factor to the metric tensor already leads to a quantum gravity field equational outcome. The "problem" with this finding of course is, that Hilbert should get all the credit for already having found a - or THE - "Theory of Everything" over 100 years ago. There wasn't much to be done, unless one counts adding a scaling factor as "much", which this author definitively does not. He sees such a "work" as a mere finding in another's paper, namely, Hilbert's "Die Grundlagen der Physik" from 1915 [1]. However, with tens of thousands of jobs at stake because they all depend on the fact that – apparently - there is no Theory of Everything yet, people obviously have problems or – to use a psychological term in order to give - at least - some of these scientists an excuse for their blindness – face an undeniable cognitive dissonant barrier to recognize that Hilbert has already done almost all the work. Worse still, it was also shown [12] that – in principle – variational kernels in the Einstein-Hilbert action of the type f[R], as they are necessary to produce the endless output of new field equations, some of the more creative and paper-productive researchers are permanently proposing, and as they are also necessary for so many other Trans-Planckian approaches (e.g. [13, 14]), are not of need, because those could always be substituted by a suitable metric scaling factor [12] without changing the total variational (Hilbert!!!) integral, which – after all – is a scalar. With the potential wall of recognition too high to be overcome by most of the string, loop gravity, trans Planckians and whatever else researchers, perhaps the quantum mechanical tunneling will help them to just – one day – miraculously diffuse through this barrier. Until then, the circus will probably produce tens of thousands more of completely useless “scientific” “contributions”, while the simple fact that the job was already done over 100 years ago continues to be hushed up.

De facto, one may even say that this author also just “produces papers”. Even worse, he always repeats the same things. This, however, is more for convenience in order to give the essentials before adding the new aspect, the author intends to consider. We see no need to hide the redundancy. Those who are already familiar with the basics, they can easily skip the introduction and first theory sections.

In here, we will apply the rather straight forward and very rich Hilbert approach… just a tiny bit expanded, respectively, generalized.

A Quantum Gravity Theory

We start with the following scaled metric tensor and force it into the Einstein-Hilbert action variational problem [2] as follows:

$$G_{\alpha\beta}=g_{\alpha\beta}\cdot F[f]\quad\rightarrow\quad\delta W=0=\delta\int\limits_V d^n x\sqrt{-G}\cdot\left(R^{*}-2\cdot\Lambda\right)\tag{1}$$

Here Λ is the cosmological constant, G denotes the determinant of the metric tensor from (1) and R^{*} gives the corresponding Ricci scalar. Performing the variation with respect to the metric $G_{\alpha\beta}$ results in:

$$0=\left(\begin{array}{c}\boxed{R_{\alpha\beta}-R\frac{g_{\alpha\beta}}{2}+\Lambda\cdot g_{\alpha\beta}}\\ -\frac{1}{2F}\left(\begin{array}{c}F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)-\\ F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\ +\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\end{array}\right)\end{array}\right)\\ +\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\\ +(n-1)\left(\frac{1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}(n-6)\right)\cdot\frac{g_{\alpha\beta}}{2}\end{array}\right)\delta G^{\alpha\beta}\tag{2}$$

and shows us that we have not only obtained the classical Einstein Theory of General Relativity [3] (see boxed terms exactly giving the Einstein Field Equations in vacuum plus the cosmological constant term), but also a set of quantum field equations for the scaling function F, clearly playing the role of the wave-function. It was shown in our previous publications [4, 5, 6, 7, 8] that these additional terms are quantum equations, fully covering the main aspects of relativistic classical quantum theory. Everything else can be obtained by a few generalizations, structural shaping and the introduction of the variation with respect to the degrees of freedom or number of dimensions [4, 5, 6, 7, 8]. So, we conclude, that we indeed have a Quantum Gravity Theory or Theory of Everything, as one also calls it, at hand, whereby it should be pointed out that (2) has to be considered the simplest possible – and still general (see [4, 5, 6, 7, 8]) - form for the corresponding quantum gravity field equations.

"Weak Gravity" and Linearity – The Transition to the Classical Quantum Theory

It was shown in [5, 6, 7, 8] that the so-called "weak gravity" condition:

$$\delta G^{\alpha\beta} = G^{\alpha\beta}\cdot\delta_0 + \overbrace{G^{ab}\delta^{\alpha\beta}_{ab}}^{\text{Gravity}} \xrightarrow{\forall\,\delta^{\alpha\beta}_{ab}\ll\delta_0} = \frac{g^{\alpha\beta}}{F}\cdot\delta_0\,, \tag{3}$$

together with a setting for the scaling function F[f] as follows:

$$F[f] = \begin{cases} C_F\cdot\left(f + C_f\right)^{\frac{4}{n-2}} & n\neq 2 \\ C_F\cdot e^{f\cdot C_f} & n = 2 \end{cases}. \tag{4}$$

leads to a significant simplification and scalarization of the quantum gravity field equations (2), namely:

$$0 = R - \frac{F'}{2F}\left((n-1)\left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}\right) - nf_{,d}g^{cd}g^{ab}g_{ac,b}\right). \tag{5}$$

This equation is completely linear in f, which not only has the characteristics of a quantum function, but – for a change – gives us the opportunity to metrically see what QUANTUM actually means, namely, a volume jitter to the metric of the system in question… at least this is one quantum option, because we have already seen others, like the perturbated kernel (e.g. see [8]).

Interestingly, for metrics without shear elements:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix};\quad g_{ii,i} = 0\,, \tag{6}$$

and applying the solution for F[f] from (4) the derivative terms in (5), which is to say:

$$(n-1)\left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}\right) - nf_{,d}g^{cd}g^{ab}g_{ac,b}\,. \tag{7}$$

converge to the ordinary Laplace operator, namely:

$$\begin{aligned} & R^{*} = 0 \quad\rightarrow\quad 0 = F\cdot R + F'\cdot(1-n)\cdot\Delta f \\ \Rightarrow\quad & 0 = \begin{cases} \left(f - C_f\right)^{\frac{4}{n-2}}\cdot C_F\left(R + \dfrac{4}{n-2}\cdot\dfrac{(1-n)}{\left(f - C_f\right)}\cdot\Delta f\right) & n > 2 \\ e^{C_f\cdot f}\cdot C_F\left(R + C_f\cdot(1-n)\cdot\Delta f\right) & n = 2 \end{cases}. \end{aligned} \tag{8}$$

We recognize the relativistic Klein-Gordon equation.

Thus, in the case of n>2 we always also have the option for a constant (broken symmetry) solution of the kind:

$$0 = f - C_{f0} \quad\Rightarrow\quad f = C_{f0}\,. \tag{9}$$

In all other cases, meaning where $f \neq C_{f0}$, we have the simple equations:

$$0=\begin{cases}\left(\mathrm{f}-C_{\mathrm{f}0}\right)\cdot\mathrm{R}+\left(1-\mathrm{n}\right)\cdot\frac{4}{\mathrm{n}-2}\cdot\Delta\mathrm{f} & \mathrm{n}>2\\ \mathrm{R}+C_{\mathrm{f}0}\cdot\left(1-\mathrm{n}\right)\cdot\Delta\mathrm{f} & \mathrm{n}=2\end{cases}. \tag{10}$$

A critical argument should now be that this equation is not truly of Klein-Gordon character as it does contain neither potential nor mass, but this author has already shown that this problem is easily solved by adding additional dimensions carrying the right properties to produce masses and potentials due to entanglement, being provided by the right scaling function F[f] (e.g. [4 – 8]).

Using these results we were able to develop a quantum gravity statistics [9, 10], formulate a Heisenberg uncertainty principle containing gravity [12] and even suggesting a path for answering the riddle of the 3 generations of elementary particles [13].

Finding Matter

Observing our variational result (2) and comparing with the classical equations from [3]:

$$\mathrm{R}_{\alpha\beta}-\frac{1}{2}\mathrm{R}\cdot\mathrm{g}_{\alpha\beta}+\Lambda\cdot\mathrm{g}_{\alpha\beta}=-\kappa\cdot\mathrm{T}_{\alpha\beta}, \tag{11}$$

where we have: $R_{\alpha\beta}$, $T_{\alpha\beta}$ the Ricci- and the energy momentum tensor, respectively, while the parameters Λ and κ are constants (usually called cosmological and coupling constant, respectively), we realize that the following terms of (2) are just the most natural energy momentum tensor elements, yielding the following identity:

$$\kappa\cdot\mathrm{T}_{\alpha\beta}=\left(\begin{array}{c}-\frac{1}{2\mathrm{F}}\left(\begin{array}{c}\mathrm{F}_{,\alpha\beta}\left(\mathrm{n}-2\right)+\mathrm{F}_{,ab}\mathrm{g}_{\alpha\beta}\mathrm{g}^{ab}+\mathrm{F}_{,a}\mathrm{g}^{ab}\left(\mathrm{g}_{\beta b,\alpha}-\mathrm{g}_{\beta\alpha,b}\right)-\\ \mathrm{F}_{,\alpha}\mathrm{g}^{ab}\mathrm{g}_{\beta b,a}-\mathrm{F}_{,\beta}\mathrm{g}^{ab}\mathrm{g}_{\alpha b,a}+\mathrm{F}_{,d}\mathrm{g}^{cd}\left(\begin{array}{c}\mathrm{g}_{\alpha c,\beta}-\frac{1}{2}\mathrm{n}\mathrm{g}_{\alpha c,\beta}-\frac{1}{2}\mathrm{n}\mathrm{g}_{\beta c,\alpha}\\ +\frac{1}{2}\mathrm{n}\mathrm{g}_{\alpha\beta,c}+\frac{1}{2}\mathrm{g}_{\alpha\beta}\mathrm{g}_{ab,c}\mathrm{g}^{ab}\end{array}\right)\end{array}\right)\\ +\frac{1}{4\mathrm{F}^2}\left(\mathrm{F}_{,\alpha}\cdot\mathrm{F}_{,\beta}\left(3\mathrm{n}-6\right)+\mathrm{g}_{\alpha\beta}\mathrm{F}_{,c}\mathrm{F}_{,d}\mathrm{g}^{cd}\left(4-\mathrm{n}\right)\right)\\ +\left(\mathrm{n}-1\right)\left(\frac{1}{2\mathrm{F}}\left(\begin{array}{c}2\Delta\mathrm{F}-2\mathrm{F}_{,d}\mathrm{g}^{cd}{}_{,c}\\ -\frac{\mathrm{n}}{\left(\mathrm{n}-1\right)}\mathrm{F}_{,d}\mathrm{g}^{cd}\mathrm{g}^{ab}\mathrm{g}_{ac,b}\end{array}\right)+\frac{\mathrm{g}^{ab}\mathrm{F}_{,a}\cdot\mathrm{F}_{,b}}{4\mathrm{F}^2}\left(\mathrm{n}-6\right)\right)\cdot\frac{\mathrm{g}_{\alpha\beta}}{2}\end{array}\right). \tag{12}$$

Einstein’s Linearization

Directly applying the results from [1], thereby using the classical linearization of the Einstein Field Equations under the assumption of weak gravitational fields and small velocities, we obtain the following field equations in vacuum:

$$\mathrm{R}_{\alpha\beta}-\frac{1}{2}\mathrm{R}\cdot\mathrm{g}_{\alpha\beta}=0\xrightarrow{\text{weak gravity}}\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha\partial x^\beta}\Psi_{\mu\nu}=\Delta_{\mathrm{MG}}\Psi_{\mu\nu}=\Delta_\eta\Psi_{\mu\nu}=0. \tag{13}$$

Thereby we have:

$$
\begin{aligned}
\psi_{\mu\nu} &= h_{\mu\nu} - h\cdot\eta_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu} \\
h &= \frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta} = \frac{1}{2}\cdot\eta_{\alpha\beta}h^{\alpha\beta} \quad \Rightarrow \quad h_{\alpha\beta}\eta^{\alpha\beta} = 2h \\
\eta^{\mu\nu}\psi_{\mu\nu} &= \eta^{\mu\nu}h_{\mu\nu} - h\cdot\eta^{\mu\nu}\eta_{\mu\nu} = 2h - h\cdot n = (2-n)\cdot h \\
g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu} + \varepsilon\left[h^2\right] \cong \eta_{\mu\nu} + h_{\mu\nu} \\
\Rightarrow g_{\mu\nu} &\cong \eta_{\mu\nu} + \psi_{\mu\nu} - \frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} - h\cdot\eta_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} + \frac{\eta^{\alpha\beta}\psi_{\alpha\beta}}{n-2}\cdot\eta_{\mu\nu}
\end{aligned}
\quad , \quad (14)
$$

with the gauge condition:

$$
\psi_{\mu\nu,}{}^{\nu} = 0, \qquad (15)
$$

and $\eta_{\mu\nu}$ denoting the metric tensor in Minkowski or Cartesian space-times.

The wave solution to (13) is given in [1, 13], but as it is not of interest here, we skip this part and move directly on to the corresponding quantized linearized field equations.

We remember that we can always extend the metric tensor in accordance with (1) and that then (13) can be expanded as follows:

$$
R^{*}{}_{\alpha\beta} - \frac{1}{2}R^{*}\cdot G_{\alpha\beta} = 0 \xrightarrow{\text{weak gravity}} F\cdot\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha \partial x^\beta}F\cdot\psi_{\mu\nu} = \Delta_{MG}F\cdot\psi_{\mu\nu} = 0, \qquad (16)
$$

because instead of (14), with a scaled metric tensor we have:

$$
\begin{aligned}
G_{\mu\nu} &= F\cdot g_{\mu\nu} \cong F\cdot\left(\eta_{\mu\nu} + \psi_{\mu\nu} + \frac{1}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu}\right) \\
F\cdot\psi_{\mu\nu} &= F\cdot h_{\mu\nu} - F\cdot h\cdot\eta_{\mu\nu} = F\cdot h_{\mu\nu} - \frac{F}{2}\cdot h_{\alpha\beta}\eta^{\alpha\beta}\cdot\eta_{\mu\nu} \\
F\cdot g_{\mu\nu} &= F\cdot\left(\eta_{\mu\nu} + h_{\mu\nu} + \varepsilon\left[h^2\right]\right) \cong F\cdot\left(\eta_{\mu\nu} + h_{\mu\nu}\right) = F\cdot\left(\eta_{\mu\nu} + \psi_{\mu\nu} + \frac{\eta^{\alpha\beta}\psi_{\alpha\beta}}{n-2}\cdot\eta_{\mu\nu}\right)
\end{aligned}
\quad . \quad (17)
$$

The Laplace operation in (16) can be expanded using the following pattern:

$$
\begin{aligned}
\Delta_G f\cdot\Phi &= \frac{1}{\sqrt{G}}\partial_\alpha\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\partial_\beta f\cdot\Phi\right) = \frac{1}{\sqrt{G}}\partial_\alpha\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\left(\Phi\cdot\partial_\beta f + f\cdot\partial_\beta\Phi\right)\right) \\
&= \frac{1}{\sqrt{G}}\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\partial_\alpha\left(\Phi\cdot\partial_\beta f + f\cdot\partial_\beta\Phi\right) + \left(\Phi\cdot\partial_\beta f + f\cdot\partial_\beta\Phi\right)\partial_\alpha\left(\sqrt{G}\cdot G^{\alpha\beta}\right)\right) \\
&= \frac{1}{\sqrt{G}}\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\left(\partial_\alpha\Phi\cdot\partial_\beta f + \partial_\alpha f\cdot\partial_\beta\Phi\right) + \left(\Phi\cdot\partial_\beta f + f\cdot\partial_\beta\Phi\right)\partial_\alpha\left(\sqrt{G}\cdot G^{\alpha\beta}\right)\right) \\
&= f\cdot\Delta_G\Phi + \Phi\cdot\Delta_G f + \frac{1}{\sqrt{G}}\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\left(\partial_\alpha\Phi\cdot\partial_\beta f + \partial_\alpha f\cdot\partial_\beta\Phi\right)\right) \\
&= f\cdot\Delta_G\Phi + \Phi\cdot\Delta_G f + \frac{2}{\sqrt{G}}\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\partial_\alpha\Phi\cdot\partial_\beta f\right)
\end{aligned}
\qquad (18)
$$

and is resulting in:

$$\begin{gathered}
R^*_{\alpha\beta}-\frac{1}{2}R^*\cdot G_{\alpha\beta}=0 \\
\xrightarrow{\text{weak gravity}} \\
F\cdot\eta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha\partial x^\beta}F\cdot\psi_{\mu\nu}=\Delta_{MG}F\cdot\psi_{\mu\nu} \\
=F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+\frac{2}{\sqrt{G}}\left(\sqrt{G}\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F\right)=0
\end{gathered} \quad . \tag{19}$$

The expansion is also necessary for the operator itself, whereby it generally holds:

$$\begin{gathered}
\Delta_G\Phi=\frac{1}{\sqrt{F^n\cdot g}}\partial_\alpha\left(\sqrt{F^n\cdot g}\cdot F^{-1}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi\right) \\
=\frac{1}{F^{\frac{n}{2}}\cdot\sqrt{g}}\partial_\alpha\left(\sqrt{g}\cdot F^{\frac{n}{2}-1}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi\right) \\
=\frac{1}{F^{\frac{n}{2}}\cdot\sqrt{g}}\left(\sqrt{g}\cdot F^{\frac{n}{2}-2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi+F^{\frac{n}{2}-1}\cdot\partial_\alpha\left(\sqrt{g}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi\right)\right) \\
=\frac{1}{F^2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi+\frac{1}{F^1\cdot\sqrt{g}}\cdot\partial_\alpha\left(\sqrt{g}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi\right) \\
=\frac{1}{F^2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi+\frac{\Delta_g\Phi}{F^1}=0 \\
\Rightarrow \\
\frac{F_{,\alpha}}{F}\cdot g^{\alpha\beta}\cdot\partial_\beta\Phi+\Delta_g\Phi=0 \quad\Rightarrow\quad F_{,\alpha}\cdot g^{\alpha\beta}\cdot\Phi_{,\beta}+F\cdot\Delta_g\Phi=0
\end{gathered} \quad . \tag{20}$$

Applying this on (19) yields:

$$\begin{gathered}
0=F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F \\
=F\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot g^{\alpha\beta}\cdot\partial_\beta\psi_{\mu\nu}+\frac{\Delta_g\psi_{\mu\nu}}{F}\right)+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot g^{\alpha\beta}\cdot\partial_\beta F+\frac{\Delta_g F}{F}\right)+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F \\
=\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot\partial_\beta\psi_{\mu\nu}+\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot\partial_\beta F+\frac{\Delta_\eta F}{F}\right)+\frac{2}{F}\cdot\eta^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F \\
=\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\beta}+\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\frac{\Delta_\eta F}{F}\right)+\frac{2}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta} \\
=\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\frac{\Delta_\eta F}{F}\right)+\frac{3}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta} \\
\Rightarrow \\
F\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)+3\cdot\eta^{\alpha\beta}\cdot F_{,\beta}\cdot\psi_{\mu\nu,\alpha}=0
\end{gathered} \quad . \tag{21}$$

which, when compared with the classical equation (13), offers a few more options and might account for matter without the need to artificially introduce it via the Hilbert matter density or the energy momentum tensor. In [13] we have shown that for F=const, we can find classical and for $F=f[k_\sigma x^\sigma]$ quantum gravity wave solutions.

With respect to the operator expansion (21) it needs to be pointed out that the original Einstein linearization starts with local geodesic coordinates on a Cartesian map. So, in principle the Laplace operator in (13) should always be considered Cartesian and no expansion of the type (21) would be of need. However, as we can apply superposition not only to the solutions of the equation (13), but also – due to its linear character –to the equation itself, we can effectively construct many geometrical forms out of the simple cartesian one. Hence, our derivation (21).

An Alternative Path to the "Wave" Solutions / An Important By-Product

Expanding the Operator

So, we already have reproduced the classical equation and found additional quantum gravitational solutions to the Einstein-linearized field equations. Nevertheless, we here also want to watch out for other solutions. Thereby, not being all too happy with the non-linear term in F in our linearized quantum gravity field equations in the last line of (21), which is to say: $\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}$, we can get rid of this one via the following setting for F:

$$F[f]=C_F\cdot\sqrt{f+C_f}\,. \tag{22}$$

Equation (21) then results in:

$$\begin{aligned}
&F\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)+3\cdot\eta^{\alpha\beta}\cdot F_{,\beta}\cdot\psi_{\mu\nu,\alpha}=0\\
&\Rightarrow\\
&\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\frac{F'}{F}\Delta_\eta f+\frac{F'}{F}3\cdot\eta^{\alpha\beta}\cdot f_{,\beta}\cdot\psi_{\mu\nu,\alpha}\\
&=\Delta_\eta\psi_{\mu\nu}+\frac{1}{2}\cdot\frac{1}{f+C_f}\cdot\left(\psi_{\mu\nu}\cdot\Delta_\eta f+3\cdot\eta^{\alpha\beta}\cdot f_{,\beta}\cdot\psi_{\mu\nu,\alpha}\right)=0\\
&\xrightarrow{\Psi=f+C_f}\\
&2\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_\eta\Psi+3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot\psi_{\mu\nu,\alpha}=0
\end{aligned}\quad. \tag{23}$$

This means, that for a given weak gravity "tensor" $\psi_{\mu\nu}$ we have a linear equation in the scalar wave function Ψ. If of bold character or mood, one may now even conclude that the equation in the last line of (23) could be interpreted as the equivalent for the classical Klein-Gordon equation under the assumption of weak gravitational fields and small velocities, because within the terms:

$$2\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}\quad\text{and}\quad 3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot\psi_{\mu\nu,\alpha}, \tag{24}$$

we might want to recognize something like generalized mass and potential. Contraction should then give us the classical scalar Klein-Gordon equation via:

$$g^{\mu\nu}\left(2\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_\eta\Psi+3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot\psi_{\mu\nu,\alpha}\right)$$
$$\simeq\eta^{\mu\nu}\left(2\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_\eta\Psi+3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot\psi_{\mu\nu,\alpha}\right)$$
$$=2\cdot\Psi\cdot\Delta_\eta\eta^{\mu\nu}\psi_{\mu\nu}+\eta^{\mu\nu}\psi_{\mu\nu}\cdot\Delta_\eta\Psi+3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot\left(\eta^{\mu\nu}\psi_{\mu\nu}\right)_{,\alpha}=0\,. \tag{25}$$
$$\xrightarrow{2h=\eta^{\mu\nu}\psi_{\mu\nu}}$$
$$2\cdot\Psi\cdot\Delta_\eta h+h\cdot\Delta_\eta\Psi+3\cdot\eta^{\alpha\beta}\cdot\Psi_{,\beta}\cdot h_{,\alpha}=0$$

Well, indeed, we recognize the main parts of the Klein-Gordon equation, but we are anything but happy that we still have the term with the first derivatives in there. Something is missing – obviously.

Going back to (21) and remembering that the complete variational integral with the Einstein-linearization from above would read (Λ=0):

$$0=\int_V d^nx\sqrt{-G}\cdot\left(F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F\right)\delta G^{\mu\nu}$$
$$=\int_V d^nx\sqrt{-G}\cdot\left(\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\frac{\Delta_\eta F}{F}\right)+\frac{3}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\delta G^{\mu\nu}\quad. \tag{26}$$
$$=\int_V d^nx\sqrt{-g}\cdot F^{\frac{n}{2}}\left(\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\frac{\Delta_\eta F}{F}\right)+\frac{3}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\delta G^{\mu\nu}$$

Now we take into account that in the current – Einstein-linearization - approximation the variation is producing non-constant/variable terms of second order, meaning:

$$\delta G^{\mu\nu}\xrightarrow{\text{"weak gravity"}}\frac{\eta^{\mu\nu}}{F}+\delta\varepsilon\left[h^2\right], \tag{27}$$

where we – as before – ignore these higher order terms and obtain:

$$0=\int_V d^nx\sqrt{-g}\cdot F^{\frac{n}{2}}\left(\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F^2}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\frac{\Delta_\eta F}{F}\right)+\frac{3}{F}\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\frac{\eta^{\mu\nu}}{F}$$
$$\xrightarrow{n=4}\qquad\qquad. \tag{28}$$
$$=\int_V d^nx\sqrt{-g}\cdot\left(F\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)+3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\eta^{\mu\nu}$$

The last term can be integrated by parts in two different ways:

$$\int_V d^nx\sqrt{-g}\cdot\left(3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\eta^{\mu\nu}$$
$$=\begin{cases}\overbrace{\int_{\partial V}dS_\beta\cdot\left(3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F\right)\eta^{\mu\nu}}^{=0}-\int_V d^nx\sqrt{-g}\cdot\left(3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha\beta}\cdot F\right)\eta^{\mu\nu}\\ \overbrace{\int_{\partial V}dS_\alpha\cdot\left(3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu}\cdot F_{,\beta}\right)\eta^{\mu\nu}}^{=0}-\int_V d^nx\sqrt{-g}\cdot\left(3\cdot\eta^{\alpha\beta}\cdot\psi_{\mu\nu}\cdot F_{,\alpha\beta}\right)\eta^{\mu\nu}\end{cases}. \tag{29}$$

Consequently, the results for the last line of (28) read as follows:

$$
\begin{aligned}
0 &= \int_V d^n x\sqrt{-g}\cdot\left(F\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)-3\cdot\begin{Bmatrix}\eta^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha\beta}\cdot F\\ \eta^{\alpha\beta}\cdot\psi_{\mu\nu}\cdot F_{,\alpha\beta}\end{Bmatrix}\eta^{\mu\nu}\right.\\
&= \int_V d^n x\sqrt{-g}\cdot\left(F\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)-3\cdot\begin{Bmatrix}\Delta_\eta\psi_{\mu\nu}\cdot F\\ \psi_{\mu\nu}\cdot\Delta_\eta F\end{Bmatrix}\eta^{\mu\nu}\right.\\
&\Rightarrow\\
0 &= \begin{cases}\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}+\Delta_\eta F\right)-2\cdot F\cdot\Delta_\eta\psi_{\mu\nu}\\ F\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\left(\frac{F_{,\alpha}}{F}\cdot\eta^{\alpha\beta}\cdot F_{,\beta}-2\cdot\Delta_\eta F\right)\end{cases}
\end{aligned}
\quad . \tag{30}
$$

In order to get rid of the non-linear terms in F we would again need:

$$
F[f]=C_F\cdot\sqrt{f+C_f}\ . \tag{31}
$$

for the first equation in the last line of (30), leading to:

$$
\begin{gathered}
\Rightarrow\\
\frac{1}{f+C_f}\psi_{\mu\nu}\cdot\Delta_\eta f-4\cdot\Delta_\eta\psi_{\mu\nu}=0\\
\xrightarrow{\Psi=f+C_f}\\
4\cdot\Psi\cdot\Delta_\eta\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_\eta\Psi=0
\end{gathered}
\quad , \tag{32}
$$

while the second equation requires:

$$
F[f]=C_F\cdot\left(f+C_f\right)^2. \tag{33}
$$

and gives us:

$$
\begin{gathered}
\Rightarrow\\
F\cdot\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot F'\Delta_\eta f\\
=\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot\frac{\Delta_\eta f}{f+C_f}=0\,.\\
\xrightarrow{\Psi=f+C_f}\\
\Psi\cdot\Delta_\eta\psi_{\mu\nu}-2\cdot\psi_{\mu\nu}\cdot\Delta_\eta\Psi=0
\end{gathered}
\tag{34}
$$

We realize the two options (31) and (33) to coincide with our own quantum field equation linearization under “weak gravity” (3) and F satisfying (4) with n=10 and n=4, respectively. While the meaning for 4 is obviously connected with the space-time on bigger scales, the n=10-option is of importance with many string and brane theories [14]. Regarding the metric picture, however, the two options will merge to one again when finally forming the complete metric tensor (thereby taking into account all approximations and simplifications being made along the way).

Ignoring the Operator Expansion

Directly using (19) yields:

$$
R^*{}_{\alpha\beta}-\frac{1}{2}R^*\cdot G_{\alpha\beta}=F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F=0\,, \tag{35}
$$

we would have no non-linear term in F. Hence, we have the following expression under the variation Hilbert integral:

$$0=\int_V d^n x\sqrt{-G}\cdot\left(F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F\right)\delta G^{\mu\nu}. \tag{36}$$

As above, we take into account that in the Einstein-linearization the variation is producing non-constant/variable terms of second order, meaning:

$$\delta G^{\mu\nu}\xrightarrow{\text{"weak gravity"}}\frac{\eta^{\mu\nu}}{F}+\delta\varepsilon\left[h^2\right], \tag{37}$$

where – as before – we ignore these higher order terms and obtain:

$$\begin{aligned}0&=\int_V d^n x\sqrt{-g}\cdot F^{\frac{n}{2}}\left(F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F\right)\frac{\eta^{\mu\nu}}{F}\\&\xrightarrow{n=4}\\&=\int_V d^n x\sqrt{-g}\cdot F\cdot\left(F\cdot\Delta_{MG}\psi_{\mu\nu}+\psi_{\mu\nu}\cdot\Delta_{MG}F+2\cdot G^{\alpha\beta}\cdot\partial_\alpha\psi_{\mu\nu}\cdot\partial_\beta F\right)\eta^{\mu\nu}\end{aligned}. \tag{38}$$

The last term can be integrated by parts in two different ways:

$$\begin{aligned}\int_V d^n x\sqrt{-g}\cdot F\cdot\left(2\cdot G^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\eta^{\mu\nu}&=2\cdot\int_V d^n x\sqrt{-g}\cdot F\cdot\left(\frac{g^{\alpha\beta}}{F}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\eta^{\mu\nu}\\&=2\cdot\int_V d^n x\sqrt{-g}\cdot\left(g^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F_{,\beta}\right)\eta^{\mu\nu}\end{aligned}. \tag{39}$$

$$=\begin{cases}\overbrace{\int_{\partial V}dS_\beta\cdot\left(2\cdot g^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\cdot F\right)\eta^{\mu\nu}}^{=0}-2\cdot\int_V d^n x\sqrt{-g}\cdot\overbrace{\sqrt{-g^{-1}}\cdot\left(\sqrt{-g}\cdot g^{\alpha\beta}\cdot\psi_{\mu\nu,\alpha}\right)_{,\beta}}^{\Delta_g\psi_{\mu\nu}}\cdot F\eta^{\mu\nu}\\ \overbrace{\int_{\partial V}dS_\alpha\cdot\left(2\cdot g^{\alpha\beta}\cdot\psi_{\mu\nu}\cdot F_{,\beta}\right)\eta^{\mu\nu}}^{=0}-2\cdot\int_V d^n x\sqrt{-g}\cdot\overbrace{\sqrt{-g^{-1}}\cdot\left(\sqrt{-g}\cdot g^{\alpha\beta}\cdot F_{,\beta}\right)_{,\alpha}}^{\Delta_g F}\cdot\psi_{\mu\nu}\eta^{\mu\nu}\end{cases}$$

This time, as only the first path is of interest to us, the result for the last line of (38) reads as follows:

$$\begin{aligned}0&=\left(F\cdot\Delta_G\psi_{\mu\nu}+\psi_{\mu\nu}\Delta_G F-2\cdot\Delta_g\psi_{\mu\nu}\right)\eta^{\mu\nu}\\&=F\cdot\Delta_G\eta^{\mu\nu}\psi_{\mu\nu}+\psi_{\mu\nu}\eta^{\mu\nu}\Delta_G F-2\cdot\Delta_g\eta^{\mu\nu}\psi_{\mu\nu}\\&\xrightarrow{\eta^{\mu\nu}\psi_{\mu\nu}=\left(\eta^{\mu\nu}h_{\mu\nu}-\eta^{\mu\nu}h\eta_{\mu\nu}\right)=(2\cdot h-n\cdot h)=(2-n)\cdot h}\\&=(2-n)\cdot\left(F\cdot\Delta_G h+h\cdot\Delta_G F-2\cdot\Delta_g h\right)\\&\xrightarrow{n\neq 2}\\&F\cdot\Delta_G h+h\cdot\Delta_G F-2\cdot\Delta_g h=0\end{aligned}. \tag{40}$$

Here now, especially with h containing a whole metric tensor, we recognize the Klein-Gordon equation with equivalents for mass and potential. We can further simplify when expanding the G-Laplace-operator in the general way:

$$F\cdot\Delta_G h+h\cdot\Delta_G F-2\cdot\Delta_g h=0$$
$$=\frac{1}{F}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+\Delta_g h+h\cdot\left(\frac{1}{F^2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta F+\frac{\Delta_g F}{F^1}\right)-2\cdot\Delta_g h$$
$$=\frac{1}{F}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h-\Delta_g h+h\cdot\left(\frac{1}{F^2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta F+\frac{\Delta_g F}{F^1}\right)=0 \quad . \tag{41}$$
$$\Rightarrow$$
$$\Delta_g h=\frac{1}{F}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\left(\frac{1}{F^2}\cdot F_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta F+\frac{\Delta_g F}{F^1}\right)$$

Interested in avoiding the non-linear term in F, we set again:

$$F[f]=C_F\cdot\sqrt{f+C_f}\,, \tag{42}$$

and obtain:

$$\Rightarrow$$
$$\Delta_g h=\frac{F'}{F}\cdot\left(f_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g f\right)=\frac{1}{2\left(f+C_f\right)}\cdot\left(f_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g f\right)$$
$$\xrightarrow{\Psi=f+C_f} \quad . \tag{43}$$
$$\Delta_g h=\frac{1}{2\Psi}\cdot\left(\Psi_{,\alpha}\cdot g^{\alpha\beta}\cdot\partial_\beta h+h\cdot\Delta_g\Psi\right)$$

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