# Why we need a generalization of the

# Hamilton **Extremal Principle**



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Cover Picture: "The Ever-Jittering Fulcrum" (by Livia Schwarzer)

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#### 1 Abstract

By demonstrating that a general extremal principle of truly holistic character automatically contradicts the concept of particles, we gain insight into one fundamental aspect governing this universe, namely, the only thing being certain is uncertainty.

The harder we try to fix something in any way, the more this very something tends to evade the constraints. It is similar to Lenz's law in physics where the current induced by a magnetic field always acts against its source.

Completely generalized, this even leads to an adjustment the Hamilton extremal principle.

In this paper we are going to introduce and investigate this generalization.

#### 2 Introduction

#### 2.1 The Classical Hamilton Extremal Principle

From Wikipedia, the free encyclopedia (<a href="https://en.wikipedia.org/wiki/Hamilton's principle">https://en.wikipedia.org/wiki/Hamilton's principle</a>):

In physics, Hamilton's principle is William Rowan Hamilton's formulation of the principle of stationary action. It states that the dynamics of a physical system are determined by a variational problem for a functional based on a single function, the Lagrangian, which may contain all physical information concerning the system and the forces acting on it. The variational problem is equivalent to and allows for the derivation of the differential equations of motion of the physical system. Although formulated originally for classical mechanics, Hamilton's principle also applies to classical fields such as the electromagnetic and gravitational fields, and plays an important role in quantum mechanics, quantum field theory and criticality theories.

So, the definition of the Hamilton principle is based on its "formulation of the principle of stationary action". In simpler words, the variation of such an action should be zero or, mathematically formulated, should be put as follows:

$$\delta W = 0 = \delta \int_{V} d^{n} x \cdot \sqrt{-g} \cdot L.$$
 (1)

Here L stands for the Lagrangian, W the action and g gives the determinant of the metric tensor, which describes the system in question within an arbitrary Rieman space-time with the coordinates x. Thereby, we used the Hilbert formulation of the Hamilton principle [1] in a slightly more general form. We were able to show in [2] that the original Hilbert variation does not only produce the Einstein-Field-Equations [3] but also contains the quantum theory [2, 4, 5]. It should be noted that, while the original Hilbert paper [1] stated with the Ricci scalar R as the integral kernel, which is to say L=R, we here used a general Langrangian, because – as we will show later in this appendix – this generality – in principle – is already contained inside the original Hilbert formulation. Even, as strange as it may sound at this point, general kernels with functions of the Ricci scalar f(R) [6] are already included (see sub-section "Why Don't We See f(R)-Lagrangians in this Universe?") in the Hilbert approach.

#### 2.2 The Hamilton Principle on Shaky Grounds

#### But what if we'd live in a universe, where the only thing which is certain was uncertainty?

This author, always used the analogy of a moving fulcrum to demonstrate his uneasiness with the formulation (1).

In [7] we were able to show that the Hamilton principle itself hinders us to localize any system or object at a certain position. We also see that this contradicts the concept of particles. Everything seems to be permanently on the move or – rather – ever-jittering. The quite surprising insight was rigorously derived in our book [7] in chapter 10. Verbally it could be summed up very briefly as follows:

- a) Try to define a particle and by doing so, you realize that you have to give the thing no matter how you want to make it look like a position. After all it has to be somewhere... the something or whatever "happening" you intend to place in your space-time. Thereby it does not matter whether one considers the real space-time or a theoretical model.
- b) You realize that this position is just another degree of freedom, which you have to treat as such, because otherwise you are just violating the classical/established physical laws.
- c) So, you take the most general principle established physics has, which is the Hamilton minimum principle in its most general form, the Einstein-Hilbert action (1) and you subject your particle, or whatever else it is you wanted to allocate a location to, to it
- d) When doing so correctly, which is to say, taking the position of the very particle into account, too, the particle literally dissolves in front of you and becomes just a wave
- e) Hence: there are no particles in this universe. It's all just waves

But if this ever-jittering fulcrum was one of the fundamental properties of our universe, should we then not take this into account when formulating the laws of this very universe? Shouldn't we better write (1) as follows:

$$\delta W \to 0 \cong \delta \int_{V} d^{n} x \cdot \sqrt{-g} \cdot L ?$$
 (2)

And while we are at it, should we not start to investigate an even more general principle like:

$$\delta W \to f(W, x, g_{\alpha\beta}) = \delta \int_{V} d^{n} x \cdot \sqrt{-g} \cdot L?$$
 (3)

It will be demonstrated in the appendix by my partner and friend Norbert Schwarzer, why such a generalization is justified.

The interesting aspect about this is that this investigation was already – partially – done by (surprise, surprise) e.g. Hilbert and Einstein. But instead of explaining it in this way, they have "hidden" their generalization inside other concepts like the introduction of a cosmological constant or – oh yes – the postulation of matter and its introduction via an ominous and purely postulated parameter L<sub>M</sub>, which is to say, an Lagrange matter term.

Consequently, it should be made abundantly clear that neither this author nor anybody else (nor Dr. Martin, see appendix) has "invented" this extended principle. It is just, as we will also see in connection with the chapter "Why Don't We See f(R)-Lagrangians in This Universe?" another way to add in variational anomalies into the classical extremal and variational process like the cosmological

constant or matter. It is also just another way to add linear independency to the classical theory in order to – for instance – tackle the 3 generations problem of elementary particles [11, 15, 16, 17] by the means of the Bianchi identity.

#### 3 Theory

#### 3.1 Principle of the Ever-Jittering Fulcrum

Taking the example of the concept of the particle, which dissolves itself, the more one tries to push it into existence, we realize that there is just nothing in this universe which we are able to "place inside a box and make the box to shrink to an infinitesimal size". The moment we try, we have to give the box a position, hence coordinates, and those are also degrees of freedom, subjectable to the Hamilton principle, being realized in the Hilbert variation process, leading to a fundamental uncertainty of the position of our box.

Thereby, it does not matter what we want to place inside the box. It could be anything and thus, everything, no matter what it is, the moment we intend to localize it, becomes subjectable to the Hamilton process and ends up as wave with an uncertain position.

In other words: A fulcrum can have not perfectly well-defined position in space (figure 1).

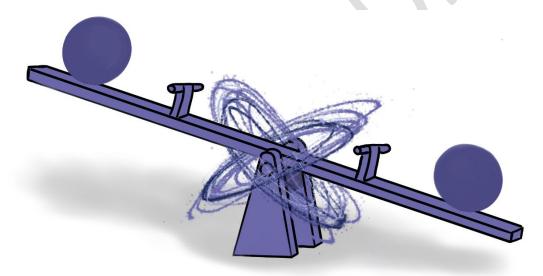


Fig. 1: Symbolizing the "Principle of the Ever-Jittering Fulcrum" (artist: Livia Schwarzer)

Applied on the Hamilton principle itself (see appendix), we end up with the necessity to also include such an uncertainty into this very principle. This can be done in a variety of ways and it is most interesting that one of the outcomes is just matter. Here it can almost be seen as an irony that even Einstein and Hilbert saw this when deriving their theoretical apparatus, but instead of thinking about it fundamentally, both resorted to the solution of just postulating what they rightfully discovered as missing. This way, the matter was introduced as the momentum-energy tensor in Einstein's General Theory of Relativity [3] and as matter Lagrange density in Hilbert's variational approach [1].

#### 4 Conclusions

"If there is one thing certain in this universe, it is uncertainty!"

Taking this recognition to heart leads to a generalized Hamilton principle, an answer to the problem of the three generations of particles and its generalization to all systems, a quantum gravity theory, a destruction of the concept of particles, an understanding for the occurrence of matter, a theory of the

relativity of perspectivity, an approach for the modelling of socio-economic space-times, an explanation for the phenomenon of coherent domains and structured water...

... and so much more.

#### 5 Appendix

### 5.1 The Classical Hamilton Extremal Principle and how to obtain Einstein's General Theory of Relativity With Matter (!) and Quantum Theory... also With Matter (!)

The famous German mathematician David Hilbert [1], even though applying his technique only to derive the Einstein-Field-Equations for the General Theory of Relativity [3] in four dimensions, - in principle - extended the classical Hamilton principle to an arbitrary Rieman space-time with a very general variation by not only — as Hamilton and others had done — concentration on the evolution of the given problem or system in time, but with respect to all its dimensions. His formulation of the Hamilton extremal principle looked as follows:

$$\delta W = 0 = \delta \int_{V} d^{n}x \left( \sqrt{-g} \cdot \left( R - 2\Lambda + L_{M} \right) \right). \tag{4}$$

There we have the Ricci scalar of curvature R, the cosmological constant  $\Lambda$ , the Lagrange density of matter  $L_M$  and the determinant g of the metric tensor of the Rieman space-time  $g_{\alpha\beta}$ . For historical reasons, it should be mentioned that Hilbert's original work [1] did not contain the cosmological constant, because it was added later by Einstein in order to obtain a static universe, but this is not of any importance here. The evaluation of the so-called Einstein-Hilbert action (4) brought indeed the Einstein General Theora of Relativity [3], but it did not produce the other great theory physicists have found, which is the Quantum Theory. It was not before Schwarzer, about one hundred years after the publication of Hilbert's paper [1], extended Hilbert's approach by considering scaling factors to the metric tensor and showed that quantum theory already resides inside the sufficiently general General Theory of Relativity [2, 4, 7, 8, 9]. We will not discuss the reason why this simple idea has not been tried out by other scientists before, but we may still express our amazement about the fact that a simple extension of the type:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f], \qquad (5)$$

solves one of the greatest problems in science<sup>1</sup>, namely the unification of physics and that it took science more than 100 years to come up with the idea. Using the symbol G for the determinant of the scaled metric tensor  $G_{\alpha\beta}$  from (5) of the Rieman space-time we can rewrite the Einstein-Hilbert action from (4) as follows:

$$\delta W = 0 = \delta \int_{V} d^{n}x \left( \sqrt{-G} \cdot F^{q} \cdot \left( R^{*} - 2\Lambda + L_{M} \right) \right). \tag{6}$$

could also be possible and still converges to the classical form for  $F \rightarrow 1$ . Here, which is to say in this paper, we will only consider examples with q=0, but for completeness and later investigation we shall mention that a comprehensive consideration of variational integrals for the cases of general q are to be found in [4]. Performing the variation in (6) with respect to the metric  $G_{\alpha\beta}$  and remembering that the Ricci curvature of (e.g. [7] appendix D) changes the whole variation to:

<sup>&</sup>lt;sup>1</sup> This does not mean, of course, that we should not also look out for generalizations of the scaled metric and investigate those as we did in [10].

$$\delta W = \delta \int_{V} d^{n}x \left( \sqrt{-G} \cdot F^{q} \cdot \left( \left( \frac{R}{F} - \frac{1}{2F^{2}} \left( (n-1) \left( \frac{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}} \right) \right) - 2\Lambda + L_{M} \right) \right) - 2\Lambda + L_{M} \right), \quad (7)$$

$$-(n-1) \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^{3}} (n-6)$$

results in:

$$0 = \left(R^*_{\alpha\beta} - \frac{1}{2}R^* \cdot G_{\alpha\beta}\right) \left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F}\right)\right)$$

$$= \left(R_{\alpha\beta} - \frac{1}{2F} \left(R^*_{\alpha\beta} - \frac{1}{2}R^* \cdot G_{\alpha\beta}\right) \left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F}\right)\right)\right)$$

$$= \left(R_{\alpha\beta} - \frac{1}{2F} \left(R_{\alpha\beta} - \frac{1}{2}R^*_{\alpha\beta} - \frac{1}{2}R_{\alpha\beta} - \frac{1}{2}R_{\alpha\beta} - \frac{1}{2}R_{\beta\alpha} - \frac{1}{2}R_{\alpha\beta} - \frac{1$$

when setting q=0 and assuming a vanishing cosmological constant. With a cosmological constant we have to write:

For better recognition of the classical terms, we have reordered a bit and boxed the classical vacuum part of the Einstein-Field equations (double lines) and the cosmological constant term (single line). Everything else can be – no, represents (!) - matter or quantum effects or both.

Thus, we also – quite boldly – have set the matter density  $L_M$  equal to zero, because we see that already our simple metric scaling brings in quite some options for the construction of matter. It will be shown elsewhere [10] that there is much more which is based on the same technique.

#### 5.2 The Alternate Hamilton Principle

We might bring forward three reasons why we could doubt the fundamentally of the Hamilton principle even in its most general form of the generalized Einstein-Hilbert action:

- a) The principle was postulated and never fundamentally derived.
- b) When rigidly demanding the extremal condition, the extremum should become an object being dependent on all coordinates. Some kind of position appears, which defines or rather "makes out" the extremum. Treating these position parameters as new attributes, the variation should be refined with respect to those and thus, the whole task increases its dimensionality. This was first rigorously derived by this author in [13, 14] and later repeated by him with some more vigor in [7]. No matter how often one repeats the process, there is always an uncertainty about the final number of dimensions, in principle increasing towards infinity. This entangles with another of principle, namely the one of the "infinite orthogonality" (using an expression of Dr. David Martin, second author of [7]), which we will investigate in another paper [12]. Hence, the process is never truly complete and the result can never be a 100% stable extremum.
- c) Even the formulation of this principle in its classical form (4) results in a variety of options where factors, constants, kernel adaptations etc. could be added, so that the rigid setting of the integral to zero offers some doubt in itself. A calculation process which offers a variety of add-ons and options should not contain such a dogma. The result should be kept open and general. One of the authors of [7] (Dr. David Martin) verbally proposed this as the "tragedy of the jittering fulcrum" and we therefore named this principle David's principle of the ever jittering fulcrum. Rigorously put mathematically by this author, it demands:

$$\delta_{g_{\alpha\beta}} W \simeq ? \simeq \delta_{g_{\alpha\beta}} \int_{V} d^{n} x \sqrt{-g} \times R$$

$$\delta_{G_{\alpha\beta}} W \simeq ? \simeq \delta_{G_{\alpha\beta}} \int_{V} d^{n} x \sqrt{-G} \times R^{*}$$
(10)

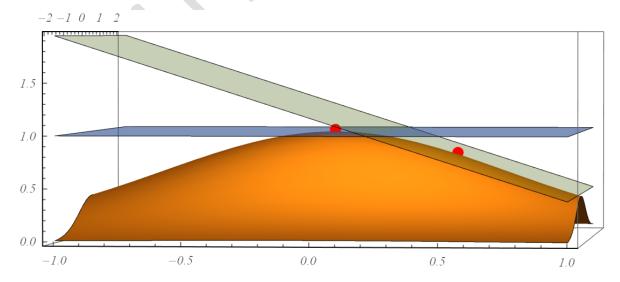


Fig. A1: David's principle of the Ever Jittering Fulcrum cannot accept a dogmatic insistence on a zero outcome of the Einstein-Hilbert action (4) or (generalized and also bringing about the Quantum Theory) (6). Instead it should allow for all states and not just the extremal position (see the two red dots and the corresponding tangent planes in the picture).

One of the simplest generalizations of the classical principle could be the linear one, which is illustrated in figure A1. It could be constructed as follows:

$$\int\limits_{V}d^{n}x\sqrt{-g}\times)\!\!/\!\!(^{\alpha\beta}\cdot g_{\alpha\beta}=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int\limits_{V}d^{n}x\sqrt{-g}\times R\;. \tag{11}$$

Thereby we have used the classical form with the unscaled metric tensor, respectively without setting the factor apart from the rest of the metric. Performing of the variation on the right-hand side and setting

$$\chi^{\alpha\beta} = H \cdot \delta g^{\alpha\beta} \tag{12}$$

or – for the reason of – maximum generality even:

$$\chi^{\alpha\beta} = H^{\alpha\beta}_{ab} \cdot \delta \gamma^{ab} = H \cdot \delta g^{\alpha\beta} \tag{13}$$

just gives us the same result as we would obtain it when assuming a non-zero cosmologic constant, because evaluation yields:

$$\int_{V} d^{n}x \sqrt{-g} \times H \cdot \delta g^{\alpha\beta} \cdot g_{\alpha\beta} = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} ,$$

$$\Rightarrow 0 = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} ,$$
(14)

respectively:

$$\int_{V} d^{n}x \sqrt{-g} \times H_{ab}^{\alpha\beta} \cdot \delta \gamma^{ab} \cdot g_{\alpha\beta} = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} 
\Rightarrow 0 = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta}$$
(15)

Simply setting  $H=-\Lambda$  (c.f. single-line boxed term in equation (9)) demonstrates this.

Nothing else is the usage of a general functional term T, being considered a function of the coordinates of the system (perhaps even the metric tensor) in a general manner, as follows:

$$\int_{V} d^{n}x \sqrt{-g} \times T = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_{V} d^{n}x \sqrt{-g} \times R.$$
 (16)

As before, performing of the variation on the right-hand side and setting

$$T = T_{\alpha\beta} \cdot \delta g^{\alpha\beta} \tag{17}$$

gives us something which was classically postulated under the variational integral, namely the classical energy matter tensor. This time, however, it simply pops up as a result of the principle of the jittering fulcrum and is equivalent to the introduction of the term  $L_M$  under the variational integral. Evaluation yields:

$$\begin{split} &\int_{V} d^{n}x \sqrt{-g} \cdot T_{\alpha\beta} \cdot \delta g^{\alpha\beta} = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ &\Rightarrow 0 = \int_{V} d^{n}x \sqrt{-g} \times \left( R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - T_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{split} \tag{18}$$

So, we see, that in introducing a cosmological constant and in postulating a matter term, even Einstein and Hilbert already – in principle - "experimented" with a non-extremal setting for the Hamilton extremal principle.

Apart from linear dependencies and other functions or functional terms, we could just assume a general outcome like:

$$f(W) = f\left(\int_{V} d^{n}x \sqrt{-g} \times R\right) = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_{V} d^{n}x \sqrt{-g} \times R$$
 (19)

This, however, would not give us any substantial hint where to move on, respectively, which of the many possible paths to follow. We therefore here start our investigation with the assumption of an eigen result for the variation as follows:

This leads to:

$$\int_{V} d^{n}x \sqrt{-g} \left( R_{\kappa\lambda} \delta g^{\kappa\lambda} - R \cdot \left( \frac{1}{2} \cdot g_{\kappa\lambda} \delta g^{\kappa\lambda} + \chi \right) \right) = 0$$
(21)

As the term  $\chi$  could always be expanded into an expression like:

$$\mathcal{K} = H \cdot \mathbf{g}_{\kappa\lambda} \delta \mathbf{g}^{\kappa\lambda} \tag{22}$$

we obtain from (21):

$$0 = \int_{V} d^{n}x \sqrt{-g} \left( R_{\kappa\lambda} \delta g^{\kappa\lambda} - R \cdot \left( \frac{1}{2} + H \right) g_{\kappa\lambda} \delta g^{\kappa\lambda} \right)$$

$$= \int_{V} d^{n}x \sqrt{-g} \left( R_{\kappa\lambda} - R \cdot \left( \frac{1}{2} + H \right) g_{\kappa\lambda} \right) \delta g^{\kappa\lambda}$$

$$\Rightarrow R_{\kappa\lambda} - R \cdot \left( \frac{1}{2} + H \right) g_{\kappa\lambda} = 0$$
(23)

We realize that the term H can be a general scalar even if we would demand the term K to be a constant.

The complete equation when assuming a scaled metric tensor of the form (5) would read:

$$\begin{pmatrix} R_{\alpha\beta} - \frac{1}{2F} \begin{pmatrix} F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} \\ + F_{,a} g^{ab} \left( g_{\beta b,\alpha} - g_{\beta \alpha,b} \right) - F_{,\alpha} g^{ab} g_{\beta b,a} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ + F_{,d} g^{cd} \left( g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} + \frac{1}{2} n g_{\alpha \beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \end{pmatrix} \\ + \frac{1}{4F^2} \left( F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n) \right) \\ - \begin{pmatrix} R - \frac{1}{2F} \left( (n-1) \left( \frac{22\Delta F - 2F_{,d} g^{cd}}{2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}} \right) - n F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \end{pmatrix} \cdot \left( \frac{1}{2} + H \right) g_{\alpha\beta}$$

and in the case of metrics with constant components this equation simplifies to:

$$\begin{pmatrix}
R_{\alpha\beta} - \frac{1}{2F} \left( F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} \right) \\
+ \frac{1}{4F^{2}} \left( F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n) \right) \\
- \left( R - \frac{(n-1)}{2F} \left( 2g^{ab} F_{,ab} + \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F} (n-6) \right) \right) \cdot \left( \frac{1}{2} + H \right) g_{\alpha\beta}
\end{pmatrix} = 0 .$$
(25)

#### 5.2.1 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (10) correspond to more stable states than non-extremal solutions and in fact we will find this in connection with the 3-generation problem, which we have derived and discussed elsewhere [11].

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