Solving the 1/137-Riddle?

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Abstract

In this paper we will derive the fine structure constant α=1/137.035999177 [A1] from the Bekenstein-Hawking thought experiment [A1 – A4], by applying an extremal principle with respect to the number of dimensions [A5, A6].

We find that the fine structure or Sommerfeld constant, as it's also been called, apparently is just a connector between the current model of science, seeing reality of something inside a 4-dimensional space-time and a multi-dimensional concept with also the dimensionality itself being subjected to a general variational, extremal principle.

With this approach we are able to give the fine structure constant as a numerical construct, consisting of natural mathematical constants in dependence on the number of dimensions needed for the storage of one bit. Here is our result for black holes with one bit being coded by just one dimension:

$$\frac{1}{\alpha}=\frac{16\cdot\pi}{C^2}\cdot e \quad \text{with:} \quad C=\pm 0.9985385435007366\,. \tag{1}$$

Abstract References

[A1] https://en.wikipedia.org/wiki/Fine-structure_constant

[A2] N. Schwarzer, "The World Formula: A Late Recognition of David Hilbert 's Stroke of Genius", Jenny Stanford Publishing, ISBN: 9789814877206, pp. 170

[A3] J. D. Bekenstein, "Black holes and entropy", Phys. Rev. D 7:2333-2346 (1973)

[A4] J. D. Bekenstein, "*Information in the Holographic Universe*", Scientific American, Volume 289, Number 2, August 2003, p. 61

[A5] N. Schwarzer, "The Funny Connection between the Pauli Extremal Principle and the Stupidity of Man", 2025, a SIO publication, www.siomec.de

[A6] N. Schwarzer, "Mathematical Psychology – The World of Thoughts as a Quantum Space-Time with a Gravitational Core", Jenny Stanford Publishing, ISBN: 9789815129274

What is the Fine Structure Constant (AI generated text)?

"The fine-structure [or Sommerfeld] constant, denoted by α, is **a fundamental dimensionless physical constant that quantifies the strength of the electromagnetic interaction between elementary charged particles**. Its CODATA recommended value is α = 0.0072973525643(11), corresponding to a reciprocal value of 1/α = 137.035999177(21). This constant is defined in terms of the elementary charge (e), the speed of light (c), the reduced Planck constant ($\hbar$), and the electric constant (ε_0) as $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. The value of α is determined experimentally, with high-precision measurements relying on techniques such as electron anomalous magnetic moment measurements and photon recoil in atom interferometry, which have achieved uncertainties at the level of parts per trillion. The fine-structure constant plays a crucial role in quantum electrodynamics (QED), where it represents the coupling strength between electrons and photons, and its precise measurement serves as a stringent test of the Standard Model of particle physics."

For Entertainment and as Introduction: A Brief Story

A Nobel prize winner in chemistry, learning about my Quantum Gravity approach recently wrote me the following lines:

„I do grant you that the advantage of the closed form is to be able to see analytical patterns and relationships that might have been hidden before."

Here is my answer

Dear Prof. X,

regarding your friendly and motivating text piece „I do grant you that the advantage of the closed form is to be able to see analytical patterns and relationships that might have been hidden before." (which doesn't mean at all that the rest is unfriendly, of course), I have to admit that I already came across some very interesting of such "analytical patterns and relationships". Most are too complex to find a short way to explain them (my usual problem without the math – my sincere apologies) but here is one I find both, short and intriguing:

When investigating a very simple metric model about the dimensional size of black holes in connection with the Bekenstein-Hawking thought experiment (throwing bits into black holes), a certain number appears which – as a unitless number – has to be quite fundamental, universal and independent on physical laws, because, as said, it is just a number.

In my purely metric or geometric model, based on the equations on the 1-pager [I1], the number is 1/(16*pi*e) (with e being Euler's number) and not only is it quite persistent in its appearances, but it also is – for my taste – a little bit too close to the so-called Sommerfeld fine structure constant alpha=1/137.035999206 to be just a coincidence.

You know, of course, that the question about the origin of this constant is one of the most fascinating riddles in science since Sommerfeld discovered it more than one hundred years ago.

Best wishes

Norbert

His answer: "Well yes, this is the way to attract attention to your comprehensive formalism."

[I1] N. Schwarzer, "Do We Have a Theory of Everything – One Pager", 2025, a SIO science paper, www.siomec.de, https://www.siomec.com/pub/2025/b17

The Bekenstein-Hawking Thought Experiment

How Some Dimensions Need Space

It was shown in [2] that for spaces of n-spherical symmetry the radius increase with increasing dimension follows the bit-wise growth of a black hole when always demanding the radius for the n-sphere to couple at its extremal dimensional condition. This means that the number of dimensions for the very n-sphere is chosen such that either surface or volume are extremal. The corresponding radius shows the Bekenstein-Hawking behavior [3, 4]:

$$r_f = 2\cdot L\cdot\sqrt{\pi\cdot\left(n+\sqrt{n\cdot(1+n)}\right)}. \tag{2}$$

Here n gives the dimension, while L is just a scaling factor.

While the classical evaluation is given in the appendix of this paper, we here want to point out a small flaw or inconsistency within the classical derivation and intent to correct it. The basic assumption in Bekenstein's experiment is the construction of a bit-like information, thrown into a black hole by choosing the size of a photon (its wavelength) equal to the Schwarzschild radius. Repeating the evaluation with an uncertainty to this assumption leads to quite some consequences and will later become important within this paper. Thereby the derivation of this refined equation is performed as follows:

At first, following Bekenstein with a slight adjustment, we start with the assumption that the photon's right size should be a wavelength λ of the Schwarzschild radius r_s times an yet unknown parameter μ. Knowing that the energy of the photon would be $E=h*\nu$, with denoting ν the frequency and h giving the Planck constant, and plugging in the equation for the Schwarzschild radius of the photon related mass change Δm (with reduced Planck constant $\hbar$ and the Newton constant G):

$$\begin{gathered}\frac{\Delta r_s\cdot c^4}{2G}=\Delta m\cdot c^2=\leftarrow\boxed{E=h\cdot\nu}\rightarrow=\frac{h\cdot c}{\lambda}=\frac{h\cdot c}{\mu\cdot r_s}\\ \Rightarrow\frac{\Delta r_s\cdot c^4}{2G}=\frac{h\cdot c}{\mu\cdot r_s}\Rightarrow\Delta r_s\cdot r_s=2\frac{h\cdot G}{\mu\cdot c^3}=4\pi\frac{\hbar\cdot G}{\mu\cdot c^3}=4\pi\cdot\frac{\ell_P^2}{\mu}\Rightarrow\Delta r_s=4\pi\cdot\frac{\ell_P^2}{\mu\cdot r_s}\end{gathered}, \tag{3}$$

we can derive ΔA as follows:

$$\begin{aligned}\Delta A &= 4\pi\left(\left(\Delta r_s+r_s\right)^2-r_s^2\right)=4\pi\left(2\Delta r_s\cdot r_s+\left(\Delta r_s\right)^2\right)\\ &=32\cdot\pi^2\cdot\frac{\ell_P^2}{\mu}+64\cdot\pi^3\cdot\frac{\ell_P^4}{\mu\cdot r_s^2}\end{aligned}. \tag{4}$$

Now we assume that we construct a whole black hole just bit by bit and that the latter in the end consists of q bits leading to the identity:

$$q\cdot\Delta A=4\cdot\pi\cdot r_s^2=q\cdot\left(32\cdot\pi^2\cdot\frac{\ell_P^2}{\mu}+64\cdot\pi^3\cdot\frac{\ell_P^4}{\mu\cdot r_s^2}\right). \tag{5}$$

Solving with respect to the Schwarzschild radius measured in units of the Planck length, results in:

$$\frac{r_s}{\ell_P}=2\cdot\sqrt{\frac{\pi}{\mu}\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}. \tag{6}$$

Purely Geometric Evaluation - Keeping Things Simple

While equation (2) is derived from the Bekenstein thought experiment [3, 4] (see appendix) and therefore lacks a rigorous purely mathematical explanation, we now want to extract its fundamental content directly from a geometrical consideration. We start with the volume of the n-sphere:

$$V = \frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot r^n . \tag{7}$$

Derivation with respect to n leads to:

$$\frac{\partial V}{\partial n} = \frac{\pi^{\frac{n}{2}}}{n \cdot \Gamma\left[\frac{n}{2}\right]} \cdot r^n \left(\gamma - H_n\left[\frac{n}{2}\right] + \ln[\pi] + 2 \cdot \ln[r] \right). \tag{8}$$

Here $H_n[x]$ denotes the harmonic number of x and γ gives the Euler Gamma constant with approximately γ=0.57722…

Demanding the volume to be extremal, we have to set:

$$\frac{\partial V}{\partial n} = 0 = \gamma - H_n\left[\frac{n}{2}\right] + \ln[\pi] + 2 \cdot \ln[r], \tag{9}$$

obtain the following solution for the radius r:

$$r = e^{\frac{1}{2}\left(H_n\left[\frac{n}{2}\right] - \gamma - \ln[\pi]\right)}. \tag{10}$$

Of course, our evaluation is slightly incorrect, because we did not assume an n-dependency for the radius r when performing the evaluation in (8). This will be corrected later as it only complicates the situation now and holds us up from moving fast forward in order to obtain a rough overview.

Please note that due to $H_n[0]=0$ for n=0 the radius for the zero-dimensional sphere does not vanish, but is:

$$r[0] = e^{\frac{1}{2}\left(H_n\left[\frac{0}{2}\right] - \gamma - \ln[\pi]\right)} = e^{-\frac{1}{2}(\gamma + \ln[\pi])} = 0.422751 . \tag{11}$$

Setting the result (10) into (7) gives us extremal n-sphere volumina as follows:

$$\begin{aligned} V &= \frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot \left(e^{\frac{1}{2}\left(H_n\left[\frac{n}{2}\right] - \gamma - \ln[\pi]\right)} \right)^n = \frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot e^{\frac{n}{2}\left(H_n\left[\frac{n}{2}\right] - \gamma - \ln[\pi]\right)} \\ &= \frac{\left(\pi \cdot e^{\left(H_n\left[\frac{n}{2}\right] - \gamma - \ln[\pi]\right)} \right)^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \end{aligned} . \tag{12}$$

The case of n=0 results in:

$$V = \frac{\left(\pi \cdot e^{\left(H_n[0] - \gamma - \ln[\pi]\right)} \right)^0}{\Gamma[1]} = 1 . \tag{13}$$

Meaning, the dimensionless extremal n-sphere still has a finite volume and a radius.

In the case of general curved geometries we may assume to be able to describe them as generalized n-tori of extremal sub-domains and end up with a volume equation as follows:

$$V=\sum_{i=1}^{N}\prod_{j=1}^{n_{ij}}T\left[n_{ij}\right]\cdot\frac{\pi^{\frac{n_{ij}}{2}}\cdot r^{n_{ij}}}{\Gamma\left[\frac{\left(n_{ij}+2\right)}{2}\right]}=\sum_{i=1}^{N}\prod_{j=1}^{n_{ij}}T\left[n_{ij}\right]\cdot\frac{\left(\pi\cdot e^{\left(H_n\left[\frac{n_{ij}}{2}\right]-\gamma-\ln[\pi]\right)}\right)^{\frac{n_{ij}}{2}}}{\Gamma\left[\frac{\left(n_{ij}+2\right)}{2}\right]}. \tag{14}$$

We see that the volume then solely depends on the symmetry of the system and the number of dimensions of the corresponding manifolds.

In the case of n-Ellipsoids the result would be a less distinct, because here the volume is given through:

$$V=\frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]}\cdot\prod_{j=1}^{n}a_j\,, \tag{15}$$

With a_j denoting the half-axes of the ellipsoid.

The Relation Between the Bekenstein- and the n-Sphere Picture

From the derivation of our equations (2) (see appendix) and (6) it becomes clear that they are purely classical Schwarzschild geometric products. Hence, this theory plays its role in a 4-dimensional space-time. The n-sphere derivation thereafter, however, is a multi- and variable dimensional approach. That the two still deliver similar distributions regarding the dependency of radius and dimension for the extremal n-spheres and Schwarzschild radius and bits for the classical black holes may have a deeper meaning.

In trying to find the "connector" as a simple factor, we divide equation (6) by (10) and square the quotient, resulting in:

$$\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{2}\right]-\gamma-\ln[\pi]\right)}}. \tag{16}$$

To some it might say nothing that the result of this ratio for q=n and q→∞ is 157.914/μ, but this author finds it a little bit too close to the reciprocal of the so-called Sommerfeld fine structure constant with α=1/137.035999177.

In the limiting case q→∞ we can even give an accurate solution to the quotient above, reading:

$$\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{2}\right]-\gamma-\ln[\pi]\right)}}=\frac{16\cdot\pi^2}{\mu}. \tag{17}$$

Assuming, as Bekenstein and Hawking did, that μ should not be too different from 1, we may wonder whether the connection of the two worlds, the one of the extremal n-spheres, n-tori and so on and the other with our 4-dimensions of space and time, has something to do with this strange dimensionless constant α=1/137.0…. As a matter of fact, with μ=1.1523517277636182 we obtain exactly 1 over the fine structure constant α.

When trying to explain the meaning of a μ different from 1 and thus, different from the Bekenstein assumption, we could just assume that the photon should not have the size of the Schwarzschild radius of the black hole, but should actually be bigger, as the bit-wise feeding of the black hole should make it Schwarzschild sized when it reaches the black hole and not many diameters away. After all, there is some blue shift to the photon to be expected when it travels towards the black hole. Usually, this blue shift would be infinite from any position outside to the event horizon, but only when the photon really need to travel until the very end, which is to say to the event horizon of the black hole. What, however, when this is not of need? What when the blue shift only needs to be of such strength that it makes to shrink the wavelength of the photon from $\mu * r_s$ down to r_s? Assuming the observer to be at infinity with respect to the black hole the redshift can be given via:

$$\frac{\lambda_\infty}{\lambda_a}=\frac{\mu\cdot r_s}{r_s}=\mu=\frac{1}{\sqrt{1-\frac{r_s}{r_a}}}=\frac{1}{\sqrt{1-\frac{r_s}{A\cdot r_s}}}. \tag{18}$$

We find that the parameter A would be A=4.04958, meaning that the absorption point r_a would be about 4 times the radius of the black hole outside the massive object. In this case, the equation (17) could be rewritten as follows:

$$\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{2}\right]-\gamma-\ln[\pi]\right)}}=\frac{1}{\alpha}. \tag{19}$$

Not aiming for the reproduction of fine structure constant but eliminating the nominator in (17) with a blue shift produced μ directly, which is to say, demanding:

$$\frac{\lambda_\infty}{\lambda_a}=\frac{\mu\cdot r_s}{r_s}=\mu=16\cdot\pi^2=\frac{1}{\sqrt{1-\frac{r_s}{r_a}}}=\frac{1}{\sqrt{1-\frac{r_s}{A\cdot r_s}}}, \tag{20}$$

we'd obtain an absorption point very near the event horizon, with an A of A=1.0000401031 or exactly:

$$A=\frac{256\cdot\pi^4}{256\cdot\pi^4-1}. \tag{21}$$

We wonder, having just found that extremal spheres have minimum radius even for the case n=0 (c.f. equation (11)), whether there even is an event horizon and a true black hole or whether we rather have to deal with extremal n-geometrical objects.

Brief Consideration of n-Tori

Instead of n-spheres we now want to investigate various n-tori structures in order check out the effects of the additional degrees of freedom these objects might bring to the game.

As an example we consider an n-torus with two equally-dimensional manifolds but not necessary equal radii:

$$V=\prod_{j=1}^{2}\frac{\pi^{\frac{n_i}{2}}\cdot r^{n_j}}{\Gamma\left[\frac{(n_j+2)}{2}\right]}=\prod_{j=1}^{2}\frac{\left(\pi\cdot e^{\left(H_n\left[\frac{n_j}{2}\right]-\gamma-\ln[\pi]\right)}\right)^{\frac{n_j}{2}}}{\Gamma\left[\frac{(n_j+2)}{2}\right]}. \tag{22}$$

Derivation with respect to n_1 and n_2 gives:

$$0=\partial_{n_1}V+\partial_{n_2}V$$
$$=\frac{\pi^{\frac{n_1+n_2}{2}}\cdot r^{n_1}\cdot r^{n_2}}{2\Gamma\left[\frac{(n_1+2)}{2}\right]\Gamma\left[\frac{(n_2+2)}{2}\right]}\left(2\gamma-H_n\left[\frac{n_1}{2}\right]-H_n\left[\frac{n_2}{2}\right]+2\ln[\pi]+2\cdot\ln[r_1\cdot r_2]\right). \tag{23}$$

Setting $n_1=n_2=n/2$ and $r_2=\beta*r_1=\beta*r$ leads us to the following equation for the radius r:

$$0=2\left(\gamma-H_n\left[\frac{n}{4}\right]+\ln[\pi]+\ln[r]+\ln[\beta]\right).$$
$$\Rightarrow r=e^{\frac{1}{2}\left(H_n\left[\frac{n}{4}\right]-\gamma-\ln[\pi]-\ln[\beta]\right)} \tag{24}$$

Now, as in (19), taking (6) as dominator, dividing by our result r[n] from (24), we obtain:

$$\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{4}\right]-\gamma-\ln[\pi]-\ln[\beta]\right)}}=\frac{32\cdot\pi^2\cdot\beta}{\mu}. \tag{25}$$

Now we simply assume that the bit-wise build-up does not refer to a black hole but an arbitrary object with radius r_f and that we can substitute the Planck length by a general scale L (see equation (2)) in accordance with:

$$\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\left(\frac{r_f}{r[n]\cdot L}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{4}\right]-\gamma-\ln[\pi]-\ln[\beta]\right)}}=\frac{32\cdot\pi^2\cdot\beta}{\mu} \tag{26}$$

and we see that we can have a rather flexible "connector" of information driven growth to various geometries in a multitude of n-tori realms. In the example here, we might just assume μ=1 and β chosen such that it satisfies:

$$\frac{1}{\alpha}=\lim_{q=n\to\infty}\left(\frac{r_f}{r[n]\cdot L}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(H_n\left[\frac{n}{4}\right]-\gamma-\ln[\pi]-\ln[\beta]\right)}}=32\cdot\pi^2\cdot\beta, \tag{27}$$

which would connect the fine structure constant to an internally structured n-tori object with two equally dimensional, but differently sized domains. Their asymmetry corresponds to the fine structure constant.

Correction of the Evaluations With Respect to Variable Radii and the Connection to α

In this section we now strictly avoid any incorrection with respect to the dimensional dependency of parameters and repeat some important evaluations from above accordingly.

The n-Extremal Sphere

In contrast to our simplified evaluation (8), we now explicitly consider an n-dependent radius and start with the volume function for an n-sphere, reading:

$$V=\frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]}\cdot r[n]^{n}. \tag{28}$$

Derivation with respect to n leads to:

$$\frac{\partial V}{\partial n}=\frac{\pi^{\frac{n}{2}}}{n\cdot\Gamma\left[\frac{n}{2}\right]}\cdot r[n]^{n-1}\left(\left(\gamma-H_{n}\left[\frac{n}{2}\right]+\ln[\pi]+2\cdot\ln[r[n]]\right)r[n]+2\cdot n\cdot r'[n]\right). \tag{29}$$

Demanding the volume to be extremal, we now obtain the following differential equation of first order in the dimensions n for the radius r[n]:

$$\frac{\partial V}{\partial n}=0=\left(\gamma-H_{n}\left[\frac{n}{2}\right]+\ln[\pi]+2\cdot\ln[r[n]]\right)r[n]+2\cdot n\cdot r'[n], \tag{30}$$

resulting in the following solution for the radius r[n]:

$$r[n]=\frac{e^{\frac{C_n}{2n}+\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{n}}}{\sqrt{\pi}}. \tag{31}$$

Thereby, this time, we even have obtained an arbitrary constant C_n. Setting this constant zero, we note again that for n=0 the radius is not zero, but exactly what we already have obtained above, namely:

$$r[0]=e^{\frac{1}{2}(-\gamma-\ln[\pi])}=\frac{e^{-\frac{\gamma}{2}}}{\sqrt{\pi}}=0.422751. \tag{32}$$

Surprisingly, leaving the constant finite, we would even obtain r[0]=∞.

The n-p-Extremal Ellipsoid

Now we generalize our sphere to an n-p-spheroid with p axes being different from r, leading to the following volume formula:

$$V=\frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]}\cdot\prod_{j=1}^{n}a_{j}=\frac{\pi^{\frac{n}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]}\cdot r^{n}\cdot\beta^{p}[n]. \tag{33}$$

The difference of the two sorts of axes is defined by a factor β. The result for the derivate with respect to the number of dimensions n reads:

$$\frac{\partial V}{\partial n}=\frac{\pi^{\frac{n}{2}}}{n\cdot\Gamma\left[\frac{n}{2}\right]}\cdot r^{n}\beta[n]^{p-1}\left(\left(\gamma-H_{n}\left[\frac{n}{2}\right]+\ln[\pi]+2\cdot\ln[r]\right)\beta[n]+2p\cdot\beta'[n]\right), \tag{34}$$

$$\frac{\partial V}{\partial n}=0=\left(\gamma-H_{n}\left[\frac{n}{2}\right]+\ln[\pi]+2\cdot\ln[r]\right)\beta[n]+2p\cdot\beta'[n], \tag{35}$$

and gives the following solution for the radius factor β[n]:

$$\beta[\mathrm{n}]=\frac{C_{\mathrm{n}}}{\mathrm{r}^{\frac{n}{\mathrm{p}}}\cdot\pi^{\frac{n}{2\cdot\mathrm{p}}}}\cdot\mathrm{e}^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{\mathrm{p}}}. \tag{36}$$

Now Corrected: The Relation Between the Bekenstein- and the n-Sphere Picture

Now, as before, we compare the purely classical Schwarzschild geometric result from the Bekenstein-Hawking thought experiment (2), living in a 4-dimensional space-time and our n-sphere derivation as a multi- and variable dimensional approach. Again we point out that the two still deliver similar distributions regarding the dependency of radius and dimension for the extremal n-spheres and Schwarzschild radius and bits for the classical black holes. This may have a deeper meaning.

Looking for the "connector" as a simple factor, we divide equation (6) by (31) and square the quotient, resulting in:

$$\left(\frac{\mathrm{r_s}}{\mathrm{r}[\mathrm{n}]\cdot\ell_{\mathrm{P}}}\right)^2=\frac{4\cdot\pi^2\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot\mathrm{e}^{\left(\frac{C_{\mathrm{n}}}{n}+2\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{n}\right)}}. \tag{37}$$

This time, the ratio for q=n and q→∞ is almost 137/μ and we wonder why we are so close to the reciprocal of the so-called Sommerfeld fine structure constant with α=1/137.035999177.

In the limiting case q→∞ we can even give an accurate solution to the quotient above, reading:

$$\lim_{\mathrm{q=n}\to\infty}\left(\frac{\mathrm{r_s}}{\mathrm{r}[\mathrm{n}]\cdot\ell_{\mathrm{P}}}\right)^2=\lim_{\mathrm{q=n}\to\infty}\frac{4\cdot\pi^2\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot\mathrm{e}^{\left(\frac{C_{\mathrm{n}}}{n}+2\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{n}\right)}}=\frac{16\cdot\pi^2}{\mu}\cdot\mathrm{e}. \tag{38}$$

As before, assuming, as Bekenstein and Hawking did, that μ should not be too different from 1, we ask for a factorial connection of the two worlds, the one of the extremal n-spheres, n-tori and so on and the other with smallest information units, bit, namely, in our 4-dimensions of space and time. We find that 16*e*π=136.636 are already pretty close to those ominous 1/α=137.0… Thus, with a μ=(1-ε)*π and an ε=0.0029207771434275253 equation (38) would have delivered us exactly 1 over the Sommerfeld fine structure constant α.

With the μ so close to π, there is a great motivation to see an entanglement of the two worlds and to see it in form of Sommerfeld's constant.

What Does it Actually Mean to "Catch a Photon"?

One option to explain the meaning of a μ different from 1 and thus, different from the Bekenstein assumption, not connected to the red- or blue-shift of the infalling photons, as applied above, thereby assuming that the observer is directly at the event horizon and thus, sees the photon's wavelength in the moment when it falls into the black hole, could just be associated with the following geometric consideration:

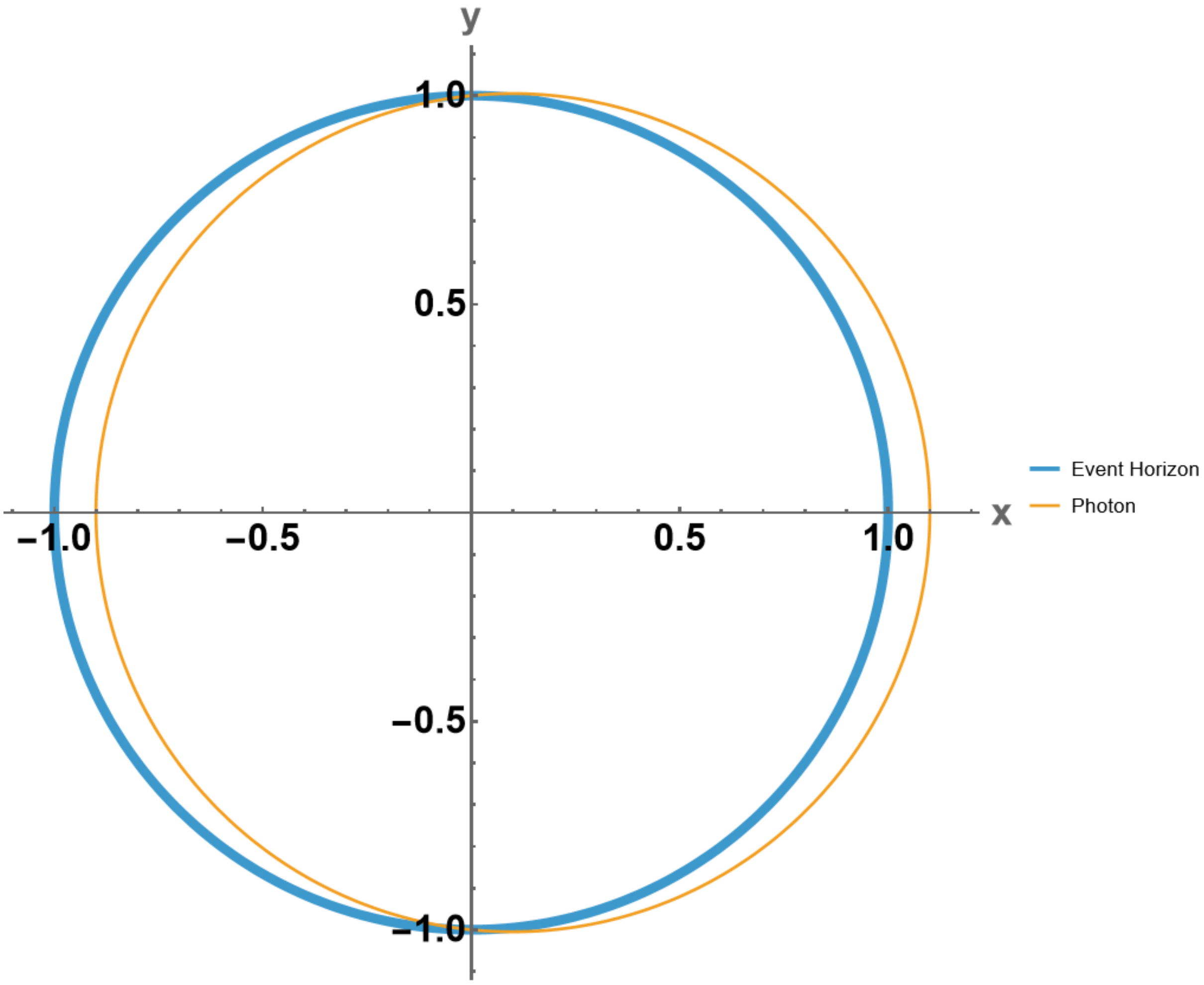

Fig. 1: Photon encircling a black hole. The wavelength of the photon is exactly the circumference of the black hole and consequently does it partially leave it when doing its voyage.

Let us take just one photon and assume that there is nothing in the black hole that could absorb the photon. Instead, the "poor thing" is just circling around the event horizon as shown in figure 1 (orange line). As the photon has a wavelength and an amplitude, we demand two conditions:

a) The wavelength should not be bigger that the circumference of the black hole so that the photon can finish one cycle of oscillation when having circled the black hole once. This shall be the new bit condition.
b) The photon should not be seen anywhere from the outside when doing its circling.

For illustration we show a set of photons not having the right size and thus, partially leaving the black hole (coming out of the event horizon, c.f. figure 2).

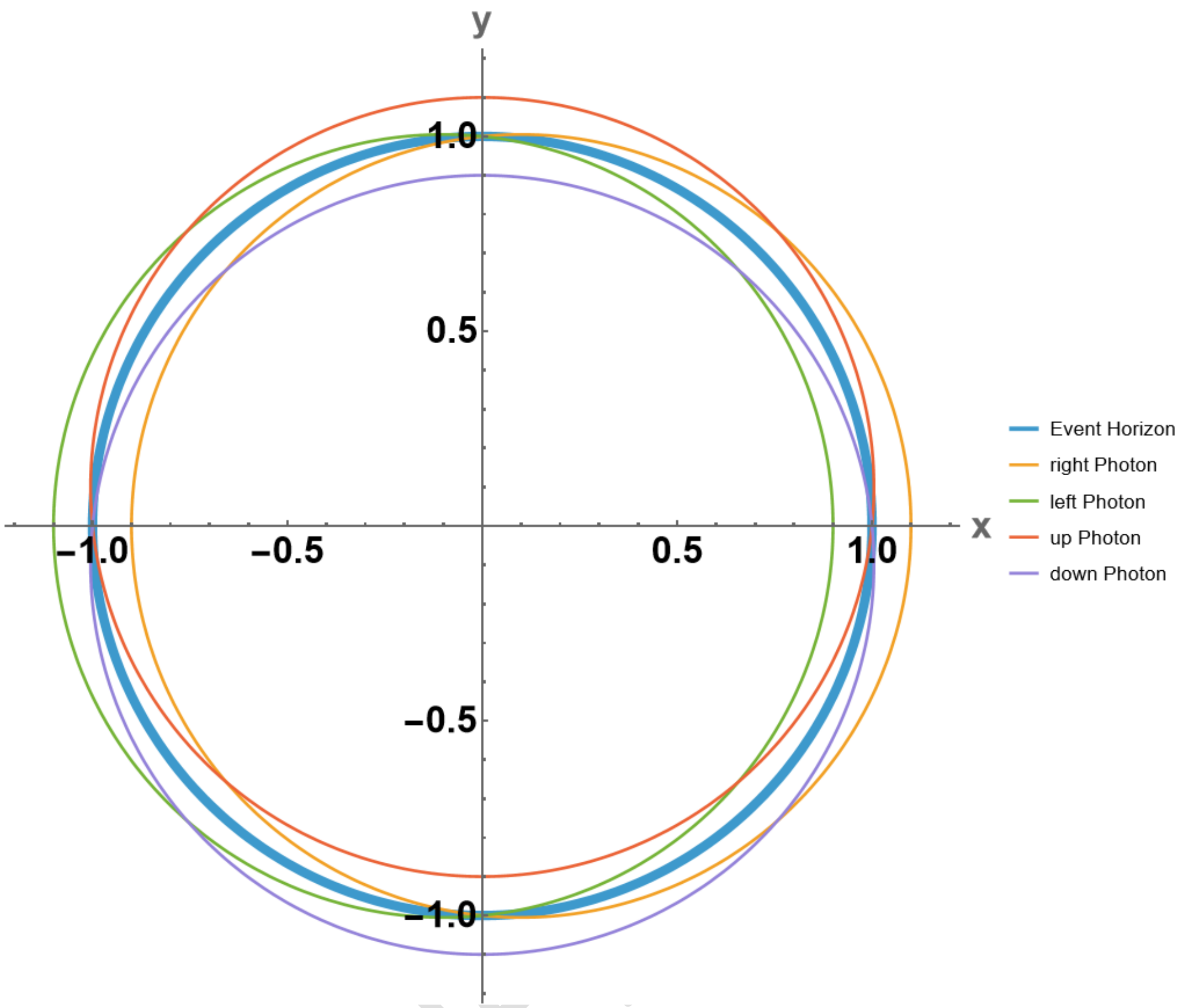

Fig. 2: 4 Photons with a wavelength equal to the circumference of the event horizon encircling a black hole. It oscillations (after all the photons are waves) bring them partially outside the black hole, which should not happen. In order to avoid this, the photons need to be slightly smaller.

"Rigorous" Derivation of Sommerfeld's Constant – A: The Sphere-Option

The photon with the right size, which we here simplify with an oscillation of type:

$$P(\varphi) = A \cdot \sin(\varphi), \tag{39}$$

would be one with a wavelength slightly smaller than the circumference of the black hole (figure 3).

In order to obtain the correct wavelength, we need to evaluate the length of the curved photon which can be given via:

$$\begin{aligned} L &= \lambda \cdot \frac{1}{2} \int_0^{2\pi} \sqrt{1 + P'(\varphi)^2} \cdot d\varphi = \lambda \cdot \frac{1}{2} \int_0^{2\pi} \sqrt{1 + A^2 \cdot \cos^2(\varphi)} \cdot d\varphi \\ &= \lambda \cdot \left(E\left[-A^2\right] + \sqrt{1 + A^2} \cdot E\left[\frac{A^2}{1 + A^2}\right] \right) \end{aligned} . \tag{40}$$

Here the function E[x] denotes the elliptic integral of the second kind. Our definition of the length guarantees that for A→0, which is associated with a completely flat wave or vanishing oscillation, we obtain the result of $\lambda * \pi$, which is just the circumference of the black hole, because a completely flat photon does not anywhere stick out of the event horizon.

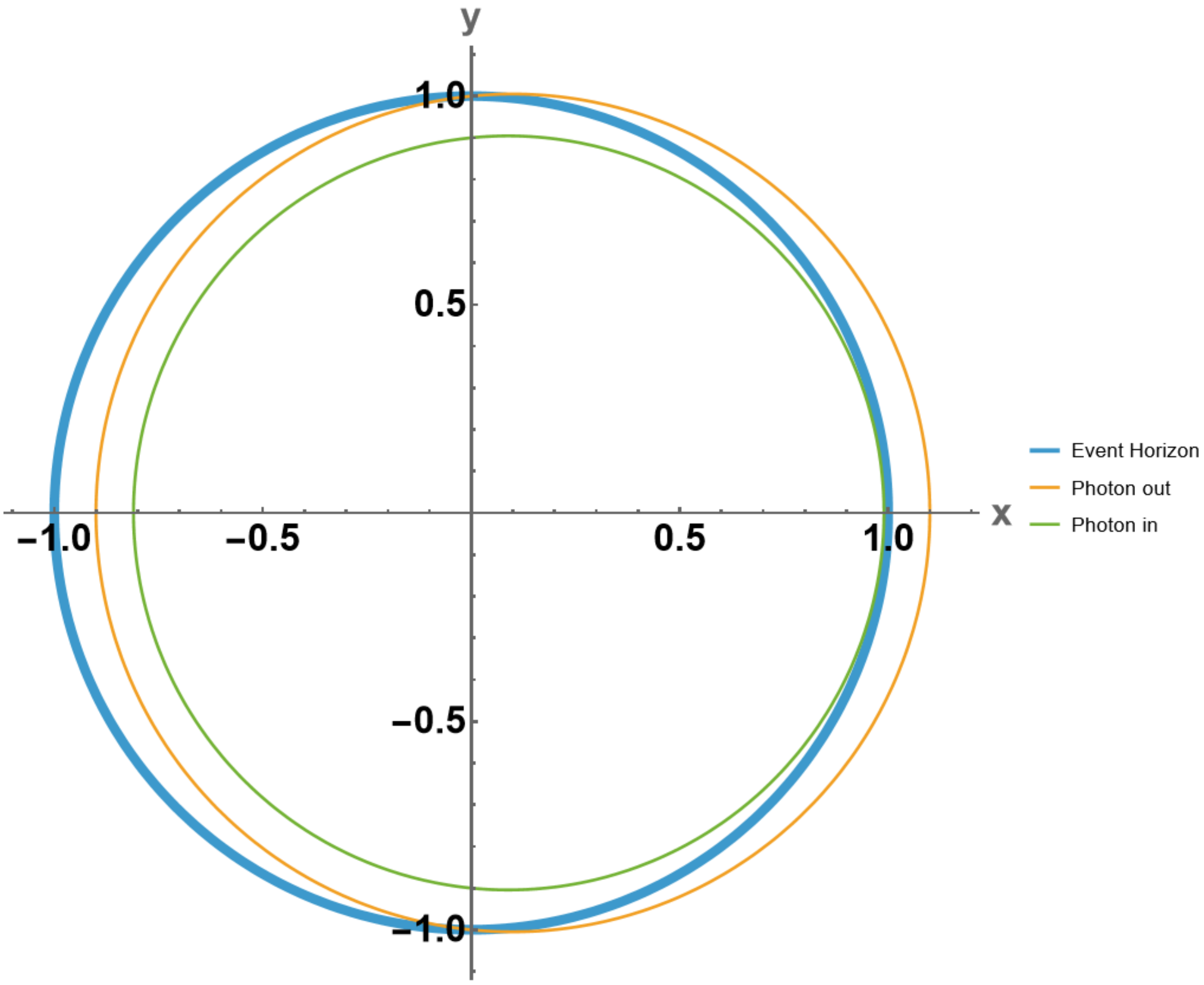

Fig. 3: Two photons encircling a black hole. Now the wavelength of one photon (green) is a little bit shorter than the circumference of the black hole so that it always stays inside when performing its circles behind the event horizon. The other photon's wavelength (orange) is exactly the circumference of the black hole and consequently it does not always stay inside when doing its voyage. We assume that this second option should be ruled out by a proper choice of μ.

In any other (normal) case, we have to demand μ to be:

$$\mu = \pi \cdot \varepsilon = \frac{\pi}{E\left[-A^2\right] + \sqrt{1+A^2} \cdot E\left[\frac{A^2}{1+A^2}\right]}, \tag{41}$$

which gives us the amplitude:

$$A = \pm 0.10836541304457664, \tag{42}$$

and the final formula for the derivation of the fine structure constant as follows:

$$\frac{1}{\alpha}=\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi^2\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(\frac{C_n}{n}+2\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{n}\right)}}=\frac{16\cdot\pi^2}{\mu}\cdot e$$
$$=\frac{16\cdot\pi^2}{\dfrac{\pi}{E\left[-A^2\right]+\sqrt{1+A^2}\cdot E\left[\dfrac{A^2}{1+A^2}\right]}}\cdot e=16\cdot\pi\cdot e\cdot E\left[-A^2\right]+\sqrt{1+A^2}\cdot E\left[\frac{A^2}{1+A^2}\right]. \tag{43}$$

Of course, this is not a rigorous, which is to say fully theoretical, derivation of the constant as it required the empirical calculation of the photon's amplitude, where, not having any information about its natural value, we needed to plug in the fine structure constant to make everything fit. If, however, we would be able to derive also the amplitude in a first principle, rigorous manner, our task of obtaining Sommerfeld's constant in a completely analytical way would be complete.

Maybe this will be possible by applying our solution to the photon [20, 21].

Extension to Cases of q≠n

It needs to be pointed out that our "rigorous" derivation from above is restricted to the situation where de facto one bit can be coded by one dimension. In non-Bekenstein cases, meaning where we are not dealing with Black holes, we may need more dimensions to actually store one bit. Let us assume we require k dimensions for this process. In this case the equation (43) changes as follows:

$$\frac{1}{\alpha}=\lim_{q=n\to\infty}\left(\frac{r_s}{r[n]\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi^2\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\cdot e^{\left(\frac{C_n}{k*n}+2\frac{\ln\left[\Gamma\left[1+\frac{k*n}{2}\right]\right]}{k*n}\right)}}=\frac{16\cdot\pi^2}{\mu\cdot k}\cdot e\,. \tag{44}$$

Now, with the system potentially being more complex than a black hole, the photon can probably be absorbed in a proper way and would not need to circle the object, which means that we do not need the factor π in the denominator. Choosing k=3 as an example, we find μ to be:

$$\mu=1.0441389205244094\,. \tag{45}$$

"Rigorous" Derivation of Sommerfeld's Constant – B: The Spheroid-Option

Now we assume the photon-absorbing dimensionally extremal system to be not perfectly spherically symmetric and apply our result for the n-p-extremal spheroid (36). Evaluation of the effective (average) radius over the whole volume would be done via:

$$r_{n-eff} = \left(r^n \cdot \beta^p[n]\right)^{\frac{1}{n}} = \left(r^n \cdot \left(\frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2\cdot p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{n}},$$
$$r_{p-eff} = \left(r^p \cdot \beta^p[n]\right)^{\frac{1}{p}} = \left(r^p \cdot \left(\frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2\cdot p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{p}} \tag{46}$$

whereby we distinguish the two options of the effective radius for all dimensions $r_{n\text{-eff}}$ and the effective radius for just the β-dimensions $r_{p\text{-eff}}$. Choosing p=n-const it does not matter which one of the effective radii we chose for our limiting procedure in comparison with the Bekenstein-Hawking experiment, because in the limit for n→∞ we obtain the same result, namely:

$$\frac{1}{\alpha} = \lim_{q=n\to\infty} \left(\frac{r_s}{r_{n-eff} \cdot \ell_P} \right)^2 = \lim_{q=n\to\infty} \frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\left(r^n \cdot \left(\frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2\cdot p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{n}}} = \frac{16\cdot\pi^2}{\mu\cdot C_n^2}\cdot e$$
$$\frac{1}{\alpha} = \lim_{q=n\to\infty} \left(\frac{r_s}{r_{n-eff} \cdot \ell_P} \right)^2 = \lim_{q=n\to\infty} \frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\left(r^p \cdot \left(\frac{C_n}{r^{\frac{n}{p}} \cdot \pi^{\frac{n}{2\cdot p}}} \cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{n}{2}\right]\right]}{p}} \right)^p \right)^{\frac{1}{p}}} = \frac{16\cdot\pi^2}{\mu\cdot C_n^2}\cdot e \,. \tag{47}$$

This leads us to a simple μ=π with the assumption of the circling photon (figures 1 and 2) but without the need for any discussion about the width of a photon and the constant:

$$C_n = \pm 0.9985385435007366\,. \tag{48}$$

Extension to Cases of q≠n

As before considering the non-Bekenstein case, meaning where we may need more dimensions to actually store one bit and assuming that we require k dimensions for this process the equations (47) change as follows (again setting p=n-const which now changes to p=k*n-const):

$$\frac{1}{\alpha}=\lim_{q=n\to\infty}\left(\frac{r_s}{r_{n-eff}\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\left(r^{k*n}\cdot\left(\frac{C_n}{r^{\frac{k*n}{p}}\cdot\pi^{\frac{k*n}{2\cdot p}}}\cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{k*n}{2}\right]\right]}{p}}\right)^p\right)^{\frac{1}{k*n}}}=\frac{16\cdot\pi^2}{\mu\cdot k\cdot C_n^2}\cdot e$$

$$\frac{1}{\alpha}=\lim_{q=n\to\infty}\left(\frac{r_s}{r_{n-eff}\cdot\ell_P}\right)^2=\lim_{q=n\to\infty}\frac{4\cdot\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}{\mu\left(r^{p}\cdot\left(\frac{C_n}{r^{\frac{k*n}{p}}\cdot\pi^{\frac{k*n}{2\cdot p}}}\cdot e^{\frac{\ln\left[\Gamma\left[1+\frac{k*n}{2}\right]\right]}{p}}\right)^p\right)^{\frac{1}{p}}}=\frac{16\cdot\pi^2}{\mu\cdot k\cdot C_n^2}\cdot e \quad . \qquad (49)$$

Here, too, we assume that the system, being more complex than a black hole, can absorb the photon in a proper way and the poor thing would not need to circle the object, which means that we do not need the factor π in the denominator. Hence, we have μ=1. Choosing k=3 as an example again, we find C_n to be:

$$C_n=\pm 1.0218311604782904 \, . \qquad (50)$$

Conclusions

We have derive the fine structure constant α as connector between the current model of science, seeing reality of something inside a 4-dimensional space-time and a multi-dimensional concept with also the dimensionality itself being subjected to a general variational, extremal principle.

Thereby we applied the Bekenstein-Hawking thought experiment, where photons are swallowed by black holes in a bit-wise manner, in two ways:

a) The classical picture, just as Bekenstein and Hawking did
b) As dimensionally extremal spheres or spheroids.

and derived α as a ratio of the two descriptions of the same process.

With this we were able to give the fine structure constant as a number consisting of natural mathematical constants in the following form:

$$\frac{1}{\alpha}=\frac{16\cdot\pi^2}{\pi\cdot C_n^2}\cdot e=\frac{16\cdot\pi}{C_n^2}\cdot e \, . \qquad (51)$$

In other, non-black-hole-like cases, where we can assume systems of higher complexity and potentially higher numbers of dimensions being of need to code one bit than in black holes, we end up with the following equation:

$$\frac{1}{\alpha}=\frac{16\cdot\pi}{k*C_n^2}\cdot e \, , \qquad (52)$$

where k gives the numbers of dimensions being of need to store one bit.

Appendix: About the Dimensional Size of Systems

In classical systems science there is no way to derive the necessary dimension of a system in a truly fundamental and neutral (mathematically based) manner. Thus, systems are often "defined" as it pleases the creator of the simulation or as there are restrictions in ability and calculation power for the "digital twin" of the natural system one intends to model. As this holds for any system, this is also true – of course – for the unconscious or conscious mind and thus, of great interest here.

We start with the conjecture that not just the system's inner properties and corresponding governing equations but also the system's size (number of degrees of freedom or dimensions) can be derived from a suitable minimum principle. Our starting point shall this time be the Einstein-Hilbert action with a generalized Lagrange density function $\Phi_R[R]$ and a yet undefined variation, which we write as follows:

$$\delta_? W = 0 = \delta_? \int_V d^n x \left(\sqrt{-g} \cdot \Phi_R[R]\right). \tag{53}$$

Please note that we could also write this for a scaled metric tensor as elaborated in the previous appendices in order to work out the connection to quantum theory, respectively, in order to make it show itself directly via a set of wrapping and wave functions $F_i[f_i]$ and f_i within the usual variational calculus.

In order to adjust undefined parameters and finalize the character of the variational task (52), we intend to consider a fundamental problem and here determine the size of a black hole in a completely new way. Classically the size of a black hole is given by the Schwarzschild radius, which itself is determined by the mass m of the black hole via: $r_s = \frac{2 \cdot m \cdot G}{c^2}$ (G… Newton's constant, c… speed of light in vacuum). This Schwarzschild radius, however, was never derived from a first principle, but was adjusted as a parameter to the Schwarzschild metric [14] in order to give the correct limit to the Newton gravitational law.

Here now we want to derive the Schwarzschild radius via a suitable version of (52). In order to do so, we first need to repeat Bekenstein's thought experiment of black holes.

The Bekenstein Bit-Problem

One of the most famous and equally puzzling problems in General Theory of Relativity is the Bekenstein-Bit problem, where it was found that black holes can store information, but so far it is been seen as a mystery how these objects actually do this. In [2, 7, 8] we have shown that bit-like information is been stored as dimensions and that each bit becomes one dimension. For convenience we are here repeating parts of the original evaluation.

In the early seventies J. Bekenstein [3, 4] investigated the connection between black hole surface area and information. Thereby he simply considered the surfaces change of a black hole which would be hit by a photon just of the same size as the black hole. His idea was that with such a geometric constellation the outcome of the experiment would just consist of the information whether the photon fell into the black hole or whether it did not. Thus, it would be a 1-bit information. His calculations led him to the funny proportionality of area and information. He found that the number of bits, coded by a certain black hole, is proportional to the surface area of this very black hole if measured in Planck area ℓ_P^2. In fact, the dependency how one bit of information changes the area of the black hole (ΔA) reads:

$$\Delta A = 32 \cdot \pi^2 \cdot \ell_P^2 + 64 \cdot \pi^3 \cdot \frac{\ell_P^4}{r_s^2}. \quad (54)$$

Thereby the derivation of this equation is performed as follows. At first we start with the assumption that the photon's right size should be a wavelength λ of the Schwarzschild radius r_s. Knowing that the energy of the photon would be E=h*ν, with denoting ν the frequency and h giving the Planck constant, and plugging in the equation for Schwarzschild radius of the photon related mass change Δm (with reduced Planck constant $\hbar$ and the Newton constant G):

$$\begin{aligned} &\frac{\Delta r_s \cdot c^4}{2G} = \Delta m \cdot c^2 = \leftarrow \boxed{E = h \cdot \nu} \rightarrow = \frac{h \cdot c}{\lambda} = \frac{h \cdot c}{r_s} \\ &\Rightarrow \frac{\Delta r_s \cdot c^4}{2G} = \frac{h \cdot c}{r_s} \Rightarrow \Delta r_s \cdot r_s = 2\frac{h \cdot G}{c^3} = 4\pi \frac{\hbar \cdot G}{c^3} = 4\pi \cdot \ell_P^2 \Rightarrow \Delta r_s = 4\pi \cdot \frac{\ell_P^2}{r_s}, \end{aligned} \quad (55)$$

we can derive ΔA as follows:

$$\begin{aligned} \Delta A &= 4\pi\left(\left(\Delta r_s + r_s\right)^2 - r_s^2\right) = 4\pi\left(2\Delta r_s \cdot r_s + \left(\Delta r_s\right)^2\right) \\ &= 32 \cdot \pi^2 \cdot \ell_P^2 + 64 \cdot \pi^3 \cdot \frac{\ell_P^4}{r_s^2} \end{aligned}. \quad (56)$$

Ignoring the extremely small second term in the last line, one could just assume our black hole to be constructed of many such bit surface pieces. Thus, we could write:

$$q \cdot \Delta A = q \cdot 32 \cdot \pi^2 \cdot \ell_P^2 = 4 \cdot \pi \cdot r_s^2 \quad \Rightarrow \quad r_s^2 = q \cdot 8 \cdot \pi \cdot \ell_P^2, \quad (57)$$

where r_s gives the radius of the black hole. We see that our black hole radius is proportional to the square root of the bits q thrown into it.

Now we want to compare the dependency $r_s[q]$ with the radii $r_{max}[N]$ resulting in maximum volume of n-spheres for a certain number of space-time dimensions N=n+1.

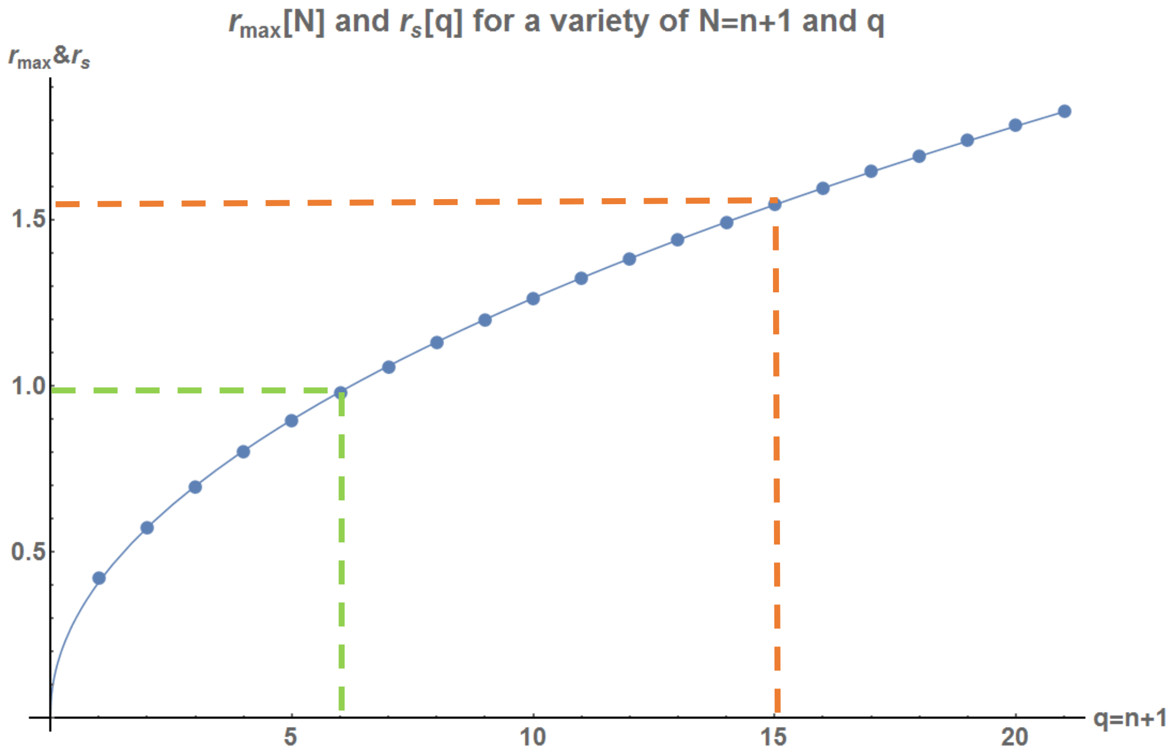

Fig. A1: Radius r_{max} for which at a certain number of dimensions the n-sphere has maximum volume in dependency on N=n+1 compared with the increase of the Schwarzschild radius r_s of a black hole in dependence on the number of bits q thrown into it. We find that q=N=n+1. As examples we pick the situation with a radius slightly bigger than 1.5 (whatever unit). We obtain maximum volume for a sphere in 15 dimensions (orange dotted line). Picking a radius slightly below 1, however, gives us a 6-dimensional sphere which can have maximum volume at such a size (green dotted line).

We find a perfect fit (s. figure A1) to the r_{max}[N]-dependency for q=N with the following function:

$$r_s = U \cdot \left(0.014948 + 0.3951244 \cdot \sqrt{q}\right); \quad U^2 = 8 \cdot \pi \cdot \ell_P^2, \tag{58}$$

where U denotes a unit-factor which was set U=1 in figure A1.

Our finding does not only connect the intrinsic dimension of a black hole with its mass respectively its surface, but also, at least partially, gives an explanation to the hitherto unsolved problem of "what are the micro states of a black hole giving it temperature and allowing it to store information". According to the evaluation in this section, these microstates are just various states of dimensions realized within the black hole in dependence on the number of bits it contains (and thus, its mass). The bigger the number of bits, the higher the intrinsic dimensions the black hole has. In fact, the connection even is a direct one and only seems to deviate from the simple direct proportionality for very low numbers of masses[1], respectively Schwarzschild radii r_s, respectively numbers of bits q the black hole has swallowed.

This finding also gives us a direct connection between a principle mathematical law (the maximum volume as function of the dimension for a given radius of an n-sphere) to the number of bits a black

[1] Besides, this deviation is also suggested by the Bekenstein finding summed up in equation (54), where we could assume the second term to become of importance at lower numbers of r_s.

hole contains, to the mass or Schwarzschild radius of this very black hole and the number and character of microstates the black hole actually uses to internally code the bits.

It has to be pointed out that the expression "intrinsic dimension" truly stands for the part of space for $r<r_s$, which is to say, the space behind the event horizon. As for the outside, the solution of a Schwarzschild object in n+1-dimensional space-times is given via:

$$g_{\alpha\beta}^{N} = \begin{pmatrix} -c^2 \cdot f[r] & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{f[r]} & 0 & 0 & \cdots & 0 \\ 0 & 0 & r^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2 \varphi_1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & g_{aa} \end{pmatrix}$$

$$g_{44} = r^2 \cdot \sin^2 \varphi_1 \cdot \sin^2 \varphi_2; \quad g_{aa} = r^2 \cdot \prod_{j=1}^{a-2} \sin^2 \varphi_j; \quad a = N-1 = n; \cdot \tag{59}$$

$$f[r] = 1 - \frac{r_s^{N-3}}{r^{N-3}} = 1 - \frac{r_s^{n-2}}{r^{n-2}}$$

As we find that the Newton laws of gravity, however, require N=4, it has to be assumed that in a region near the event horizon the dimension of the black hole decreases to the known 4 dimensions so that Newton's laws of gravity are properly mirrored to us (as outside observers). The corresponding derivation was given in [8] (chapter 11).

A System-Immanent Scale

Note: the correct solution for the evaluation of the Schwarzschild radius r_s as function of the bits thrown into a black hole object (if using the results from [15]) would be:

$$r_s = 2 \cdot \ell_P \cdot \sqrt{\pi \cdot \left(q + \sqrt{q \cdot (1+q)}\right)}\ ! \tag{60}$$

We find a perfect fit to the n-spheres with maximized volume to a given radius (dots in figure A1) with a Planck length of $\ell_P = 0.07881256452824544$ (s. figure A2, which is almost perfectly equal to the fit in figure A1).

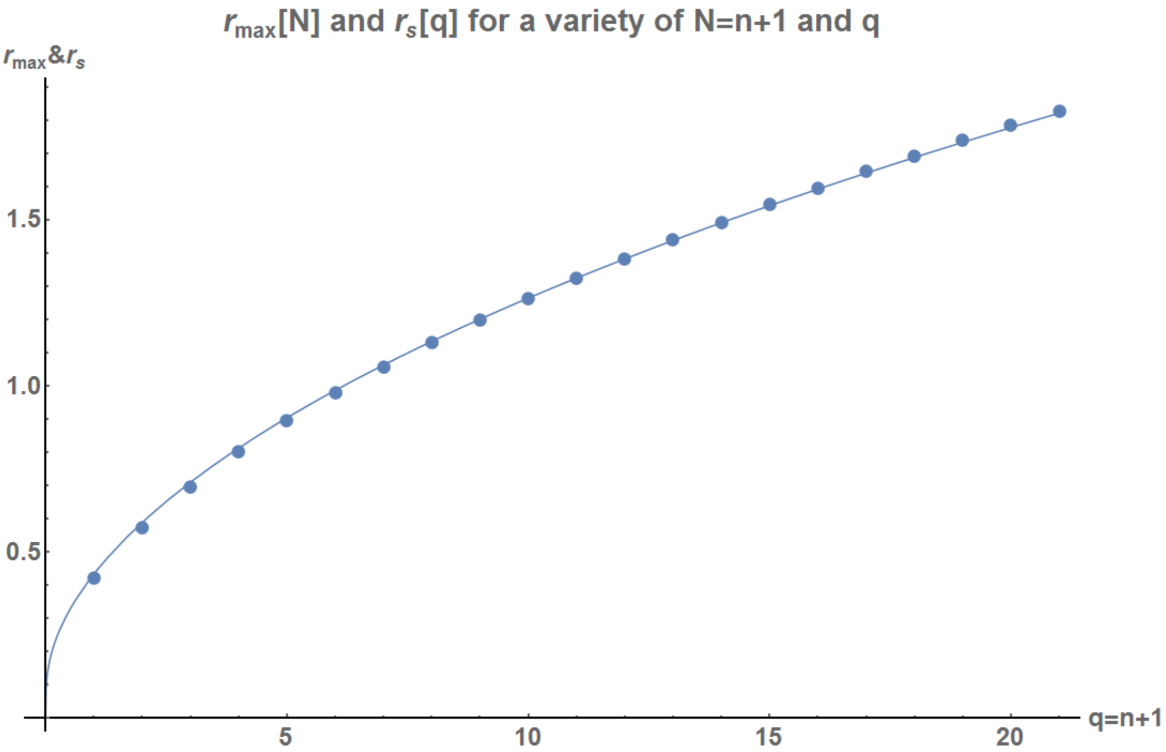

Fig. A2 (Please note that we have applied a slightly different fit than it was applied in fig. A1): Radius r_{max} for which at a certain number of dimensions the n-sphere has maximum volume in dependence on N=n+1 compared with the increase of the Schwarzschild radius r_s of a black hole in dependence on the number of bits q thrown into it by using (59). We find that q=N=n+1.

But what would be the unit for this Planck length?

Well, it was already shown in [8] that by using the results from [15] and the "volume integral" (52) with:

$$\delta_n W = 0 = \delta_n \int_V d^n x \sqrt{-g} \, , \tag{61}$$

for n-spheres (in [15] with T[n]=1):

$$V_N = V_{n+1} = T[n] \cdot \frac{\pi^{\frac{(n)}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot r^n \, , \tag{62}$$

we have evaluated the very dimension n to which at a given radius r_s the volume of a n-sphere has a maximum. The r_s are put into the calculation as plain "natural" numbers, meaning an r_s=1 is just a 1 and that is it. We might name this unit a "mathematical meter" or just "mams" (plural for "mathematical meters"). Transformation to our usual units, like meters, requires the introduction of a factor T[n]=U^n.

With $U = \frac{[\ell_P]_{in\_meters}}{[\ell_P]_{in\_mams}} = \frac{1.616255(18) \times 10^{-35} \text{ meters}}{0.07881256452824544}$, for instance, we can easily change to our meters. Nevertheless it appears somehow astonishing that there seems to exist a fundamental "natural" unit, being completely based on a mathematical - geometrical - extremal principle (the maximum volume of n-spheres as functions of their dimensions for certain radii). It is also interesting that these dimensions are so nicely correlated to the number of bits a black hole has swallowed. In fact, using the unit of mams, the number of spatial (n-sphere) dimensions is perfectly equal to the number of swallowed bits.

Thus, in the case that black holes would in fact store their content as dimensions and the Einstein-Hilbert-Action being extended with respect to the number of dimensions in addition to the metric, we immediately also get an absolute scale for our black hole system in which the number 1 is “made out” of 12.6883 Planck length and where a 3-sphere has a radius of 0.6969979737167096 mams.

Back to the Optimum Size Question for any System

When observing the integral in (52), we see that – in principle – we seek for a maximum volume for a given dimension or, taking the radius of a Schwarzschild object, look for the corresponding dimension making the volume integral an extremum. As the determinant g of the Schwarzschild metric is just equal to the one of a n-sphere with the additional time-dimension to be integrated, we can easily use the volume integral result of n-spheres, which reads:

$$V_N = V_{n+1} = \frac{\pi^{\frac{(n)}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot r^n . \tag{63}$$

Please note that due to the time-coordinate t we have V_{n+1} instead of V_n. Thereby the integration via t in (62) is assumed to be performed such that it would give 1. In general, we might take care about this part of the integration via a proportional constant T we could even consider to be n-dependent T[n] and thus, equation (61):

$$V_N = V_{n+1} = \overbrace{T[n]}^{=U^n} \cdot \frac{\pi^{\frac{(n)}{2}}}{\Gamma\left[\frac{(n+2)}{2}\right]} \cdot r^n . \tag{64}$$

Now we evaluate the various dimensions for which, for a given radius r of the n-sphere, we would obtain extrema. The results were already given in figures A1 and A2. There we have illustrated the resulting r_{max} as functions of the dimensions N=n+1 (note: n=n-sphere dimension, N=t+n-sphere dimension).

Now we just compare our findings with the original question of extracting a minimum principle for the dimensional size of a given system with our generalized starting point for the variational task (52) and conclude that:

$$\begin{aligned} &\delta_? W = 0 = \delta_? \int_V d^n x \left(\sqrt{-g} \cdot \Phi_R[R]\right) \\ &\Rightarrow \delta_n W = 0 = \delta_n \int_V d^n x \left(\sqrt{-g} \cdot \left[\Phi_R[R] = 1\right]\right) = \delta_n \int_V d^n x \sqrt{-g} \end{aligned} . \tag{65}$$

Thus, the determination of the optimum size of a system we intend to consider, investigate or analyze can just be found by a dimensional variation of the volume integral of that very system. In the case of spherical symmetries, this then leads to equations of the form:

$$\Rightarrow \delta_{n} W = 0 = \delta_{n} \int_{V} d^{n}x \left(\sqrt{-g} \cdot \left[\Phi_{R}[R] = 1 \right] \right) = \delta_{n} T[n] \cdot \frac{\pi^{\frac{(n)}{2}}}{\Gamma\left[\frac{(n+2)}{2} \right]} \cdot r^{n}. \tag{66}$$

Along the way we also can extract suitable fundamental scales for our system.

The First Bit Requires the Highest Mass = The First Thought is the Most Difficult

From (59) we can now extract the minimum Schwarzschild radius for the storage of one bit, which would be equal to 5.508 Planck length and corresponds to 11.16 times the Planck mass. This is a huge amount of mass and thus, also energy, one needs to safely store just one single bit. Luckily, the situation improves the more bits one intends to store, as for instance the one millionth bit only requires about $5.5*10^{-3}$ Planck masses. Please note that, of course, one might also store bits within spin arrangements of electrons. Then a 1-bit information would be connected with a single electron, whose mass and spin energy is many magnitudes below the Planck mass. This spin storage, however, cannot be seen as a storage of a classical binary bit, because in fact it resembles a quantum bit. Apparently, the safe storage of a pure and truly binary information requires an - almost - macroscopic massive structure. Here the black hole probably provides the smallest possible mass ensemble there can be to arrange such a storage for a certain bit. The limit is given at about 11 times the Planck mass. Only from this mass onward black holes can store binary information… at least until the Hawking radiation leads to a destruction of our black hole 1-bit storage system.

And what would then happen to the stored information? Well, this brings us to the question how safe is information within our universe [16].

Byproducts: A few Fundamental Questions

About the Relativity of System-Scales

We saw that – similar to the Bekenstein or Bekenstein-Hawking problem (see reference [17]) – we add bits via dimensions to our metric structure (here a black hole). Assuming our metric system to be a black hole (which we here only use to have a simple as possible math), we can even obtain a ratio of the black hole's radius r_s to the smallest structure this black hole can resolve. Taking the result for ℓ_P - the Planck length - from the Bekenstein thought experiment, we find the ratio between the Schwarzschild radius r_s of a black hole and ℓ_P:

$$\frac{r_s}{\ell_P} = 2 \cdot \sqrt{\pi \cdot \left(q + \sqrt{q \cdot (1+q)} \right)}. \tag{67}$$

In our universe the Planck length ℓ_P is considered to be the smallest length possible to resolve. What if the ratio (66), we found for black holes, actually is a more fundamental law? At any rate, it appears logic to assume that more bits could be coded or stored by an object of bigger size and smaller internal structure, thereby leaving more options to describe something with these structures. Thus, the equation (66) makes intuitive sense, but could it also be just the other way round? Could it be that to an object of given size the number of bits (being equivalent to its dimensions as we see in figures A1, A2) it contains, determines the smallest scale – the Planck length – of the object, too? And, if referring to the "Mathematical Psychology", the system presents a thinking entity, does this also mean that thoughts have a physical scale?

From inside and taking the Planck length as measure, the increase of information to this very object, subject or entity would look like an increase of its size. Now assuming the inside of the black hole to be a general system, the inhabitants of this system may see this system as their very own universe and would register the increase of information as a growth of their "universe", measured in the Planck length of that very system-universe. When learning, we seem to feel the increase of mind. May be this perception is just what is actually really going on.

Does More Information Always Mean More Mass?

Quantum computer scientists have already pointed out that, with our current way of storing information, we will one day reach a limit with respect to the number of atoms we can apply for the storing process and the energy being needed to keep the information stable (maintained). Citing from the abstract of [18], we have the following situation:

"Currently, we produce ~10^{21} digital bits of information annually on Earth. Assuming a 20% annual growth rate, we estimate that after ~350 years from now, the number of bits produced will exceed the number of all atoms on Earth, ~10^{50}. After ~300 years, the power required to sustain this digital production will exceed 18.5 × 10^{15} W, i.e., the total planetary power consumption today, and after ~500 years from now, the digital content will account for more than half Earth's mass, according to the mass-energy–information equivalence principle. Besides the existing global challenges such as climate, environment, population, food, health, energy, and security, our estimates point to another singular event for our planet, called information catastrophe."

It has to be pointed out that when looking for possible inner Schwarzschild solutions [19], we also found that there are solutions, where the mass decreases with the increase of the object size. It may well be that such strange states are not only realized in black holes, but could perhaps also help to overcome our future information storage problem.

Generalization to General Spheres?

In the sub-sections above we saw that, when taking the equation for the Laplace length ℓ_P from the Bekenstein thought experiment [3, 4]), we find the following ratio between r_s (Schwarzschild radius of a black hole) and ℓ_P (c.f. equation (66)):

$$\frac{r_s}{\ell_P} = 2 \cdot \sqrt{\pi \cdot \left(q + \sqrt{q \cdot (1+q)}\right)}. \tag{68}$$

Most interestingly, we also found that the solution to the extremal volume problem for a fixed radius r_f for n-spheres results in the same dependency when variating with respect to the number of dimensions of those n-spheres. We obtain (see dots in figures A1 and A2) excellent fits, when applying an approach like:

$$r_f = L \cdot \sqrt{\pi \cdot \left(n + \sqrt{n \cdot (1+n)}\right)}. \tag{69}$$

Thereby we have the characteristic (system-dependent) length scale L.

This automatically gives us a connection between the size-parameter r_f of any system of spherical symmetry and its theoretical capability to store information. A perfect mathematical n-sphere thereby follows the rule (68) almost perfectly, while other systems may do so only from certain critical sizes onwards, but, nevertheless, we think we can draw the conclusion that the information storage capacity of given systems – if showing enough spherical symmetry – can be extracted from (68). Then the structural size-parameter r_f determines the number of storable bits n in dependence on the system-immanent length parameter L.

From this, one even may deduce that r_f and L could be substituted by other system characteristics. While in (68) their dimension is length, we should not exclude mass, time, charges, energies and so on.

Other Geometries

The simplest generalization of (64) can be given for an ensemble of N tori of dimensions n_j for the sub-n_j-spheres of the individual torus. The volume integral would then yield:

$$W=\int_V d^n x\left(\sqrt{-g}\right)=\prod_{j=1}^{n} V_j\left(n_j,r_j\right)=\prod_{j=1}^{n} T\left[n_j\right]\cdot\frac{\pi^{\frac{n_j}{2}}}{\Gamma\left[\frac{\left(n_j+2\right)}{2}\right]}\cdot\left(r_j\right)^{n_j}. \tag{70}$$

This could be further generalized for a sum of tori and leaves us with a great variety of pure volume (radii) and dimension variations.

As tori can be seen as combined n_{ij}-spheres with n_{ij} giving the dimension of the sub-spheres constructing each torus (64) can be generalized as follows when assuming N tori with dimensions n_i and sub-spheres of dimensions n_{ij}:

$$W=\sum_{i=1}^{N} W_i=\sum_{i=1}^{N}\int_{V_i} d^{n_i} x\left(\sqrt{-g}\right)=\sum_{i=1}^{N}\prod_{j=1}^{n_{ij}} T\left[n_{ij}\right]\cdot\frac{\pi^{\frac{\left(n_{ij}\right)}{2}}}{\Gamma\left[\frac{\left(n_{ij}+2\right)}{2}\right]}\cdot\left(r_{ij}\right)^{n_{ij}}. \tag{71}$$

Assuming that also complex symmetries of systems could be constructed out of sums of tori, we realize that the variational options are manifold:

$$\delta W=0=\delta\left(n_{ij},r_{ij}\right)\sum_{i=1}^{N}\prod_{j=1}^{n_{ij}} T\left[n_{ij}\right]\cdot\frac{\pi^{\frac{\left(n_{ij}\right)}{2}}}{\Gamma\left[\frac{\left(n_{ij}+2\right)}{2}\right]}\cdot\left(r_{ij}\right)^{n_{ij}} \tag{72}$$

and leave us with a great variety of options for an optimum sized system in the case of complex symmetries.

Consequences from the Bekenstein Thought Experiment Regarding the Solutions to the Quantum-Einstein-Field-Equations

In [2, 8] we have shown that the classical n-dimensional Schwarzschild solution could be applied to construct internally structured n-dimensionally black holes, while outside we still have the usual 4-dimensional solution from [14] with the classical Schwarzschild metric. This, however, would not explain how the black hole can code any information.

With the help of the new metric solutions evaluated in [19], namely, to just give an example, in the three-dimensional case with coordinates t, r and an angle:

$$g_{\alpha\beta} = C_1 \cdot f[t]^4 \cdot \begin{pmatrix} -c^2 & 0 & 0 \\ 0 & t^2 & 0 \\ 0 & 0 & t^2 \cdot \sin(r)^2 \end{pmatrix}, \qquad (73)$$
$$f[t] = \sqrt{t^{\pm i \cdot c - 1}} \cdot C_f$$

(please note that r has become an angle while t took over the position of the radius),

$$g_{\alpha\beta} = C_1 \cdot f[t]^4 \cdot \begin{pmatrix} -c^2 & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^2 \cdot \sin(r)^2 \end{pmatrix} \qquad (74)$$
$$f[t] = e^{\pm \frac{i \cdot c}{2 \cdot \rho} \cdot t} \cdot C_f$$

(this represents a shell-like object) we want to solve also this problem. A generalization of this type solution is been given in appendix N. Thereby, we found that the Schwarzschild singularity could be avoided (fig. A3).

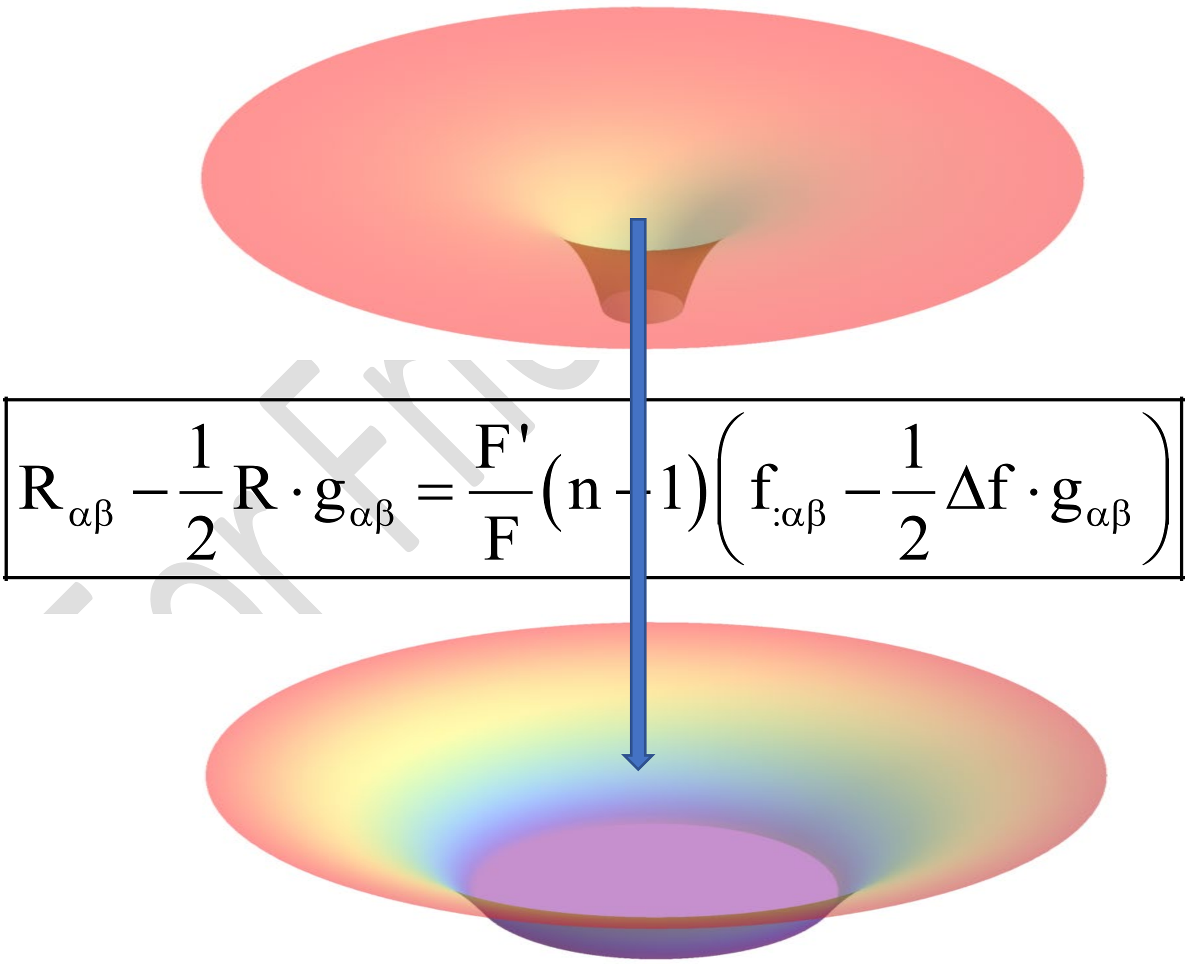

Fig. A3: "From the classical Schwarzschild solution to a Quantum Black Hole" [19]

At first, however, we should note that also the n-dimensional Schwarzschild solutions from [8], section 3.8 (c.f. solution (58) in here) would provide plenty of options to code information, because there are enough degrees of freedom regarding the thicknesses of the individual x-dimensional layers of the onion-like Schwarzschild object, which was proposed there (see also [19]). Similar assumptions could be made for the Robertson-Walker approach introduced in [19], but apart from again mentioning the onion-layer structured ogre-mind from the movie Schreck, we will not further consider these possibilities in here.

In the case of photonic inner solutions as also suggested in [19] one might assume some kind of standing waves inside the black hole, but as we currently don't have the math to realize such structures, we postpone the investigation of this possibility.

Thus, we here concentrate on solutions (72), (73) as potential inner solutions to a black hole. As we see that the parameter ρ clearly is a length, we want to derive its properties. For the general case this was already done in appendix N. Nevertheless, we repeat it here for the setting (72) and (73). From basic quantum theory we know that a particle at rest has the time dependency:

$$f[t]=e^{\pm i\cdot\frac{m\cdot c^2}{\hbar}\cdot t}\cdot C_f, \tag{75}$$

with m giving the rest mass of the particle and $\hbar$ denoting the reduced Planck constant. Comparing with the f[t]-function from the metric solution (73), we find:

$$\frac{m\cdot c}{\hbar}=\frac{1}{2\cdot\rho}. \tag{76}$$

Inserting the Schwarzschild radius $r_s=\frac{2\cdot m\cdot G}{c^2}$ (G… Newton's constant, c… speed of light in vacuum), thereby substituting the rest mass m, leaves us with:

$$\frac{r_s\cdot c^3}{2\cdot\hbar\cdot G}=\frac{1}{2\cdot\rho}\quad\Rightarrow\quad\rho=\frac{1}{r_s}\cdot\left(\frac{c^3}{\hbar\cdot G}\right)^{-1}=\frac{\ell_P^2}{r_s}. \tag{77}$$

Here ℓ_P denotes the Planck length. By inserting (66) into (76) we obtain:

$$\rho=\frac{\ell_P^2}{r_s}=\frac{\ell_P^2}{2\cdot\ell_P\cdot\sqrt{\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}}=\frac{\ell_P}{2\cdot\sqrt{\pi\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}}. \tag{78}$$

Thus, while for a black hole the number of bits thrown into it leads to an almost perfectly square-root-like increase of the Schwarzschild radius in accordance with equation (66), the ρ-parameter of the (73)-objects decreases with the number of bits. (73)-objects would have the same ρ-parameter, which we may see as a size, as a black hole only for Schwarzschild radii r_s equal to the Planck length. In other words, for growing black holes with radii bigger than the Planck length the corresponding equally heavy (73)-objects would be significantly smaller than the black holes.

So, we ask: Could the (73)-objects be used as building blocks for the black holes, residing inside it, which is to say behind the event horizon?

Assuming that the black hole's surface is made out of metric spherical objects of the type (73) and further assuming that each of these objects in the surface of the black hole, which is to say at $r=r_s$ (which also happens to be the event horizon), requires its own surface space of something like $C_\rho*\rho^2$, we can directly evaluate the number of such (73)-objects, we from now on name ρ-spheres, are residing inside the event horizon with increasing numbers of bits thrown into the black hole. Assuming that the mass is always additive, the total mass m of the black hole must then be distributed among the N ρ-spheres, which changes (76) to:

$$\frac{r_s \cdot c^3}{2\cdot\hbar\cdot G}=\frac{1}{2\cdot N\cdot\rho}\quad\Rightarrow\quad N\cdot\rho=\frac{1}{r_s}\cdot\left(\frac{c^3}{\hbar\cdot G}\right)^{-1}=\frac{\ell_P^2}{r_s}\quad\Rightarrow\quad \rho=\frac{\ell_P^2}{N\cdot r_s}. \tag{79}$$

Also having to satisfy the following equation for the N ρ-spheres sitting on the surface, we have to solve the following equation:

$$\begin{gathered}N\cdot C_\rho\cdot\rho^2=4\cdot\pi\cdot r_s^2\\ \Rightarrow C_\rho\cdot\frac{\ell_P^2}{4\cdot\pi\cdot N\cdot\left(q+\sqrt{q\cdot(1+q)}\right)}=(4\cdot\pi)^2\cdot\ell_P^2\cdot\left(q+\sqrt{q\cdot(1+q)}\right).\\ \Rightarrow N=\frac{1}{64\cdot\pi^3\cdot\left(q+\sqrt{q(1+q)}\right)^2}\end{gathered} \tag{80}$$

Wie realize, that such a structure could not be used to store any information, because the number of ρ-spheres should have to increase with the number of bits and not decrease as it does. Things are improving the moment we allow a combination of ρ-spheres and (72)-objects (the latter we shall call t-spheres) to make up our inner black hole. We propose the following (simplest of the many possibilities) structure:

A) In the center of the black hole sits a ρ-spheres of "radius-parameter" ρ given in (76) and thus, $\rho=\frac{\ell_P^2}{r_s}$, which is to say, the bigger the Schwarzschild radius r_s of the black hole, the smaller its core. In fact, for infinite masses the core would become a singularity.
B) This single ρ-sphere core is surrounded by t-spheres (72) and the number of those t-spheres, which a black hole can bind, is proportional to the number of bits the black hole has swallowed.
C) Taking the Bekenstein-condition, this demands an average size for the t-spheres, being bound by the black hole or the black hole's surface, to be such that its projected surface would be equal to ℓ_P^2. In other words, we could assume the average radius of the t-spheres (the ones bound to the black hole) to be equal to $\ell_P/\sqrt{\pi}$.

With such a structure, it is very well possible that in fact black holes have no singularity and follow our scheme of inner-outer-solution, but one cannot detect any difference to the classical Schwarzschild solution from the outside, because the inner-parts are always hidden behind the event horizon.

But does this help us to solve the Bekenstein information problem?

Yes, it does.

We can imagine many t-sphere objects (of number N=q) sitting on the surface of the black hole. As the generalized solution to (72) would read:

$$\begin{gathered}g_{\alpha\beta}=C_1\cdot f[t]^4\cdot\begin{pmatrix}-c^2 & 0 & 0\\ 0 & A^2\cdot t^2 & 0\\ 0 & 0 & B^2\cdot t^2\cdot\sin(r)^2\end{pmatrix},\\ f[t]=\sqrt{t^{\pm\frac{i\cdot c}{A}-1}}\cdot C_f\end{gathered} \tag{81}$$

we see that each t-sphere could not only store information via a certain sign within the exponent, but also via the free parameter B.

The Size of the Electron?

Applying (78) and assuming a ρ-sphere-structure for the electron, gives us:

$$\frac{r_s \cdot c^3}{2 \cdot \hbar \cdot G} = \frac{1}{2 \cdot N \cdot \rho} \quad \Rightarrow \quad N \cdot \rho = \frac{1}{r_s} \cdot \left(\frac{c^3}{\hbar \cdot G} \right)^{-1} = \frac{\ell_P^2}{r_s} \quad . \quad \Rightarrow \quad \rho = \frac{\ell_P^2}{N \cdot r_s} = \frac{1.93 \times 10^{-13}}{N} \text{meter} \tag{82}$$

Setting N=1 we would end up with a ρ-sphere of $\rho = 1.93 \times 10^{-13}$ meter for the “pure” or “naked” electron.

Global References

[1] W. Pauli, "Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren" (1925), Zeitschrift für Physik 31: 765–783, Bibcode:1925ZPhy...31..765P. doi:10.1007/BF02980631

[2] N. Schwarzer, “Mathematical Psychology – The World of Thoughts as a Quantum Space-Time with a Gravitational Core”, Jenny Stanford Publishing, ISBN: 9789815129274

[3] J. D. Bekenstein, “Black holes and entropy”, Phys. Rev. D 7:2333-2346 (1973)

[4] J. D. Bekenstein, “*Information in the Holographic Universe*”, Scientific American, Volume 289, Number 2, August 2003, p. 61

[5] N. Schwarzer, “Fluid Universe – The Way of Structured Water; Mathematical Foundation” , 2025, a mathematical foundations book

[6] W. Wismann, D. Martin, N. Schwarzer, “Creation, Separation and the Mind…”, 2024, RASA strategy book, ISBN 979-8-218-44483-9

[7] N. Schwarzer, "The World Formula: A Late Recognition of David Hilbert ‘s Stroke of Genius", Jenny Stanford Publishing, ISBN: 9789814877206

[8] N. Schwarzer: “The Math of Body, Soul and the Universe”, Jenny Stanford Publishing, ISBN 9789814968249

[9] N. Schwarzer, „Einstein had it… Part XXXV: The 2-Body Problem and the GTR-Origin of Mass”, Self-published, Amazon Digital Services, 2018, Kindle

[10] N. Schwarzer, “Science Riddles – Riddle No. 11: What is Mass?”, Self-published, Amazon Digital Services, 2021, Kindle, ASIN: B07SSF1DFP

[11] N. Schwarzer, “THE PSA-PAPER - The Fundamental Theory”, Feb. 2024, classified RASA strategy publication

[12] N. Schwarzer, “The Quantum Black Hole”, Part 7b of “Medical Socio-Economic Quantum Gravity”, Self-published, Amazon Digital Services, March 2021, Kindle

[13] P. A. M. Dirac, “The Quantum Theory of the Electron”, Published 1 February 1928.DOI: 10.1098/rspa.1928.0023

[14] K. Schwarzschild, "Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der Einsteinschen Theorie" [“On the gravitational field of a ball of incompressible fluid following Einstein's theory”] , (1916), Sitzungsberichte der Königlich-Preussischen Akademie der Wissenschaften (in German), Berlin: 424–434

[15] N. Schwarzer, “Science Riddles – Riddle No. 8: Could the Schwarzschild Metric Contain its own Quantum Solution?”, Self-published, Amazon Digital Services, 2018, Kindle

[16] L. Susskind, The Black Hole War: “My battle with Stephen Hawking to make the world safe for quantum mechanics”, Little, Brown 2008

[17] N. Schwarzer, “Is there an ultimate, truly fundamental and universal Computer Machine?”, Self-published, Amazon Digital Services, 2019, Kindle

[18] M. M. Vopsen, “The information catastrophe”, AIP Advances 10, 085014 (2020); https://doi.org/10.1063/5.0019941

[19] N. Schwarzer, “The Quantum Black Hole”, Part 7b of “Medical Socio-Economic Quantum Gravity”, Self-published, Amazon Digital Services, March 2021, Kindle

[20] N. Schwarzer, „The Photon“, www.amazon.com/dp/B06XGC4NDM

[21] N. Schwarzer, “The Theory of Everything - Quantum and Relativity is everywhere – A Fermat Universe”, Pan Stanford Publishing, (2020), ISBN-10: 9814774472