

The Fine Structure Constant just a Connector Between Standard Model and a Multidimensional Reality?

By Dr. rer. nat. habil. Norbert Schwarzer

The Problem

The Sommerfeld fine structure constant is one of the most important constants in physics. It has the funny value of $\alpha=1/137.035999177$, is dimensionless (so, really just a number) and appears in so many different applications and effects that one definitively may consider it quite universal.

But nobody can truly explain where it is coming from.

Just for fun, let's assume that our Standard model is not correct. I mean, it is not wrong, but also not quite right... in a sense that where the perfect model might give straight answers and illustrative interpretation in a direct way, we need adjusting parameters and some intricate, really not very intuitive workarounds to describe the existing model and its results. After all, the perfect model of reality is reality itself and hence, why not checking for an option to move a bit closer to this very reality?

A Strange Idea

Let's pick the Bekenstein-Hawking thought experiment [1, 2] and throw bits into black holes. They, which is to say Bekenstein and Hawking, have shown that each bit increases the surface of the black hole by the tiny amount of one Planck length squared ℓ_p^2 . Expending this experiment, one could think about constructing a whole black hole out of such bits. The growth rate for the Schwarzschild radius r_s (marking the event horizon) of the black hole per bit q is then described by the following equation:

$$\frac{r_s}{\ell_p} = 2 \cdot \sqrt{\frac{\pi}{\mu} \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}. \quad (1)$$

For convenience we assume to measure the Schwarzschild radius in units of the Plank length ℓ_p . The meaning of the parameter μ will be discussed later.

So far, everything is quite classical and we face the usual problem of having no idea what codes those bits inside the black hole.

Dimensionally Extremal n-Spheres

Now we assume that there is a different model, which could code such bits and still shows the same or a sufficiently similar growth rate for the Schwarzschild radius. We find such a model in form of dimensional extremal n-spheres where the radius per dimension n follows the function (e.g. [3 – 7]):

$$r[n] = \frac{e^{\frac{C_n}{2n} + \frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{n}}}{\sqrt{\pi}}. \quad (2)$$

For bigger numbers $q=n$ we can perfectly fit the two functions with just one scaling parameter and their ratio produces the following limit for $q=n \rightarrow \infty$:

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{4 \cdot \pi^2 \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}{\mu \cdot e^{\frac{C_n}{n} + 2 \cdot \frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{n}}} = \frac{16 \cdot \pi^2 \cdot e}{\mu}. \quad (3)$$

The attentive reader will have noticed that for $\mu=1$ (the Bekenstein-Hawking setting by the way) this value is pretty close to the inverse of the fine structure constant times π , namely:

$16 \cdot \pi^2 \cdot e = \pi \cdot 136.636$. Thus, in order to exactly produce the inverse of the fine structure constant we would require $\mu=0.997079 \cdot \pi$.

The deviation from the Bekenstein choice $\mu=1$ could be explained with the correct selection of the size of the photon so that it really can only carry one bit of information into the black hole. We have to take into account that an observer in a certain distance of the black hole will have to choose the size of this photon such that it will have about the size of the black hole when being absorbed and not in the far distance. When traveling towards the black hole namely the photon undergoes a severe blue shift. De facto, the blue shift will be infinitive for an observer in the infinity and the absorption happening at the event horizon, but it becomes finite for positions slightly outside the black hole. In our case, placing the origin in the distant infinity the absorption point A should be exactly at $A=1.0000532542 \cdot r_s$. In this case, the result of (3) exactly reads:

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{4 \cdot \pi^2 \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}{\mu \cdot e^{\frac{C_n}{n} + 2 \cdot \frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{n}}} = \frac{16 \cdot \pi^2 \cdot e}{\mu} = 137.035999177 = \frac{1}{\alpha}. \quad (4)$$

Ignoring any possible connection with the fine structure constant and demanding an outcome like:

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{4 \cdot \pi^2 \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}{\mu \cdot e^{\frac{C_n}{n} + 2 \cdot \frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{n}}} = \frac{16 \cdot \pi^2 \cdot e}{\mu} = 1 \quad (5)$$

the result for the absorption point would be even closer to the black hole, namely, $A=1.00000539552 \cdot r_s$.

But what happens if the Bekenstein assumption of $\lambda=r_s$ is not correct?

What if the photon could only be caught by the black hole and stored as a 1-bit information when it has a certain size, which is slightly different than the Bekenstein assumption. Let us assume that the catching process requires the photon to circle the black hole. Then, a simple geometric consideration shows us that the shortest wavelength should be:

$$\lambda = 2 \cdot \pi \cdot (r_s + \varepsilon). \quad (6)$$

Thereby the number 2 as factor before the Pi is easily explained via our geometrical consideration of the shortest wavelength circling the black hole with its circumference of $2 \cdot \pi \cdot r_s$, while the ε stands

for the assumption that the photon starts the circling process a tiny bit away from the event horizon. Sucking the latter into an equivalent μ , gives us the following substitute for the Bekenstein equation (1):

$$\begin{aligned} \frac{r_s}{\ell_p} &= 2 \cdot \sqrt{\frac{\pi}{\mu} \cdot (q + \sqrt{q \cdot (1+q)})} = 2 \cdot \sqrt{\frac{\pi}{2\pi\mu'} \cdot (q + \sqrt{q \cdot (1+q)})} \\ &\Rightarrow \frac{r_s}{\ell_p} = \sqrt{\frac{2}{\mu'} \cdot (q + \sqrt{q \cdot (1+q)})} \end{aligned} \quad (7)$$

and subsequently, instead of (3):

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{2 \cdot \pi \cdot (q + \sqrt{q \cdot (1+q)})}{\mu' \cdot e^{\frac{C_n+2}{n} \ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}} = \frac{8 \cdot \pi \cdot e}{\mu'}. \quad (8)$$

Still keen on obtaining the fine structure constant via:

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{2 \cdot \pi \cdot (q + \sqrt{q \cdot (1+q)})}{\mu' \cdot e^{\frac{C_n+2}{n} \ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}} = \frac{8 \cdot \pi \cdot e}{\mu'} = 137.035999177 = \frac{1}{\alpha}, \quad (9)$$

gives us $\mu' = 0.49854$ and this, as it is smaller than 1, does not allow us to find an absorption point with a suitable blue shift, because there would be no blue shift. On the other hand, demanding:

$$\lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{2 \cdot \pi \cdot (q + \sqrt{q \cdot (1+q)})}{\mu' \cdot e^{\frac{C_n+2}{n} \ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}} = \frac{8 \cdot \pi \cdot e}{\mu'} = 1 \quad (10)$$

leaves us with $8 \cdot \pi \cdot e = \mu'$ and the corresponding absorption point of:

$$A = \frac{64 \cdot e^2 \cdot \pi^2}{64 \cdot e^2 \cdot \pi^2 - 1} \quad (11)$$

or $A = 1.00021430 \cdot r_s$, being equivalent to $\varepsilon = 0.00021430 \cdot r_s$ or:

$$\varepsilon = \frac{r_s}{64 \cdot e^2 \cdot \pi^2 - 1}. \quad (12)$$

Finding the Right Dimensionality

Naturally, we may also assume that the number of the fine structure constant just comes out of the quotient in the limit of (3), but instead of n moving towards infinity, we just have a certain dimensionality and need to solve the following equation:

$$\left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \frac{4 \cdot \pi^2 \cdot (q + \sqrt{q \cdot (1+q)})}{\mu \cdot e^{\frac{C_n+2}{n} \ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}} = 137.035999177 = \frac{1}{\alpha}. \quad (13)$$

Thereby we might assume non-black-hole objects and also consider the fact that one bit does not only need one dimensions to be coded but k dimensions. In this case (13) changes to:

$$\left(\frac{r_s}{r[n] \cdot \ell_p} \right)^2 = \frac{4 \cdot \pi^2 \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}{\mu \cdot e^{\frac{C_n}{k \cdot q} + 2 \frac{\ln \left[\Gamma \left[1 + \frac{k \cdot q}{2} \right] \right]}{k \cdot q}}} = 137.035999177 = \frac{1}{\alpha}. \quad (14)$$

Setting $\mu=1$, we find that reasonable results can only be expected for $k \leq 3$ and the C_n chosen such that we always end up in positive integer n .

This aspect will be discussed in [8].

Dimensionally Extremal n-p-Spheroids

Now we generalize our n -spheres to spheroids with p axes being different from r , leading to the following volume formula:

$$V = \frac{\pi^{\frac{n}{2}}}{\Gamma \left[\frac{(n+2)}{2} \right]} \cdot \prod_{j=1}^n a_j = \frac{\pi^{\frac{n}{2}}}{\Gamma \left[\frac{(n+2)}{2} \right]} \cdot r^n \cdot \beta^p[n], \quad (15)$$

Where the difference of the two sorts of axes is defined by a factor $\beta = \beta[n]$. The result for the n -extremal condition reads:

$$\frac{\partial V}{\partial n} = \frac{\pi^{\frac{n}{2}}}{n \cdot \Gamma \left[\frac{n}{2} \right]} \cdot r^n \beta[n]^{p-1} \left(\left(\gamma - H_n \left[\frac{n}{2} \right] + \ln[\pi] + 2 \cdot \ln[r] \right) \beta[n] + 2p \cdot \beta'[n] \right), \quad (16)$$

$$\frac{\partial V}{\partial n} = 0 = \left(\gamma - H_n \left[\frac{n}{2} \right] + \ln[\pi] + 2 \cdot \ln[r] \right) \beta[n] + 2p \cdot \beta'[n], \quad (17)$$

and gives the following solution for the radius factor $\beta[n]$:

$$\beta[n] = \frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2 \cdot p}}} \cdot e^{\frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{p}}. \quad (18)$$

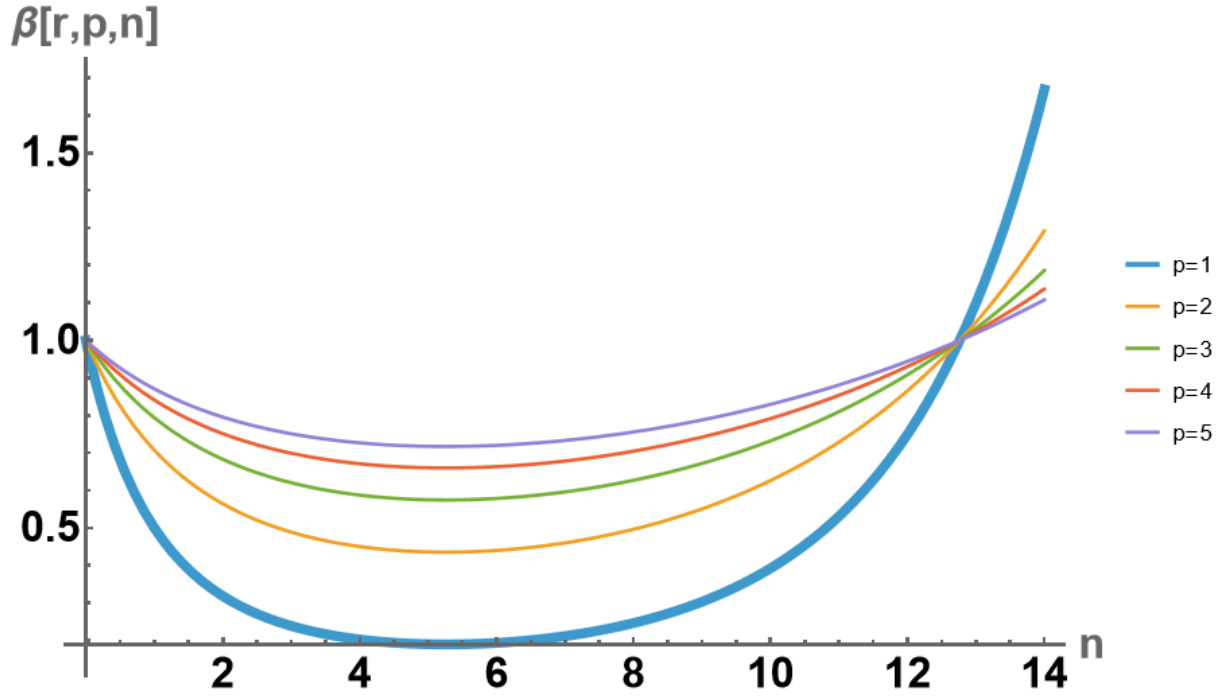


Fig. 1: Factor β for various p in accordance with our equation (17) for n -(volume)-extremal- p -axis sphere-like ellipsoids.

Evaluation of the effective (average) radius over the whole volume would be done via:

$$r_{\text{eff}} = (r^n \cdot \beta^p[n])^{\frac{1}{n}} = \left(r^n \cdot \left(\frac{C_n}{r^p \cdot \pi^{\frac{n}{2-p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{n}}. \quad (19)$$

Choosing $p=n$ -const we result in the following limiting equation:

$$\frac{1}{\alpha} = \lim_{q=n \rightarrow \infty} \left(\frac{r_s}{r_{\text{eff}} \cdot \ell_p} \right)^2 = \lim_{q=n \rightarrow \infty} \frac{4 \cdot \pi \cdot (q + \sqrt{q \cdot (1+q)})}{\mu \left(r^n \cdot \left(\frac{C_n}{r^p \cdot \pi^{\frac{n}{2-p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{n}}} = \frac{16 \cdot \pi^2}{\mu \cdot C_n^2} \cdot e. \quad (20)$$

This leads us to a simple $\mu=\pi$ with the assumption of the circling photon (see [9]) and the constant:

$$C_n = \pm 0.9985385435007366. \quad (21)$$

Eccentricity

In accordance with the eccentricity for ordinary ellipse, we might define an n - p -eccentricity as follows:

$$e_{\text{ell}} = \sqrt{a^2 - b^2} \rightarrow e_{np} = r \cdot \sqrt{1 - \beta^2} = r \cdot \sqrt{\left(\frac{C_n}{r^p \cdot \pi^{\frac{n}{2-p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^2 - 1}. \quad (22)$$

This gives us the following limit for the linear eccentricity of our photon eating objects:

$$\lim_{n \rightarrow \infty} e_{np} = \lim_{n \rightarrow \infty} r \cdot \sqrt{1 - \beta^2} = \infty \quad (23)$$

and the following numerical eccentricity:

$$\varepsilon_{\text{ell}} = \frac{e_{\text{ell}}}{a} = \sqrt{a^2 - b^2} \rightarrow \varepsilon_{np} = \frac{r}{\beta} \cdot \sqrt{1 - \beta^2} = \frac{r}{\beta} \cdot \sqrt{\left(\frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2 \cdot p}}} \cdot e^{\frac{\ln \left[\Gamma \left[1 + \frac{n}{2} \right] \right]}{p}} \right)^2 - 1} \quad (24)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \varepsilon_{np} = 1$$

We realize that we are dealing with rather extreme objects of such eccentricity that in essence only the β -dimensions are dominating the scenery ("survive"), while the other dimensions become completely recessive.

Important Note

It needs to be pointed out explicitly that our results here are just one option and it only holds for systems (apparently black holes) where one dimensions suffices to code one bit. For all other possibilities the reader is referred to [8, 9].

References

- [1] J. D. Bekenstein, "Black holes and entropy", Phys. Rev. D 7:2333-2346 (1973)
- [2] J. D. Bekenstein, "Information in the Holographic Universe", Scientific American, Volume 289, Number 2, August 2003, p. 61
- [3] W. Wismann, D. Martin, N. Schwarzer, "Creation, Separation and the Mind...", 2024, RASA strategy book, ISBN 979-8-218-44483-9
- [4] N. Schwarzer, "The World Formula: A Late Recognition of David Hilbert 's Stroke of Genius", Jenny Stanford Publishing, ISBN: 9789814877206
- [5] N. Schwarzer: "The Math of Body, Soul and the Universe", Jenny Stanford Publishing, ISBN 9789814968249
- [6] N. Schwarzer, "Mathematical Psychology – The World of Thoughts as a Quantum Space-Time with a Gravitational Core", Jenny Stanford Publishing, ISBN: 9789815129274
- [7] N. Schwarzer, "Fluid Universe – The Way of Structured Water; Mathematical Foundation", 2025, a mathematical foundations book
- [8] N. Schwarzer, "Stable Islands of Dimensionality", 2025, a SIO publication, www.siomec.de
- [9] N. Schwarzer, "Solving the 1/137-Riddle?", 2025, a SIO publication, www.siomec.de