Infinite Orthogonality
Why you cannot read this text twice in the same way

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1 Abstract

In this brief paper we are going to discuss the question of additivity of solutions to quantum gravity field equations and their non-linearity. This appears as an antagonism. The concept of "infinite orthogonality"[1] is been investigated as a way to sort the contradiction out and in fact, we find that in systems with significantly high numbers of dimensions, simplifications occur which force the field equations into linear structures. This, however, does not automatically lead to an omnipresent infinity.

2 Questions

Can the simple everyday practical experience that there is just no way that we go through the same river twice, being anything but the somewhat casual formulation for the fact that in systems with huge numbers of degrees of freedom or high dimensionality, certain constellations statistically are impossible to be repeated in exactly the same way, be more than only a statistical effect?

In our macroscopic realm, we cannot repeat an experiment, a process, a happening. Something is always different and be it the surrounding universe.

This insight, even though quite obvious for the bigger systems of our mundane world-experience, leads to a dilemma when decreasing the dimensional size of systems down to a number of dimensions where statistics cannot provide us with the safe haven we are used to apply in order to comfortably hide behind entropy and irreversibility of any moment in time. Of course, we still have the second law of thermodynamics, some might argue, but this is already covered by the statistic reasoning used above. So, one may throw in dissipative effects in order to always guarantee the all-moments-are-unique rule, but here, quantum theory tells us a different story. Atoms, after all, are stable objects and as we have shown that all objects in this universe are just waves and oscillations [A1, A2, A3] and therefore have to be of dynamic character, certain things have to be repetitive… or haven't they?

Question: Is there a limit for a system to be allowed to be repetitive and what are the parameters setting this limit?

Assuming that there is such a limit and that this limit's parameter, regarding the system's dimensionality, is n.

More questions:

What then is the critical number for n and could this have something to do with the Planck units?

Does the boundary for the Einstein-Hilbert action tell us something about the system's ability to repeat itself. What parameter is of importance there n (number of dimensions) or V (volume)?

[1] The expression was coined by Dr. David Martin

And, when quantum systems QS are of small size (V or n ?), is the process of observation / measurement then the coupling of this QS into the rest of the universe (a bigger system, including the observer) and is this what the elders thought is the collapse of the wave function?

Is the universe self-observing and is this the reason why it cannot repeat itself?

And consequently:

When does a system start to be self-observing?

2.1 References to "Questions"

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[A5] N. Schwarzer, "The World Formula: A Late Recognition of David Hilbert's Stroke of Genius", Jenny Stanford Publishing, 2020, ISBN: 9789814877206

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3 Superpositivity and Linearity

When we want that solutions of differential equations behave additive we have to restrict ourselves to linear differential equations or systems of such. Interestingly, neither the original Einstein-Field-Equations [1, 2] nor the Quantum Gravity Field Equations (e.g. [3 - 10]) are linear… at least not without some additional input and boundary conditions like the assumption of a "weak gravity" (e.g. [7, 10]).

Could it be that linearity can guarantied via orthogonality?

And if so, shouldn't then – due to the huge dimensionality of the universe – infinitely many orthogonal properties, attributes, degrees of freedom and so on exist in order to always disentangle any potential non-linearity. During the evolution of a system this sure will converge towards infinitely many orthogonal options, but in the opinion of this author only after an infinite time span or infinitely many universal operations. He even showed that it completely suffices to just have a sufficiently high dimensionality in order to already end up with Dirac like linearization options [10, 11].

4 What is Infinite Orthogonality then?

One of the authors of the quite fundamental book [12], Dr. David Martin, always insisted on the existence of an infinite number of options, not just in this universe, but within every system. He named this principle “the principle of infinite dimensionality”[2]. As an illustrative example Dr. Martin often uses the following constellation:

> *“The Observer-Observed entanglement BY DEFINITION gives rise to the very definition of “incapable of being correlated or covariance” as the extinguished observation moment means that all uncorrelated options simultaneously exist including the extinction of the very observation just made. Just like you cannot take a step into the same stream twice, so to one cannot make the same observation twice. The erasure of “new” and the persistence of a “memory function” means that the observation dyad – once observed – is statically orthogonal to itself!”*

Along the way, one automatically also gets a very small problem with the so-called covariance principle, but this was already discussed elsewhere [13].

The problem some – more clever – critiques, when not resorting to ad hominem arguments right from the start, had with this concept, was that a system (perhaps even this universe) may not have infinitely many attributes to actually produce the demanded infinite dimensionality. So, how can the principle be still fulfilled even in systems (space-times) with finite numbers of dimensions?

Well, at least when leaving orthogonality aside for the time being, the answer is surprisingly simple: linearity!

5 About “Infinite Orthogonality” and Linearity

The problem some – more clever – critiques, when not resorting to ad hominem arguments right from the start, had with the concept of the “Infinite Orthogonality”, was that a system (perhaps even this universe) may not have infinitely many attributes to actually produce the demanded infinite dimensionality. So how can the principle be still fulfilled even in systems (space-times) with finite numbers of dimensions.

Well, at least when leaving orthogonality aside for the time being, the answer is surprisingly simple: linearity!

Linearity allows the superposition of solutions to a system and as even with just two solutions S_1 and S_2 to a system one might always construct a superposition like:

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[2] In order to be fair and correct, one should mention that Dr. Martin at first referred to his principle by using the expression “infinite orthogonality” or “infinite orthogonal dimensionality”, but due to the fact that in the scientific community orthogonality is mainly seen as dimensional independency and thus, with reduction to this aspect, is not of need as any linear independent set of dimensions could be orthonormalized by the Gram-Schmidt process, the principle was reduced to the current “infinite dimensionality” by this author when just considering ideal systems. Nevertheless, orthogonality has its justification in the “linearity” or “superposable” principle because of:

A) We can easily show mathematically that any non-orthogonality is nothing other than entanglement. This allows us to assume a fundamental orthogonality with entanglements instead of working with non-orthogonalities.
B) There is a statistical orthogonality when ALWAYS and very fundamentally conspiring the whole (see derivation (49) to (52)).

$$S_{tot} = a \cdot S_1 + b \cdot S_2 \quad (1)$$

one ends up with an infinite number of possibilities when a and b are arbitrary… not being orthogonal to each other though.

And how does the statistic orthogonality comes into play? Which is to say, how is it assured that nobody goes twice through the same river or what assures that every linear combination occurs only once? Well, in order to put it mathematically we just have to extend (1) to its true and practical form, which reads:

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + c \cdot S_{Universe} . \quad (2)$$

Of course, $S_{Universe}$ actually means $S_{Rest\_of\_the\_universe}$, because the states S_1, S_2 are a part of it. Each and every practical linear combination of states S_1, S_2 exists in this universe, which in the moment the linear combination is formed shall exist in state $S_{Universe\_A}$. Resolving the linear combination and reforming it some time later would not only change the universe due to the process itself, but it also happens at a different state $S_{Universe\_B}$ even when the states S_1, S_2 and their formation would - for some funny reason - not influence the rest of the world. Hence, (2) – when realized for the first time - would have to be written:

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + c \cdot S_{Universe\_A} , \quad (3)$$

while its second realization leads to:

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + c \cdot S_{Universe\_B} . \quad (4)$$

As the two situations statistically and fundamentally exclude each other one has an orthogonality and the fact that there are infinitely many such combinations leads to our "orthogonal infinity" or "infinite orthogonality" principle.

5.1 Bringing in Time

When substituting the $S_{universe}$ term by time t:

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + t , \quad (5)$$

we might even conclude that in fact the principle of infinite orthogonality holds true even for systems without the universal rest as long as these systems have time, because we would then obtain two otherwise equal states as two different moments in time, which is to say:

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + t_A , \quad (6)$$

$$S_{tot} = a \cdot S_1 + b \cdot S_2 + t_B . \quad (7)$$

On the other hand, we may just take time as what it is been used here in our equations, namely a dimension summing up "the universal rest":

$$c \cdot S_{Universe} = t , \quad (8)$$

thereby making every constellation unique.

6 The Observer – Observed System

Considering every attribute/property/degree of freedom of a system as dimension, we realize that by taking the Einstein-Hilbert action [1] as a starting point:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot \left(R - 2\Lambda + L_M \right) \right), \tag{9}$$

the number of dimensions, which were treated as constants by Einstein and Hilbert, should

a) be considered variable and
b) should therefore be part of the variation process
c) take into account the observer's dimensionality as he, too, belongs to the system as a whole
d) take into account the universe the observer and what he observes lives in as we cannot per se exclude that there is no entanglement.

This usually leads to huge numbers of dimensions even for the simplest experimental set-ups and requires the consideration of field equations of special character as an outcome of (9) under the limit of n→∞.

We are going to investigate this aspect in a set of follow up publications [14, 15, 16, 17], for convenience we are going to present one example in the appendix of this paper.

7 Conclusions

We have seen that even the most simple setup for any experiment or observation in general requires the incorporation of many dimensions as the observer has to be seen as a part of this very experiment and the observer is part of an even bigger system (usually this is a whole universe) the dimensionality of our task is pretty high. However, there is no reason to assume that it is infinite.

8 Appendix

8.1 The Classical Hamilton Extremal Principle and how to obtain Einstein's General Theory of Relativity With Matter (!) and Quantum Theory… also With Matter (!)

The famous German mathematician David Hilbert [1], even though applying his technique only to derive the Einstein-Field-Equations for the General Theory of Relativity [2] in four dimensions, - in principle - extended the classical Hamilton principle to an arbitrary Rieman space-time with a very general variation by not only – as Hamilton and others had done – concentration on the evolution of the given problem or system in time, but with respect to all its dimensions. His formulation of the Hamilton extremal principle looked as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot \left(R - 2\Lambda + L_M \right) \right). \tag{10}$$

There we have the Ricci scalar of curvature R, the cosmological constant Λ, the Lagrange density of matter L_M and the determinant g of the metric tensor of the Rieman space-time $g_{\alpha\beta}$. For historical reasons, it should be mentioned that Hilbert's original work [1] did not contain the cosmological constant, because it was added later by Einstein in order to obtain a static universe, but this is not of any importance here. The evaluation of the so-called Einstein-Hilbert action (10) brought indeed the Einstein General Theora of Relativity [2], but it did not produce the other great theory physicists have found, which is the Quantum Theory. It was not before Schwarzer, about one hundred years after the publication of Hilbert's paper [1], extended Hilbert's approach by considering scaling factors to the metric tensor and showed that quantum theory already resides inside the sufficiently general General Theory of Relativity [3 - 10]. We will not discuss the reason why this simple idea has not been tried out by other scientists before, but we may still express our amazement about the fact that a simple extension of the type:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f], \tag{11}$$

solves one of the greatest problems in science[3], namely the unification of physics and that it took science more than 100 years to come up with the idea. Using the symbol G for the determinant of the scaled metric tensor $G_{\alpha\beta}$ from (11) of the Rieman space-time we can rewrite the Einstein-Hilbert action from (10) as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right) \cdot \tag{12}$$

could also be possible and still converges to the classical form for F→1. Here, which is to say in this paper, we will only consider examples with q=0, but for completeness and later investigation we shall mention that a comprehensive consideration of variational integrals for the cases of general q are to be found in [5].

Performing the variation in (12) with respect to the metric $G_{\alpha\beta}$ and remembering that the Ricci curvature of (e.g. [7] appendix D) changes the whole variation to:

$$\begin{aligned}
\delta W &= 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right) \\
&= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(\left(\begin{array}{c} \dfrac{R}{F} - \dfrac{1}{2F^2} \left(\begin{array}{l} (n-1)\left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}{g^{cd}}_{,c}} \right) \\ -nF_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) \\ -(n-1)\dfrac{g^{ab}F_{,a}\cdot F_{,b}}{4F^3}(n-6) \end{array} \right) - 2\Lambda + L_M \right) \right),
\end{aligned} \tag{13}$$

results in:

$$\begin{aligned}
0 &= \left({R^*}_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta\left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\begin{array}{c} \left(R_{\alpha\beta} - \dfrac{1}{2F} \left(\begin{array}{c} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b} \right) - \\ F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd} \left(\begin{array}{c} g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} \\ + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \end{array} \right) \end{array} \right) \\ + \dfrac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n) \right) \end{array} \right) \delta G^{\alpha\beta} \\ + \left(\dfrac{(n-1)}{2F} \left(\begin{array}{c} \left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}{g^{cd}}_{,c}} \right) \\ -\dfrac{n}{(n-1)} F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) + \dfrac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6) - \dfrac{R}{(n-1)} \right) \cdot \dfrac{g_{\alpha\beta}}{2} \end{array} \right),
\end{aligned} \tag{14}$$

when setting q=0 and assuming a vanishing cosmological constant. With a cosmological constant we have to write:

[3] This does not mean, of course, that we should not also look out for generalizations of the scaled metric and investigate those as we did in [5, 10].

$$0=\left(R^{*}{}_{\alpha\beta}-\frac{1}{2}R^{*}\cdot G_{\alpha\beta}\right)\overbrace{\left(\frac{1}{F}\cdot\delta g^{\alpha\beta}+g^{\alpha\beta}\cdot\delta\left(\frac{1}{F}\right)\right)}^{\delta G^{\alpha\beta}}$$

$$=\left(\begin{array}{c}
\boxed{\boxed{R_{\alpha\beta}-R\frac{g_{\alpha\beta}}{2}}}+\boxed{\Lambda\cdot g_{\alpha\beta}} \\
-\frac{1}{2F}\left(\begin{array}{c}
F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)- \\
F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}
g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha} \\
+\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}
\end{array}\right)
\end{array}\right) \\
+\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right) \\
+\left(\frac{(n-1)}{2F}\left(\begin{array}{c}
2\Delta F-2F_{,d}g^{cd}{}_{,c} \\
-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}
\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}(n-6)\right)\cdot\frac{g_{\alpha\beta}}{2}
\end{array}\right)\delta G^{\alpha\beta} \quad . \qquad (15)$$

For better recognition of the classical terms, we have reordered a bit and boxed the classical vacuum part of the Einstein-Field equations (double lines) and the cosmological constant term (single line). Everything else can be – no, represents (!) - matter or quantum effects or both.

Thus, we also – quite boldly – have set the matter density L_M equal to zero, because we see that already our simple metric scaling brings in quite some options for the construction of matter. It will be shown elsewhere [10] that there is much more which is based on the same technique.

8.2 The Principle of the Ever Jittering Fulcrum and The Alternate Hamilton Principle

We might bring forward three reasons why we could doubt the fundamentally of the Hamilton principle even in its most general form of the generalized Einstein-Hilbert action:

a) The principle was postulated and never fundamentally derived.
b) When rigidly demanding the extremal condition, the extremum should become an object being dependent on all coordinates. Some kind of position appears, which defines or rather “makes out” the extremum. Treating these position parameters as new attributes, the variation should be refined with respect to those and thus, the whole task increases its dimensionality. No matter how often one repeats the process, there is always an uncertainty about the final number of dimensions, in principle increasing towards infinity. This entangles with another of David Martin’s principle, namely the one of the “infinite orthogonality”, which we have investigated in this paper (see also [14 - 17]). Hence, the process is never truly complete and the result can never be a 100% - stable - extremum.
c) Even the formulation of this principle in its classical form (10) results in a variety of options where factors, constants, kernel adaptations etc. could be added, so that the rigid setting of the integral to zero offers some doubt in itself. A calculation process which offers a variety of add-ons and options should not contain such a dogma. The result should be kept open and general. One of the authors of [12] (Dr. David Martin) proposed this as the “tragedy of the jittering fulcrum” and we therefore named this principle David’s principle of the ever jittering fulcrum. It demands:

$$\begin{aligned}\delta_{g_{\alpha\beta}} W \simeq ? \simeq \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R \\ \delta_{G_{\alpha\beta}} W \simeq ? \simeq \delta_{G_{\alpha\beta}} \int_V d^n x \sqrt{-G} \times R^* \end{aligned} . \tag{16}$$

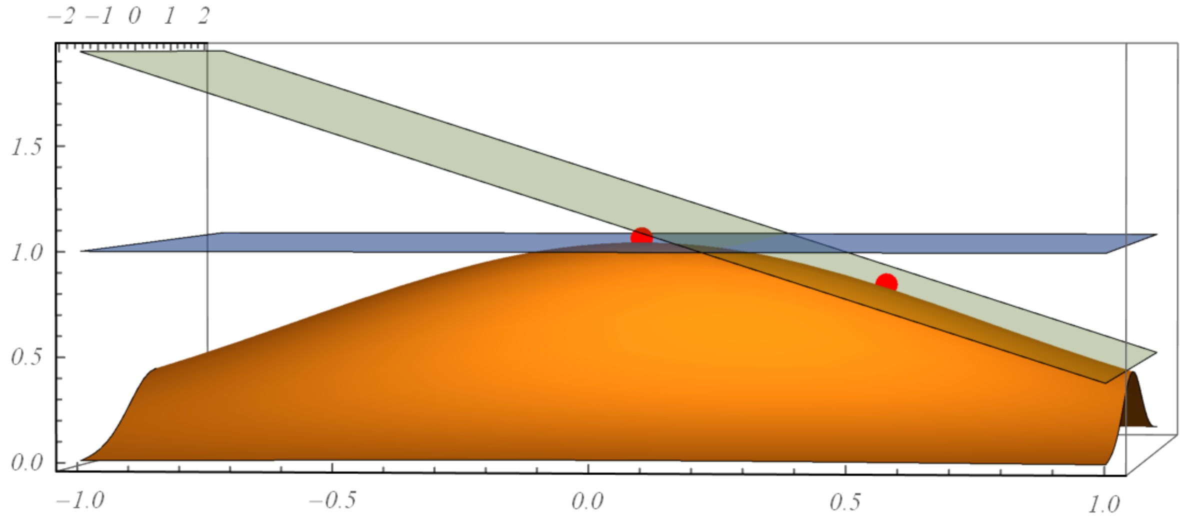

Fig. A1: David's principle of the Ever Jittering Fulcrum cannot accept a dogmatic insistence on a zero outcome of the Einstein-Hilbert action (10) or (generalized and also bringing about the Quantum Theory) (12). Instead it should allow for all states and not just the extremal position (see the two red dots and the corresponding tangent planes in the picture).

One of the simplest generalizations of the classical principle could be the linear one, which is illustrated in figure A1. It could be constructed as follows:

$$\int_V d^n x \sqrt{-g} \times \not{\lambda}^{\alpha\beta} \cdot g_{\alpha\beta} = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R \,. \tag{17}$$

Thereby we have used the classical form with the unscaled metric tensor, respectively without setting the factor apart from the rest of the metric. Performing of the variation on the right-hand side and setting

$$\not{\lambda}^{\alpha\beta} = H \cdot \delta g^{\alpha\beta} \tag{18}$$

or – for the reason of – maximum generality even:

$$\not{\lambda}^{\alpha\beta} = H^{\alpha\beta}_{ab} \cdot \delta\gamma^{ab} = H \cdot \delta g^{\alpha\beta} \tag{19}$$

just gives us the same result as we would obtain it when assuming a non-zero cosmologic constant, because evaluation yields:

$$\begin{aligned}\int_V d^n x \sqrt{-g} \times H \cdot \delta g^{\alpha\beta} \cdot g_{\alpha\beta} = \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 = \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned} , \tag{20}$$

respectively:

$$
\begin{aligned}
\int_V d^n x\sqrt{-g}\times H^{\alpha\beta}_{ab}\cdot\delta\gamma^{ab}\cdot g_{\alpha\beta} &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}\right)\delta g^{\alpha\beta} \\
\Rightarrow 0 &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}-Hg_{\alpha\beta}\right)\delta g^{\alpha\beta}
\end{aligned}
\qquad (21)
$$

Simply setting H=-Λ (c.f. single-line boxed term in equation (15)) demonstrates this.

Nothing else is the usage of a general functional term T, being considered a function of the coordinates of the system (perhaps even the metric tensor) in a general manner, as follows:

$$
\int_V d^n x\sqrt{-g}\times T=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,. \qquad (22)
$$

As before, performing of the variation on the right-hand side and setting

$$
T=T_{\alpha\beta}\cdot\delta g^{\alpha\beta} \qquad (23)
$$

gives us something which was classically postulated under the variational integral, namely the classical energy matter tensor. This time, however, it simply pops up as a result of David's principle of the jittering fulcrum and is equivalent to the introduction of the term L_M under the variational integral. Evaluation yields:

$$
\begin{aligned}
\int_V d^n x\sqrt{-g}\cdot T_{\alpha\beta}\cdot\delta g^{\alpha\beta} &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}\right)\delta g^{\alpha\beta} \\
\Rightarrow 0 &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}-T_{\alpha\beta}\right)\delta g^{\alpha\beta}
\end{aligned}
, \qquad (24)
$$

So, we see, that in introducing a cosmological constant and in postulating a matter term, even Einstein and Hilbert already – in principle - "experimented" with a non-extremal setting for the Hamilton extremal principle. And it is for this very reason that we only reluctantly used the expression "David's principle", but we think it is still fair and justified as it was Dr. Martin who first brought in the idea of the unstable fulcrum and combined it with the question of the orthogonality of events (see main part of this paper).

Apart from linear dependencies and other functions or functional terms, we could just assume a general outcome like:

$$
f(W)=f\left(\int_V d^n x\sqrt{-g}\times R\right)=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,\cdot \qquad (25)
$$

This, however, would not give us any substantial hint where to move on, respectively, which of the many possible paths to follow. We therefore here start our investigation with the assumption of an eigen result for the variation as follows:

$$
\xcancel{h}\cdot W=\xcancel{h}\cdot\int_V d^n x\sqrt{-g}\times R=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,. \qquad (26)
$$

This leads to:

$$
\int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda}-R\cdot\left(\frac{1}{2}\cdot g_{\kappa\lambda}\delta g^{\kappa\lambda}+\xcancel{h}\right)\right)=0 \qquad (27)
$$

As the term $\xcancel{h}$ could always be expanded into an expression like:

$$\not{h} = H \cdot g_{\kappa\lambda}\delta g^{\kappa\lambda} \tag{28}$$

we obtain from (27):

$$\begin{aligned} 0 &= \int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\delta g^{\kappa\lambda}\right) \\ &= \int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\right)\delta g^{\kappa\lambda} \\ &\Rightarrow R_{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda} = 0 \end{aligned} \tag{29}$$

We realize that the term H can be a general scalar even if we would demand the term $\not{h}$ to be a constant.

The complete equation when assuming a scaled metric tensor of the form (11) would read:

$$\left(\begin{array}{l} \left(\begin{array}{l} R_{\alpha\beta} - \dfrac{1}{2F}\left(\begin{array}{l} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} \\ +F_{,a}g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b}\right) - F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} \\ +F_{,d}g^{cd}\left(g_{\alpha c,\beta} - \dfrac{1}{2}ng_{\alpha c,\beta} - \dfrac{1}{2}ng_{\beta c,\alpha} + \dfrac{1}{2}ng_{\alpha\beta,c} + \dfrac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\right) \end{array}\right) \\ \qquad + \dfrac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right) \end{array}\right) \\ -\left(\begin{array}{l} R - \dfrac{1}{2F}\left((n-1)\left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}}\right) - nF_{,d}g^{cd}g^{ab}g_{ac,b}\right) \\ \qquad -(n-1)\dfrac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6) \end{array}\right)\cdot\left(\dfrac{1}{2}+H\right)g_{\alpha\beta} \end{array}\right) = 0 \tag{30}$$

and in the case of metrics with constant components this equation simplifies to:

$$\left(\begin{array}{l} \left(\begin{array}{l} R_{\alpha\beta} - \dfrac{1}{2F}\left(F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab}\right) \\ +\dfrac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right) \end{array}\right) \\ -\left(R - \dfrac{(n-1)}{2F}\left(2g^{ab}F_{,ab} + \dfrac{g^{ab}F_{,a}\cdot F_{,b}}{2F}(n-6)\right)\right)\cdot\left(\dfrac{1}{2}+H\right)g_{\alpha\beta} \end{array}\right) = 0 \cdot \tag{31}$$

8.2.1 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (16) correspond to more stable states than non-extremal solutions and in fact we will find this in connection with the 3-generation problem, which we have derived and discussed elsewhere [18].

9 Systems of High Dimensionality

In order to give an example about what happens in systems of high dimensionality, we pick the simple equation (31) and investigate the limit of n→∞:

$$
0=\lim_{n\to\infty}\frac{1}{n^{2}}\left(\begin{array}{c}\left(\begin{array}{c}R_{\alpha\beta}-\frac{1}{2F}\left(F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}\right)\\+\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\end{array}\right)\\-\left(R-\frac{(n-1)}{2F}\left(2g^{ab}F_{,ab}+\frac{g^{ab}F_{,a}\cdot F_{,b}}{2F}(n-6)\right)\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\end{array}\right)
$$

$$
=\lim_{n\to\infty}\frac{1}{n^{2}}\left(\begin{array}{c}R_{\alpha\beta}-R\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}+\frac{1}{2F}\left(F_{,\alpha\beta}-F_{,ab}g_{\alpha\beta}g^{ab}\right)\\+\frac{1}{2F}\left(3\frac{g^{ab}F_{,a}\cdot F_{,b}}{F}-2g^{ab}F_{,ab}\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\\+\frac{1}{2F^{2}}\left(2g_{\alpha\beta}F_{,c}F_{,d}g^{cd}-3F_{,\alpha}\cdot F_{,\beta}\right)\\+\frac{n}{2F}\left(\begin{array}{c}-F_{,\alpha\beta}+\frac{3F_{,\alpha}\cdot F_{,\beta}-g_{\alpha\beta}F_{,c}F_{,d}g^{cd}}{2F}\\+\left(2g^{ab}F_{,ab}-7\frac{g^{ab}F_{,a}\cdot F_{,b}}{2F}\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\end{array}\right)\\+n^{2}\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\end{array}\right)
$$

$$
=\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\quad. \qquad (32)
$$

We recognize the principle structure of the so-called Eikonal equation [11]. Thereby we have assumed that neither the Ricci terms nor the scaling function F depend on n. We will see that there are other options when investigating them in our follow up publications [14 - 17].

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