Do We Have a Theory of Everything?

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The Quantum Gravity Properties of a Scaled Metric Tensor

We start with the following scaled metric tensor (e.g. [1]):

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \tag{1}$$

and force it into the Einstein-Hilbert action [2] variational problem as follows:

$$\delta W = 0 = \delta \int_{V} d^{n} x \sqrt{-G} \cdot R^{*} \cdot$$
 (2)

Here G denotes the determinant of the metric tensor from (1) and R* gives the corresponding Ricci scalar. The complete variational task would then read (e.g. [3] appendix D):

$$\delta W = 0 = \delta \int_{V} d^{n}x \sqrt{-G} \cdot R^{*}$$

$$= \delta \int_{V} d^{n}x \left(\sqrt{-G} \cdot \left(\frac{R}{F} - \frac{1}{2F^{2}} \left((n-1) \left(2\Delta F - 2F_{,d} g^{cd}_{,c} \right) - nF_{,d} g^{cd} g^{ab} g_{ac,b} \right) - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^{3}} (n-6) \right) \right), \tag{3}$$

Performing the variation with respect to the metric $G_{\alpha\beta}$ results in:

$$0 = \begin{pmatrix} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta \alpha,b}) - \\ F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd}\begin{pmatrix} g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ + \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{pmatrix} + \frac{1}{4F^{2}}(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)) \\ + \left(\frac{(n-1)}{2F}\begin{pmatrix} 2\Delta F - 2F_{,d}g^{cd}_{,c} \\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} + \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^{2}}(n-6) - \frac{R}{(n-1)} \cdot \frac{g_{\alpha\beta}}{2} \end{pmatrix} \right)$$

A bit of reordering gives us:

$$0 = \begin{bmatrix} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta \alpha,b}) - \\ F_{,\alpha\beta}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd} \begin{bmatrix} g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ + \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{bmatrix} \right\} \delta G^{\alpha\beta}$$

$$+ \frac{1}{4F^{2}} \left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n) \right) + \left(\frac{(n-1)}{2F} \begin{pmatrix} 2\Delta F - 2F_{,d}g^{cd}_{,c} \\ - \frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} + \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^{2}}(n-6) \cdot \frac{g_{\alpha\beta}}{2} \right)$$

and shows us that we have not only obtained the classical Einstein Theory of Relativity [4] (see boxed terms exactly giving the Einstein Field Equations in vacuum), but also a set of field equations for the scaling function F.

It was shown in our previous publications [3, 5, 6, 7, 8, 9, 10] that these additional terms are clearly quantum equations fully covering the main aspects of relativistic classical quantum theory. We can briefly demonstrate this by just assuming weak gravity conditions of the following kind:

$$\delta G^{\alpha\beta} = G^{\alpha\beta} \cdot \delta_0 + \overbrace{G^{ab}}^{Gravity} \xrightarrow{\forall \delta_{ab}^{\alpha\beta} \ll \delta_0} = \frac{g^{\alpha\beta}}{F} \cdot \delta_0 , \qquad (6)$$

This reduces (5) to:

$$\begin{pmatrix}
\frac{R}{F} - \frac{1}{2F^{2}} \left((n-1) \left(2\Delta F - 2F_{,d} g^{cd}_{,c} \right) - nF_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\
- (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^{3}} (n-6)
\end{pmatrix} = 0,$$
(7)

where, demanding the condition:

$$0 = 4FF'' + (F')^{2}(n-6),$$
(8)

and satisfying it with the following wrapping approach for F=F[f] for the volume scaling of the metric tensor:

$$F[f] = \begin{cases} C_{F} \cdot (f + C_{f})^{\frac{4}{n-2}} & n-2 \neq 0, \\ C_{F} \cdot e^{f \cdot C_{f}} & n-2 = 0 \end{cases}$$
 (9)

we are able to completely linearize the field equations via:

$$0 = \left(R^*_{\alpha\beta} - \frac{1}{2}R^*G_{\alpha\beta}\right) \left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F}\right)\right)$$

$$\xrightarrow{\delta G^{\alpha\beta} = G^{\alpha\beta} \cdot \delta_0 + G^{ab} \delta_{ab}^{\alpha\beta}} \xrightarrow{\forall \delta_{ab}^{\alpha\beta} \in \delta_0} = \frac{g^{\alpha\beta}}{F} \cdot \delta_0$$

$$= \left(R^* - \frac{1}{2}R^*G_{\alpha\beta}\right) g^{\alpha\beta} \cdot \delta \left(\frac{1}{F}\right) = R^* \left(1 - \frac{n}{2}F\right) \cdot \delta \left(\frac{1}{F}\right) , \qquad (10)$$

$$= \left(R - \frac{F'}{2F} \left((n-1)\left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}\right)\right) - nf_{,d}g^{cd}g^{ab}g_{ac,b}\right) \cdot \left(1 - \frac{n}{2}\right)\delta \left(\frac{1}{F}\right)$$

$$- (n-1)\frac{g^{ab}f_{,a} \cdot f_{,b}}{4F^2} \left(4FF" + (F')^2(n-6)\right)$$

finally obtaining:

$$0 = R - \frac{F'}{2F} \Big((n-1) \Big(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \Big) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \Big).$$
 (11)

This equation is completely linear in f, which not only has the characteristics of a quantum function, but – for a change – gives us the opportunity to metrically see what QUANTUM actually means, namely, a volume jitter to the metric of the system in question... at least this is one quantum option, because we have already seen others (e.g. see [3, 5, 6, 7, 8, 9, 10]).

Interestingly, for metrics without shear elements like (which regards many metrics, like cartesian, spherical, elliptical, cylindrical and so on, used in standard quantum theory):

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-ln-l} \end{pmatrix}; \quad g_{ii,i} = 0,$$
 (12)

and applying the solution for F[f] from (9) the derivative terms in (11), which is to say:

$$(n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} .$$
 (13)

converge to the ordinary Laplace operator, namely:

$$R^{*} = 0 \rightarrow 0 = F \cdot R + F' \cdot (1-n) \cdot \Delta f$$

$$\Rightarrow 0 = \begin{cases} (f - C_{f})^{\frac{4}{n-2}} \cdot C_{F} \left(R + \frac{4}{n-2} \cdot \frac{(1-n)}{(f - C_{f})} \cdot \Delta f \right) & n > 2 \end{cases}$$

$$e^{C_{f} \cdot f} \cdot C_{F} \left(R + C_{f} \cdot (1-n) \cdot \Delta f \right) \quad n = 2$$

$$(14)$$

We recognize just the most important specially relativistic quantum equation, namely, the Klein-Gordon equation.

From there it only requires text book knowledge to obtain the Schrödinger and the Dirac equation in the usual way (e.g. see [3, 5, 6, 7, 8, 9, 10]).

As a side note, we need to point out that, in the case of n>2, we always also have the option for a constant (broken symmetry) solution of the kind:

$$0 = f - C_{f0} \quad \Rightarrow \quad f = C_{f0}. \tag{15}$$

In all other cases, meaning where $f \neq C_{f0}$, we have the simple equations:

$$0 = \begin{cases} \left(\mathbf{f} - C_{f0} \right) \cdot \mathbf{R} + \left(1 - \mathbf{n} \right) \cdot \frac{4}{\mathbf{n} - 2} \cdot \Delta \mathbf{f} & \mathbf{n} > 2 \\ \mathbf{R} + C_{f0} \cdot \left(1 - \mathbf{n} \right) \cdot \Delta \mathbf{f} & \mathbf{n} = 2 \end{cases}$$
 (16)

A critical argument should now be that this equation is not truly of Klein-Gordon character as it does not contain any potential nor mass, but the third author has already shown that this problem is easily solved by adding additional dimensions carrying the right properties to produce masses and potentials (e.g. [3, 5, 6, 7, 8, 9, 10]).

So, we conclude, that we indeed have a Quantum Gravity Theory or Theory of Everything, as one also calls it, at hand.

References

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