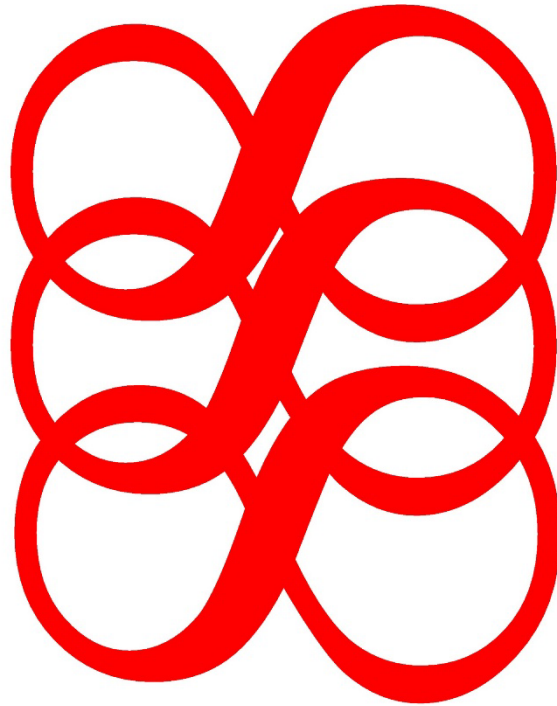


Infinite Orthogonal Dimensionality

**Part III: What Happens in a True
Dimensional Infinity?
The Answer is Shocking**



**by
Dr. Norbert Schwarzer**

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Infinite Orthogonal Dimensionality

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1 Abstract

In this paper we proceed with our discussion of the question of universal infinity and linearity.

This time we concentrate on the effect of a truly infinite dimensionality with respect to our quantum Einstein field equations.

We will see that something very strange is happening... something that forces us to consider a certain classical Quantum Theory, the Dirac theory, namely, in a much more general way.

Furthermore, the newly found insights allow us a glimpse behind the event horizon of the Planck level and the tools we developed along the way may even enable us to investigate what was before the beginning of time and what is behind the wonder of consciousness.

2 Introduction

It was shown in numerous of our previous publications [1–9] (also see appendix of this paper) that a scaled metric of the kind:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \quad (1)$$

if subjected to a generalized non-extremal Hilbert principle (c.f. appendix of this paper) via the Hilbert variational process [10] leads to the following quantum gravity field equations:

$$0 = \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \quad (2)$$

$$= \left(\begin{array}{c} R_{\alpha\beta} - \frac{1}{2F} \left(\begin{array}{c} F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} \\ + F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} \\ + F_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha \beta} g_{ab, c} g^{ab} \right) \end{array} \right) \\ + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \end{array} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ - \left(\begin{array}{c} R - \frac{1}{2F} \left((n-1) \left(\overbrace{2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab, c}}^{=2\Delta F - 2F_{,d} g^{cd, c}} \right) - n F_{,d} g^{cd} g^{ab} g_{ac, b} \right) \\ - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \end{array} \right) \end{array} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta}$$

Inserting a wrapping function $F[f]$ and a bit of reordering leads to:

$$\begin{aligned}
0 &= \left(R_{\alpha\beta}^* - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha \beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right) \right. \\
&\quad - \left(R - \frac{F'}{2F} \left(\begin{aligned} &(n-1) (2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- n f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\
&\quad + \frac{1}{4F^2} \left(\begin{aligned} &f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \\ &+ g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\begin{aligned} &(F')^2 (4-n) - 2FF'' \\ &+ (n-1) \cdot \left(\frac{1}{2} + H \right) (4FF'' + (F')^2 (n-6)) \end{aligned} \right) \end{aligned} \right) \right) \\
&\quad \left. \left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha \beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right) \right) \right. \\
&\quad - \left(R - \frac{F'}{2F} \left(\begin{aligned} &(n-1) (2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- n f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\
&\quad + \frac{1}{4F^2} \left(\begin{aligned} &f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \\ &+ g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\begin{aligned} &(F')^2 \left(4 - n + (n-1) \cdot \left(\frac{1}{2} + H \right) (n-6) \right) \\ &+ 2FF'' \left(2(n-1) \cdot \left(\frac{1}{2} + H \right) - 1 \right) \end{aligned} \right) \end{aligned} \right) \right) \right) \cdot \delta G^{\alpha\beta} \quad (3)
\end{aligned}$$

These equations clearly have the characteristics of a quantized General Theory of Relativity [11] (see [1–9] for proof).

When observing (2), we realize that our metric scaling has brought in the dimensional parameter n in an explicit way and completely independent of the unscaled metric tensor $g_{\alpha\beta}$. This fact we are going to use in order to find out what happens in big systems, which is to say, systems with huge numbers or even “infinite” linearly independent¹ dimensions.

¹ We will see that the aspect of linear independency is going to be important.

3 The Surprise

Taking the full field equations (2), inserting a function $F=F[f]$, reordering in the following form:

$$0 = \left(\begin{aligned} & 2 \frac{F}{F'} \cdot \mathbf{R}_{\alpha\beta} - \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \end{aligned} \right) \\ & + g_{\alpha\beta} \left(\left(\begin{aligned} & (n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right. \right. \\ & \left. \left. - n f_{,d} g^{cd} g^{ab} g_{ac, b} \right) - 2 \frac{F}{F'} \cdot \mathbf{R} \right) \cdot \left(\frac{1}{2} + H \right) \\ & + \frac{1}{2F \cdot F'} f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \\ & + g_{\alpha\beta} \frac{1}{2F \cdot F'} f_{,a} f_{,b} g^{ab} \left(\begin{aligned} & (F')^2 \left(4 - n + (n-1)(n-6) \cdot \left(\frac{1}{2} + H \right) \right) \\ & + 2FF'' \left(2(n-1) \cdot \left(\frac{1}{2} + H \right) - 1 \right) \end{aligned} \right) \end{aligned} \right), \quad (4)$$

and setting

$$F[f] = C_{f1} (f + C_{f0})^{-2} \quad (5)$$

gives us:

$$0 = \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) \\ & - \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \end{aligned} \right) \\ & + g_{\alpha\beta} \left(\begin{aligned} & (n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \\ & - n f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \\ & - g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \end{aligned} \right). \quad (6)$$

Please note the red symbols. These are the Ricci tensor and the Ricci scalar as they appear in the classical Einstein field equations [11], only that here also quantum terms for a wave function f have appeared and we clearly recognize the characteristics of a quantum gravity field theory.

We realize that for systems with high dimensionality the term:

$$0 = \left((f + C_{f0})^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) - \left(f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,a} g^{ab} g_{\beta b, a} - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \right) \right) \cdot (f + C_{f0}) \cdot \left(-g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) + g_{\alpha\beta} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac, b} \right) \cdot \left(\frac{1}{2} + H \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \right) \quad (7)$$

becomes recessive, while the nonlinear term:

$$-g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \quad (8)$$

dominates the scene.

In other words, our field equations (6) converge to the following result in a limiting procedure for high numbers of n:

$$0 = \lim_{n \rightarrow \infty} \left((f + C_{f0})^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) - \left(f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,a} g^{ab} g_{\beta b, a} - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \right) \right) \cdot (f + C_{f0}) \cdot \left(-g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) + g_{\alpha\beta} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac, b} \right) \cdot \left(\frac{1}{2} + H \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \right) \Rightarrow \boxed{0 = f_{,a} f_{,b} g^{ab}} \quad (9)$$

Thereby, we assumed that the Ricci terms, which is to say $\left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right)$ do not grow linearly or faster with n.

What this result is telling us is the following:

Apparently, in systems with huge numbers of degrees of freedom, which is to say high dimensionality and scaling functions F not being dependent on the dimensionality n, the space or space-time must appear flat because the limiting procedure $n \rightarrow \infty$ makes the term $\left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right)$ disappear.

4 About the Weakness of Gravity

Is this also the reason why gravity is so weak?

Meaning, when assuming the universe has a huge number of dimensions and is—on bigger scales at least—governed by scaling functions not depending on n , then consequently, the term

$\left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right)$ becomes small in comparison with $f_{,a} f_{,b} g^{ab}$, which clearly appears as a

matter term in (9). In other words, in systems with increasing numbers of parameters/attributes/degrees of freedom/dimensions the matter and other interactions start to become dimensionally dominant, while the Ricci-determined gravity moves into the background.

From there we may even estimate the number of degrees of freedom the universe or the sub-system, in which we live in, think it is the universe, and experience such a weak gravity, possesses. In order to illustrate the weakness of the gravitational force in comparison with the other forces, we want to use a nice and rather entertaining explanation we found on the internet (www.reddit.com/r/askscience/comments/173rdxn/why_or_how_is_gravity_weak/?rdt=65394):

So, what do we mean when we say it's "weak"? One way to look at it is looking at the gravitational constant vs Coulomb's Constant. Both gravitational force and electric force have the same basic format. Force due to gravity looks like this:

$$F_{\text{grav}} = G \cdot \frac{m_1 \cdot m_2}{r^2} \quad (10)$$

where G is the gravitational constant and the electrostatic force equation looks like this:

$$F_{\text{elec}} = k \cdot \frac{q_1 \cdot q_2}{r^2} \quad (11)$$

where q 's are "charges" and k is Coulomb's constant.

This isn't a direct comparison, since they have different units, but still illustrative. The gravitational constant is $\sim 6.6 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ while Coulomb's Constant is $\sim 9 \times 10^9 \text{ Nm}^2/\text{C}^2$. So, a lot of the units are the same between these constants- they both have Newtons and meters squared on the top, while gravitational constant has kilograms squared on the bottom and Coulomb's has Coulomb's squared down there. So, something you can do is think about taking two electrons, placing them close, and compare the gravitational force between them to the electrostatic force between them- and you'll see it's not even close. For instance, take two electrons, place them a mm apart. The electrostatic repulsion between them is quite small, only $2 \times 10^{-22} \text{ N}$, but the gravitational attraction between those two electrons? Only $5.5 \times 10^{-65} \text{ N}$. So, the force between the electrons due to electrostatic forces is $\sim 10^{43}$ times larger. That is why we say gravity is weak.

Or put another way: Think about the Sun. It is absolutely massive. All of the mass of the Sun pulling on you (assuming you have about 70 kg's of mass) with gravity, is only about 415 milli-Newtons of force. AKA- you couldn't even feel it. But imagine instead if we stripped all of the electrons out of a human's body (again, this would kill you instantly, but you know), left that body on Earth, and took all the electrons to the Sun. How strong would that force of attraction be? There are about 2×10^{28} protons in a 70 kg human body, and the same number of electrons. If we took just the electrons out of your body, and places them as far away as the Sun is from you now, it would pull with a force of $\sim 4 \times 10^6$ Newtons. Or put into perspective, that would be about the same force as if you tied 10 Statue of Liberties to yourself and let them dangle.

So, yes, gravity is "weak" but interestingly, it is also the most important force over long distances. Electrostatic force doesn't impact the motion of planets and stars and galaxies much at all. Why?

Because the universe is almost completely electrically neutral. So, there is basically no electrical attraction or repulsion between the Earth and Sun, since they both are electrical neutral, so that gravity, as "weak" as it is, dominates. But when things are close (and I mean, really close) even when they're neutral, electrostatic forces dominate again. For instance, two atoms which are both neutral, when they get close, the electrons of those atoms are closer together than the protons, so there will be a repulsion there. In fact, when you "touch" something, what is causing you to "make contact" is electron/electron repulsion.

Seeing that in equation (9) there is a n^2 -difference between the gravity term $\left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right)$

and the nonlinear term in f , reading $f_{,a} f_{,b} g^{ab}$, because we have:

$$0 = \lim_{n \rightarrow \infty} \left(f + C_{f0} \right) \cdot \left(\begin{aligned} & \left(f + C_{f0} \right)^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) \\ & - \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} & g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} \\ & - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha \beta, c} \end{aligned} \right) \end{aligned} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & + g_{\alpha\beta} \left((n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \\ & - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \end{aligned} \right), \quad (12)$$

$$= \lim_{n \rightarrow \infty} \left(\begin{aligned} & \left(f + C_{f0} \right)^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \\ & - \left(\begin{aligned} & -2f_{,\alpha\beta} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} g_{\alpha c, \beta} \end{aligned} \right) \\ & + (f + C_{f0}) \cdot \left(\begin{aligned} & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & - g_{\alpha\beta} \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \\ & + (f + C_{f0}) \cdot n \cdot \left(\begin{aligned} & - \left(f_{,\alpha\beta} - \frac{f_{,d} g^{cd}}{2} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha \beta, c}) \right) \\ & + g_{\alpha\beta} \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b} \right) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \\ & + \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n (3 + 2H) - \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n^2 (1 + 2H) \end{aligned} \right), \quad (13)$$

we might want to abbreviate the last equation as follows:

$$\begin{aligned}
0 &= \lim_{n \rightarrow \infty} \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \\ & + (f + C_{f0}) \cdot \overbrace{\left(\begin{aligned} & - \left(-2f_{,\alpha\beta} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \right) \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} g_{\alpha c, \beta} \end{aligned} \right)}^{M_1} \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & - g_{\alpha\beta} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \\ & + (f + C_{f0}) \cdot n \cdot \overbrace{\left(\begin{aligned} & - \left(f_{,\alpha\beta} - \frac{f_{,d} g^{cd}}{2} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha\beta, c}) \right) \\ & + g_{\alpha\beta} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b}) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right)}^{M_2} \\ & + \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n (3 + 2H) - \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n^2 (1 + 2H) \end{aligned} \right) \\
&= \lim_{n \rightarrow \infty} \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} + (f + C_{f0}) \cdot M_1 \\ & + n \cdot \left((f + C_{f0}) \cdot M_2 + \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} (3 + 2H) \right) - \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n^2 (1 + 2H) \end{aligned} \right) \cdot \quad (14)
\end{aligned}$$

This can be written as:

$$\begin{aligned}
0 &= \lim_{n \rightarrow \infty} \left(\begin{aligned} & g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} + \frac{(f + C_{f0}) \cdot M_1 - g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{(f + C_{f0})^2} \\ & + n \cdot \left(\frac{(f + C_{f0}) \cdot M_2 + \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} (3 + 2H)}{(f + C_{f0})^2} \right) - \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2(f + C_{f0})^2} n^2 (1 + 2H) \end{aligned} \right) \cdot \quad (15) \\
&= \lim_{n \rightarrow \infty} \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} + \Psi_0 + n \cdot \Psi_1 - n^2 \cdot \Psi_2 \right)
\end{aligned}$$

4.1 Another Infinity Option

But what happens when taking (2) and demanding linearity with respect to F in the scalar-F-term times the metric tensor, which we here marked in green:

$$\begin{aligned}
0 &= \left(R_{\alpha\beta}^* - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \right. \right. \\ & \quad \left. \left. + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \right) \right) \right) \right. \\ & \quad - \left(R - \frac{F'}{2F} \left(\begin{aligned} & (n-1) (2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ & - n f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & \quad \left. + \frac{1}{4F^2} + g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\begin{aligned} & f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \\ & (F')^2 \left(4 - n + (n-1) \cdot \left(\frac{1}{2} + H \right) (n-6) \right) \\ & + 2FF'' \left(2(n-1) \cdot \left(\frac{1}{2} + H \right) - 1 \right) \end{aligned} \right) \right) \right) \delta G^{\alpha\beta} \end{aligned} \quad , \quad (16)
\end{aligned}$$

via a function of the type:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4(2H(n-1)+n-2)}{(n-2)(2H(n-1)+n-3)}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \cup n = \frac{3+2H}{1+2H} \end{cases} \quad ? \quad (17)$$

In this case (2) yields the following field equations:

$$\begin{aligned}
0 &= \left(R_{\alpha\beta}^* - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \right. \right. \\ & \quad \left. \left. + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \right) \right) \right) \right. \\ & \quad - \left(R - \frac{F'}{2F} \left(\begin{aligned} & (n-1) (2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ & - n f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & \quad \left. + \frac{1}{4F^2} f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \right) \right) \delta G^{\alpha\beta} \end{aligned} \quad , \quad (18)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(R_{\alpha\beta} - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right. \right. \\
& \quad \times \left(f_{,\alpha\beta}(n-2) + f_{,ab}g_{\alpha\beta}g^{ab} + f_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta\alpha,b}) - f_{,\alpha}g^{ab}g_{\beta b,a} \right) \\
& \quad \left. \left. - f_{,\beta}g^{ab}g_{\alpha b,a} + f_{,d}g^{cd} \begin{pmatrix} g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ + \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{pmatrix} \right) \right) \\
& = - \left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \delta G^{\alpha\beta} \\
& \quad \times \left((n-1)(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) - nf_{,d}g^{cd}g^{ab}g_{ac,b} \right) \\
& \quad + f_{,\alpha} \cdot f_{,\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2(f+C_f)^2} \delta G^{\alpha\beta}. \quad (19)
\end{aligned}$$

Assuming non-n-dependent Ricci terms R and $R_{\alpha\beta}$ and performing the limiting procedure for $n \rightarrow \infty$ this gives the following result:

$$\begin{aligned}
0 = & \left(\left(R_{\alpha\beta} - \frac{2}{(f+C_f)} \left(f_{,\alpha\beta} - \frac{f_{,d}}{2}g^{cd}(g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right) + 2 \frac{f_{,\alpha} \cdot f_{,\beta}}{(f+C_f)^2} \right) \delta G^{\alpha\beta} \\
& - \left(R - \frac{2}{(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - f_{,d}g^{cd}g^{ab}g_{ac,b}) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \delta G^{\alpha\beta}. \quad (20)
\end{aligned}$$

We see that in this case the Ricci curvature terms do not vanish and that this time, in addition to the linear terms in f , the $f_{,\alpha} \cdot f_{,\beta}$ -term and not $f_{,a}f_{,b}g^{ab}g_{\alpha\beta}$ haunts the field equations.

4.1.1 The Case of $n=3$

It should be noted that in the case of $n=3$ $F[f]$ becomes an exponential function:

$$F[f] = C_F \cdot e^{f \cdot C_f} \quad (21)$$

and the setting (17) results in the following field equations:

$$\begin{aligned}
0 &= \left(R_{\alpha\beta}^* - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\left(R_{\alpha\beta} - \frac{C_f}{2} \left(\begin{aligned} &f_{,\alpha\beta} + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{ac, \beta} - \frac{3}{2} g_{ac, \beta} - \frac{3}{2} g_{\beta c, \alpha} \\ &+ \frac{3}{2} g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right) \delta G^{\alpha\beta} \right. \\
&\quad \left. - \left(R - \frac{C_f}{2} \left(\begin{aligned} &2(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- 3f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} + \frac{C_f^2}{4} f_{,\alpha} \cdot f_{,\beta} \right) \right) \delta G^{\alpha\beta} \quad (22)
\end{aligned}$$

4.2 Is This THE Answer to the Weak Gravity Question?

We realize a quadratic equation (15) in the dimensionality n leading us to the conclusion that with increasing n first the terms $g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} + \Psi_0$ became recessive, followed by $n \cdot \Psi_1$ with the $-n^2 \cdot \Psi_2$, which is to say:

$$0 = -\frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2(f + C_{f0})^2} n^2 (1 + 2H), \quad (23)$$

dominating in the end.

Our problem is that we cannot tell which one of the non-gravitational terms $n \cdot \Psi_1$, $-n^2 \cdot \Psi_2$ stands for the electromagnetic or even electroweak interaction. There might even be a Debey-shield effect in play for all fundamental interactions blocking our view ("The Relativity of Perspectivity", see [9]). However, when assuming (for a variety of reasons) a linear n -difference between the stronger forces like electrostatic and gravity, we require something like:

$$n = \text{Const} \cdot 10^{43} \quad (24)$$

in order to explain the relative weakness of the gravitational force. With the currently widely accepted assumption of about 10^{81} particles in the sub-system we consider our universe, and assuming a variety of entanglements and boundary conditions, somehow binding some of the degrees of freedom, thereby reducing the number of truly linear independent dimensions, this n does not seem to be too far off, which is to say it does not appear completely unreasonable.

However, can we even consider the number of degrees of freedom of the whole universe as being "felt" by such small things like a hydrogen atom... and thereby we mean "felt" in an instant, so that the variational Hilbert integral (see appendix) does take the whole universe into account in each and every elementary time unit?

Surely this could only be true when there would be an instant causal connection between all positions of the universe and this is impossible with the speed of light in vacuum being the fastest signal possible. Consequently, the regions in which enough dimensions have to come together and being felt by a system in order to generate the observed dependency between the various interactions, have to be much smaller.

What if we assume the Planck volumina to contribute with one dimension each?

From https://en.wikipedia.org/wiki/Fundamental_interaction we obtain the following relative strength ratios for the various interactions:

$$\frac{\text{Weak}}{\text{Gravity}} = 10^{33}; \quad \frac{\text{Strong}}{\text{Gravity}} = 10^{38}; \quad \frac{\text{Electromagnetic}}{\text{Gravity}} = 10^{36}; \quad \frac{\text{Gravity}}{\text{Gravity}} = 1. \quad (25)$$

One Planck volume would be $V_P = 4.222 \cdot 10^{-105} \text{m}^3$. In order to achieve a difference of electromagnetism to gravity via just a linear number of dimensions, we would require $n=10^{36}$. Those 10^{36} Planck volumina fit into a box of a side length of $1.61624 \cdot 10^{-23} \text{m}$. Would the difference be obtained via an n^2 -dependency, which is to say $\text{EM}/\text{Grav} = n^2 = 10^{36}$, then fewer Planck volumina are needed, namely only 10^{18} ? These fit into a box of only $1.61624 \cdot 10^{-29} \text{m}$ side length. With $\text{EM}/\text{Grav} = n^3 = 10^{36}$, the number of Planck volumina would be only 10^{12} with a box size of just $1.61624 \cdot 10^{-31} \text{m}$.

Below this level, gravity should not be so weak anymore.

At least, so one might conclude here, that with the dimensional differences of certain terms in our field equations, we may have found an explanation for the weakness of gravity in comparison with the other fundamental interactions.

When, however, taking the conjecture that the number of dimensions a unit system should feel in this universe in one unit time span in order to make sure that all have the same properties, then, of course, we need to have the critical number of dimensions realized inside each and every one of those unit systems. With the unit time span being the Planck time, the critical number of n has to be reached in each and every Planck unit. This restricts the size of the unit volumes to Planck measures, meaning to a volume of not more than $V_P = 4.222 \cdot 10^{-105} \text{m}^3$. This gives all Planck volumina not only the same number of internal n , but also the same fundamental properties and—when, for instance, following the F-setting from (15)—enforces a weak gravity into a world for high n -numbers being realized in all its Planck volumina. Some sub-systems may, of course, have chosen to stick out a bit and opted for other F , thereby realizing themselves as elementary particles, perhaps. The asymmetry of matter and antimatter could be explained this way [7].

From this automatically follows, however, that underneath the Planck-level completely different sets of fundamental natural constants are active, that the phenomenological physics is very different, and that gravity is not weak at all. The same holds for superstructures which could kind of determine their own physical laws by just steering the number n inside a given system (defined by the boundary of the Hilbert variational integral and the internal speed of communication). Consciousness may be one of those supersystems (super here just means “structured above Planck level”) that even actively uses this degree of freedom, thereby forming sheer wonderworlds... still being based on our quantum gravity laws, but on a shaky/assessable/controllable dimensional ground.

Further investigation is of need, especially with respect to conservation laws, which we have derived in our recent publications [1, 5, 7, 8] (see also appendix of this paper).

5 The Four Fundamental Interactions or the 3 Generations?

Interestingly, the equation (15) is only an equation of second order in n and would not code the four interactions, which are gravity, weak, electromagnetism, and strong (at least not if being distinguished by the exponent to the dimension parameter n). However, what we have ignored so far is the fact that these equations are no stand-alone equations but they are the kernel parts inside a Hilbert variational integral (c.f. appendix of this paper):

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-g} \cdot \mathcal{L}, \quad (26)$$

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(\left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} \\ &+ F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} \\ &+ F_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) \right. \\
&\quad \left. - \left(R - \frac{1}{2F} \left(\begin{aligned} &(n-1) (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- n F_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\
&\quad \left. \left. - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \right) \delta G^{\alpha\beta} \cdot \quad (27)
\end{aligned}$$

Now we assume that the metric shall be fixed during the variation so that only the volume remains flexible and can be varied, leading us to:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&\quad \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(\left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b, a} \\ &+ F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - F_{,\alpha} g^{ab} g_{\beta b, a} \\ &+ F_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) \right. \\
&\quad \left. - \left(R - \frac{1}{2F} \left(\begin{aligned} &(n-1) (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- n F_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\
&\quad \left. \left. - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \right) g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \quad (28)
\end{aligned}$$

This can dramatically be simplified as follows:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&\xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^* - \left(R - \frac{1}{2F} \left((n-1) \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n F_{,d} g^{cd} g^{ab} g_{ac,b} \right) - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R - \frac{1}{2F} \left((n-1) \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n F_{,d} g^{cd} g^{ab} g_{ac,b} \right) - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \cdot (29)
\end{aligned}$$

This time we have obtained a third-order equation in n :

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
&K = \left(R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right. \\
&\quad \left. + n \cdot \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right. \\
&\quad \left. - n^2 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right), \quad (30)
\end{aligned}$$

$$\begin{aligned}
& \Rightarrow \\
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
0 &= \left(\begin{aligned} & R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ & + n \cdot \left(\begin{aligned} & 7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{aligned} & 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ & - F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \end{aligned} \right) \\ & - \left(R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \left(\frac{1}{2} + H \right) \\ & - n^2 \left(\left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{aligned} & 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ & - F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \right) \left(\frac{1}{2} + H \right) - \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \\ & + n^3 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \end{aligned} \right). \quad (31)
\end{aligned}$$

Now, in fact, four interactions of different strength (here marked with different colors as black, blue, green, and red), caused by the influence of the dimensional parameter n , could be coded via our scalar field equation (31).

Separated from the integration and only demanding the kernel in (28) to guarantee the zero outcome, we would be back with:

$$\begin{aligned}
0 &= \left(\begin{aligned} & \left(\begin{aligned} & F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ & + F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \end{aligned} \right) \\ & R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} & g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} \\ & + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \end{aligned} \right) \\ & + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \end{aligned} \right) \\ & - \left(\begin{aligned} & R - \frac{1}{2F} \left(\begin{aligned} & (n-1) (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \\ & - n F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \\ & - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \end{aligned} \right) \quad (32)
\end{aligned}$$

and consequently only a quadratic polynomial in n . Reshaping gives us:

$$\begin{aligned}
0 = & \left(\begin{aligned} & \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} & -2F_{,\alpha\beta} + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ & + F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \\ & + F_{,d} g^{cd} \left(g_{\alpha c,\beta} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \end{aligned} \right) \right. \\ & \left. + \frac{1}{2F^2} (-3F_{,\alpha} \cdot F_{,\beta} + 2g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \right) \\ & - \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 3 \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & + n \left(\begin{aligned} & \left(-\frac{1}{2F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2} g^{cd} (g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right. \\ & \left. + \frac{1}{4F^2} (3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \right) \\ & + \left(\frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} - F_{,d} g^{cd} g^{ab} g_{ac,b}) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & - 7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ & + n^2 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \end{aligned} \right) \end{aligned} \right). \quad (33)
\end{aligned}$$

The dominant term for high numbers of n would then be:

$$0 = \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta}. \quad (34)$$

Together with the integration, which is to say back to (31), and for very high n—as before and similar to the outcome from (33)—a nonlinear term in F, which is to say:

$$\begin{aligned}
0 = & \lim_{n \rightarrow \infty} \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right), \\
0 = & \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \quad (35)
\end{aligned}$$

would dominate the scene of (31) (where we this time avoided the coloring).

5.1 Merging Interactions? Avoiding the Nonlinearity Makes n^3 Disappear

But what happens when taking (30) and demanding linearity in K with respect to F via a function of the type:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases} \quad (36)$$

In this case (31) becomes:

$$\begin{aligned}
& \Rightarrow \\
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
&= -n \cdot \left(\begin{aligned} & R + \frac{F'}{2F} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) \\ & \frac{F'}{2F} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} \\ & + \left(R + \frac{F'}{2F} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \\ & + n^2 \left(\left(\frac{F'}{2F} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} \right) \left(\frac{1}{2} + H \right) \right) \end{aligned} \right) \\
&= -n \cdot \left(\begin{aligned} & R + \frac{2}{(n-2)(f+C_f)} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) \\ & \frac{2}{(n-2)(f+C_f)} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} \\ & + \left(R + \frac{2}{(n-2)(f+C_f)} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \\ & + n^2 \left(\left(\frac{2}{(n-2)(f+C_f)} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} \right) \left(\frac{1}{2} + H \right) \right) \end{aligned} \right) \quad (37)
\end{aligned}$$

and could be brought into the following form:

$$0 = -n \cdot \left(\begin{aligned} & \frac{2}{(f+C_f)} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) - 2R \\ & \frac{2}{(f+C_f)} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} - R \\ & + \left(\frac{2}{(f+C_f)} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}) - 2R \right) \left(\frac{1}{2} + H \right) \\ & + n^2 \left(\left(\frac{2}{(f+C_f)} \begin{pmatrix} 2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \\ -f_{,d} g^{cd} g^{ab} g_{ac,b} \end{pmatrix} - R \right) \left(\frac{1}{2} + H \right) \right) \end{aligned} \right). \quad (38)$$

We see that the third-order term in n has vanished, and with it the nonlinearity in f . This time, as before with the complete field equations (19) and our choice of $F[f]$ via (17), the Ricci term does not become recessive with $n \rightarrow \infty$.

Inserting (17) into (28) results in:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&\xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right. \\
&\quad \times \left(\begin{aligned} &f_{,\alpha\beta}(n-2) + f_{,ab}g_{\alpha\beta}g^{ab} \\ &+ f_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta\alpha,b}) - f_{,\alpha}g^{ab}g_{\beta b,a} \\ &- f_{,\beta}g^{ab}g_{\alpha b,a} + f_{,d}g^{cd} \left(\begin{aligned} &g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ &+ \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{aligned} \right) \end{aligned} \right) \\
&\quad \left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \\
&\quad \times \left(\begin{aligned} &(n-1)(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ &- nf_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) \\
&\quad \left. + f_{,\alpha} \cdot f_{,\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2(f+C_f)^2} \right) \quad (39)
\end{aligned}$$

and consequently gives:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(\begin{aligned} &\left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right) \left(1 - \left(\frac{1}{2} + H \right) n \right) \\ &\times \left(\begin{aligned} &(n-1)(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ &- nf_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) \\ &+ f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2(f+C_f)^2} \end{aligned} \right) \cdot \delta \left(\frac{1}{F} \right). \quad (40)
\end{aligned}$$

In the case of $n \rightarrow \infty$ we would result in something like:

$$\begin{aligned}
0 &= \lim_{n \rightarrow \infty} \left[\left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right) \left(1 - \left(\frac{1}{2} + H \right) n \right) \right. \\
&\quad \times \left(\begin{aligned} &(n-1)(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ &\quad \left. - nf_{,d}g^{cd}g^{ab}g_{ac,b} \right) \end{aligned} \right) \\
&\quad \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \\
&= \lim_{n \rightarrow \infty} \left[\left(R \left(1 - \left(\frac{1}{2} + H \right) n \right) + \left(\frac{1}{2} + H \right) \left(\begin{aligned} &(n-1)(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ &\quad \left. - nf_{,d}g^{cd}g^{ab}g_{ac,b} \right) \right) \right) \right. \\
&\quad \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \right]
\end{aligned} \tag{41}$$

5.2 Dimensionality, the “Other Answer” to the 3-Generations Problem?

It should be noted that (31) could also give an alternative answer to the 3-generations problem of elementary particles [8]. Instead of the set of third-order differential equations allowing us to extract such three generations from the mathematical rule of the Bianchi identity as performed in [8], we may just interpret masses as dimensions, thereby remembering the connection of dimensions of an n -sphere and the Bekenstein-Hawking information problem [13, 14] plus the size and mass of black holes as discussed in many of our previous publications (c.f. [1–5]). In difference to the Bianchi identity and the process we went through in [8], (31) would directly be a scalar equation of third-order, and thus, saving us some of the trouble we had in [8] with the scalarization (see appendix of this paper). We would also get a third-order polynomial in n even in an environment with the classical extremal Hamilton principle as this would make (31) to:

$$\begin{aligned}
0 &= \left(\begin{aligned} &R + \frac{1}{2F} \left(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \right) - 6 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \\ &+ n \cdot \left(\begin{aligned} &7 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{aligned} &2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \\ &\quad \left. - F_{,d}g^{cd}g^{ab}g_{ac,b} \right) \end{aligned} \right) \end{aligned} \right) \\ &- \frac{1}{2} \left(R + \frac{1}{2F} \left(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \right) - 6 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \\ &- n^2 \left(\left(7 \frac{g^{ab}F_{,a} \cdot F_{,b}}{8F^2} - \frac{1}{4F} \left(\begin{aligned} &2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \\ &\quad \left. - F_{,d}g^{cd}g^{ab}g_{ac,b} \right) \right) \right) - \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \\ &+ n^3 \frac{g^{ab}F_{,a} \cdot F_{,b}}{8F^2} \end{aligned} \right) \end{aligned} \tag{42}$$

and we see that H is not of need to still have a third-order polynomial in n .

When linearizing this equation via a scaling function of the type (36), we already saw that we then obtain a second-order equation of the form:

$$0 = \begin{pmatrix} R + \frac{2}{(n-2)(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ -n \cdot \begin{pmatrix} \frac{2}{(n-2)(f+C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \\ + \left(R + \frac{2}{(n-2)(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \end{pmatrix} \\ + n^2 \left(\left(\frac{2}{(n-2)(f+C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \right) \left(\frac{1}{2} + H \right) \right) \end{pmatrix}. \quad (43)$$

In order to simplify our consideration here, we assume metrics of constants and obtain from (31):

$$0 = \begin{pmatrix} \frac{g^{ab}F_{,ab}}{F} - 6 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \\ + n \cdot \left(7 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} - \frac{g^{ab}F_{,ab}}{F} - \left(\frac{g^{ab}F_{,ab}}{F} - 6 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \left(\frac{1}{2} + H \right) \right) \\ - n^2 \left(\left(7 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} - \frac{g^{ab}F_{,ab}}{F} \right) \left(\frac{1}{2} + H \right) - \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \\ + n^3 \left(\frac{1}{2} + H \right) \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \end{pmatrix}. \quad (44)$$

Assuming F to fulfill the condition:

$$\frac{g^{ab}F_{,ab}}{F} - \frac{g^{ab}F_{,a} \cdot F_{,b}}{F^2} = 0, \quad (45)$$

we can simplify (44) as follows:

$$\begin{aligned} 0 &= \frac{g^{ab}F_{,ab}}{F} \begin{pmatrix} 1 - \frac{3}{2} + n \cdot \left(\frac{7}{4} - 1 - \left(1 - \frac{3}{2} \right) \left(\frac{1}{2} + H \right) \right) \\ - n^2 \left(\left(\frac{7}{4} - 1 \right) \left(\frac{1}{2} + H \right) - \frac{1}{4} \right) + n^3 \left(\frac{1}{2} + H \right) \frac{1}{4} \end{pmatrix} \\ &= \frac{g^{ab}F_{,ab}}{16 \cdot F} (n^2 (n-1+2H(n-3))(n-1+(2+H))) \end{aligned} \quad (46)$$

In the linear case with (36) and assumed metrics of constants, we result in:

$$0 = \frac{4g^{ab}f_{,ab}}{(n-2)(f+C_f)} \left(1 - n \cdot \left(\frac{3}{2} + H \right) + n^2 \left(\frac{1}{2} + H \right) \right). \quad (47)$$

A bit of generalization with respect to (45) like:

$$\frac{g^{ab}F_{,ab}}{F} - d \cdot \frac{g^{ab}F_{,a} \cdot F_{,b}}{F^2} = 0, \quad (48)$$

gives:

$$0 = \frac{g^{ab}F_{,ab}}{16 \cdot F} n^2 (2 - 3d + n(d(5 + 3H) - 3 - 2H))(4 + 8H + d(n - 5 + 2H(n - 7))). \quad (49)$$

Solutions to the equation with respect to n would read:

$$n = 0; \quad n = \frac{2 - 3d}{3 - 5 \cdot d + 2H - 3 \cdot d \cdot H}; \quad n = 7 - \frac{4}{d} - \frac{2}{1 + 2H}. \quad (50)$$

Inserting the typical particle at rest solution from the Dirac theory [12] (m =mass, t =time, $i=\sqrt{-1}$):

$$F = C_F \cdot e^{i \cdot C_F \cdot m \cdot t}, \quad (51)$$

we see no possibility to achieve an outcome for the three masses as obtained with the Bianchi identity in [8]. Still we have a chance to solve the scalar version of the field equation with just a suitable number of dimensions, whereby—of course—only the nontrivial solutions:

$$n = \frac{2 - 3d}{3 - 5 \cdot d + 2H - 3 \cdot d \cdot H}; \quad n = 7 - \frac{4}{d} - \frac{2}{1 + 2H} \quad (52)$$

are of any interest.

But what happens when we combine the Bianchi derivation with our dimensional investigation?

And what can we expect in a more general geometrical environment instead of our current simplification of metrics of constants?

While we are going to leave the first question for later, we here want to briefly discuss the second one.

In order to answer it, we go back to the general form (42) and abbreviate as follows:

$$0 = \left(\begin{array}{c} \overbrace{R + \frac{1}{2F} (2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}) - 6\frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2}}^{-A} \\ + n \cdot \left(\begin{array}{c} \overbrace{7\frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{array}{c} 2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \\ -F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right)}^B \\ - \frac{1}{2} \left(R + \frac{1}{2F} (2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}) - 6\frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \end{array} \right) \\ - n^2 \left(\begin{array}{c} \overbrace{\left(7\frac{g^{ab}F_{,a} \cdot F_{,b}}{8F^2} - \frac{1}{4F} \left(\begin{array}{c} 2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} \\ -F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) \right)}^C \\ - \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \end{array} \right) \\ + n^3 \underbrace{\frac{g^{ab}F_{,a} \cdot F_{,b}}{8F^2}}^D \end{array} \right). \quad (53)$$

The resulting polynomial:

$$0 = -A + n \cdot B - n^2 \cdot C + n^3 \cdot D \quad (54)$$

has three solutions in n , but so far, and in difference to our investigation in [8], we don't see how the dimensionality could be mirrored in classical solutions like the Dirac particle at rest... at least not

without a rigorous paradigm shift regarding the connection/entanglement of mass and dimensions². Nevertheless, we want to investigate the polynomial. Equation (54) can be written as follows:

$$0 = -C_0 + C_1 \cdot n - C_2 \cdot n^2 + n^3. \quad (55)$$

The general solution to a three-order polynomial could be given via the following product form:

$$(n - n_1) \cdot (n - n_2) \cdot (n - n_3) = n^3 - n^2 \cdot (n_1 + n_2 + n_3) + n \cdot (n_1 n_2 + n_1 n_3 + n_2 n_3) - n_1 n_2 n_3. \quad (56)$$

Comparing the last line of (56) with (55) and (54) gives us:

$$\begin{aligned} C_2 &= n_1 + n_2 + n_3 = \frac{C}{D} \\ &= \frac{\left(\left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{8F^2} - \frac{1}{4F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right) - \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2}}{\frac{g^{ab} F_{,a} \cdot F_{,b}}{8F^2}} \\ C_1 &= n_1 n_2 + n_1 n_3 + n_2 n_3 = \frac{B}{D} \\ &= \frac{\left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - F_{,d} g^{cd} g^{ab} g_{ac,b} \right) - \frac{1}{2} \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right)}{\frac{g^{ab} F_{,a} \cdot F_{,b}}{8F^2}} \\ C_0 &= n_1 n_2 n_3 = \frac{A}{D} = \frac{3 \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} - R - \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c})}{\frac{g^{ab} F_{,a} \cdot F_{,b}}{8F^2}} \end{aligned} \quad (57)$$

We see that in a particle at rest constellation with metrics of constants (c.f. our considerations between equations (44) and (52)), where we expect an outcome as:

$$A = C_A \cdot m^2; \quad B = C_B \cdot m^2; \quad C = C_C \cdot m^2; \quad D = C_D \cdot m^2, \quad (58)$$

there wouldn't be any mass term left in our equation (55). Hence, the third-order polynomial with respect to the dimension as obtained via (42) does not code the three generations with three different masses but obviously something else. Things are a bit more interesting, however, when assuming non-constant metrics (still without any Ricci curvature). Assuming something like:

$$\frac{g^{ab} F_{,a} \cdot F_{,b}}{F^2} = m^2; \quad \frac{g^{ab} F_{,ab}}{F} = d \cdot m^2; \quad \frac{F_{,d} g^{cd} g^{ab} g_{ab,c}}{F} = M \cdot m; \quad \frac{F_{,d} g^{cd} g^{ab} g_{ac,b}}{F} = \mu \cdot m, \quad (59)$$

² Something we should not automatically exclude here as we have seen such connections with respect to the Hawking-Bekenstein information problem [13, 14].

$$\begin{aligned}
C_2 &= \frac{\left(\frac{7}{8}m^2 - \frac{1}{4}(2d \cdot m^2 + m \cdot (M - \mu))\right) - \frac{m^2}{4}}{\frac{m^2}{8}} = \frac{5m^2 - 2(2d \cdot m^2 + m \cdot (M - \mu))}{m^2} \\
C_1 &= \frac{7m^2 - 4(2d \cdot m^2 + m \cdot (M - \mu)) - 4\left(R + \frac{1}{2}(2d \cdot m^2 + m \cdot M) - 3\frac{m^2}{2}\right)}{m^2} \\
&\xrightarrow{R=0} \frac{m^2 - 4(2d \cdot m^2 + m \cdot (M - \mu)) - 2(2d \cdot m^2 + m \cdot M)}{m^2} \quad , \quad (60) \\
&= \frac{m^2 + 4m \cdot \mu - 6(2d \cdot m^2 + m \cdot M)}{m^2} \\
C_0 &= 4 \frac{3m^2 - 2R - 2d \cdot m^2 + m \cdot M}{m^2} \xrightarrow{R=0} 4 \frac{3m^2 - 2d \cdot m^2 + m \cdot M}{m^2}
\end{aligned}$$

we see that this offers us a set of parameters to obtain systems fulfilling the scalar field equations via three possible settings for n , for a given constellation of metric $g_{\alpha\beta}$, and scaling factor F . However, the “three generations” are then not occurring in different masses, but in three sets of dimensionality n for every F - $g_{\alpha\beta}$ -constellation.

Well, we leave such considerations for later.

6 “Natural” Linearity and the Classical Extremal Principle by Just Choosing the Right $F(f)$

Again we chose the following functional wrapper for our metric tensor:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases} \quad (61)$$

We already saw that in this case, and together with the condition $\delta g^{\alpha\beta} \equiv 0$ from (28), (31) yields (just with a bit reordered structure):

$$\begin{aligned}
\Rightarrow \delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H\right)n\right) \cdot \delta\left(\frac{1}{F}\right) \\
K &= \left(R + \frac{2}{(n-2)(f + C_f)} \left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - n \cdot \left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - f_{,d}g^{cd}g^{ab}g_{ac,b}\right)\right)\right) \\
&= \left(R + \frac{4}{(n-2)(f + C_f)} \left(\Delta f - f_{,d}g^{cd}_{,c} - n \cdot \left(\Delta f - f_{,d}g^{cd}_{,c} - \frac{f_{,d}g^{cd}g^{ab}g_{ac,b}}{2}\right)\right)\right)
\end{aligned} \quad (62)$$

When now restricting ourselves to metrics of the type:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1,n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \quad (63)$$

the last line in (62) results in the metric Klein-Gordon equation with the Ricci scalar acting as the mass term as follows:

$$0 = \frac{4}{f + C_f} \cdot \frac{1-n}{n-2} \Delta f + R \xrightarrow{f=\Psi-C_f} 0 = \Delta \Psi - \frac{1}{4} \cdot \frac{n-2}{n-1} \cdot R \cdot \Psi. \quad (64)$$

We see that the term H has disappeared, which means that the classical extremal Hamilton principle automatically is obtained also from a generalized Hamilton principle in the very moment that the variation of the metric tensor is restricted to its volume part.

Hence, the question is justified whether the classical EXTREMAL Hamilton principle is just an accidental outcome of such a volume-restricted metric variation?!

7 Summing up a Few Infinity Options

We have seen that due to the fundamental degree of freedom we have with the functional wrapper of the metric scaling or volume function $F[f]$, we end up in a great variety of options also with respect to the behavior in systems with huge numbers of dimensions.

In this section we now want to sum up the most important settings.

7.1 Kernel Only

We start with the consideration of only the kernel of the Hilbert variation (26) after the variation, thereby applying the classical conjecture that it suffices to make this kernel to zero in order to fulfill the complete integral equation.

7.1.1 $F[f]$ Not Fixed, but Independent on n

Taking (3), we have seen that we have a variety of options to obtain various n -dependencies. Leaving F open and assuming that it does not depend on n , results in:

$$\begin{aligned}
0 = & \left(\begin{aligned} & \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} & -2F_{,\alpha\beta} + F_{,ab}g_{\alpha\beta}g^{ab} - F_{,\beta}g^{ab}g_{\alpha b,a} \\ & + F_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha}g^{ab}g_{\beta b,a} \\ & + F_{,d}g^{cd} \left(g_{\alpha c,\beta} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \right) \end{aligned} \right) \right. \\ & \left. + \frac{1}{2F^2}(-3F_{,\alpha} \cdot F_{,\beta} + 2g_{\alpha\beta}F_{,c}F_{,d}g^{cd}) \right) \\ & - \left(R + \frac{1}{2F}(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}) - 3\frac{g^{ab}F_{,a} \cdot F_{,b}}{2F^2} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & + n \left(\begin{aligned} & \left(-\frac{1}{2F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2}g^{cd}(g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right. \\ & \left. + \frac{1}{4F^2}(3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta}F_{,c}F_{,d}g^{cd}) \right) \\ & + \left(\frac{1}{2F}(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} - F_{,d}g^{cd}g^{ab}g_{ac,b}) \right) \\ & \left. - 7\frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & + n^2 \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \end{aligned} \right) \end{aligned} \right) \quad (65)
\end{aligned}$$

and consequently only a quadratic polynomial in n .

For the limit of $n \rightarrow \infty$ the dominant term is:

$$0 = \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta}. \quad (66)$$

7.1.2 $F[f]$ Fixed Such That the Nonlinear F -Term in the Ricci Scalar Vanishes

Setting $F[f]$ as:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2, \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases}, \quad (67)$$

(2) evolves to:

$$\begin{aligned}
0 = & \left[R_{\alpha\beta} - \frac{F'}{2F} \cdot \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \right. \right. \\ & \left. \left. - g_{\alpha\beta} \frac{F'}{2F} \cdot \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \right. \right. \\ & \left. \left. + g_{\alpha\beta} \left(\frac{F'}{2F} \cdot \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac, b} \right) - R \right) \cdot \left(\frac{1}{2} + H \right) \right. \right. \\ & \left. \left. + \frac{1}{4F^2} f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \right. \right. \\ & \left. \left. + g_{\alpha\beta} \frac{1}{4F^2} f_{,a} f_{,b} g^{ab} \left((F')^2 \left(4 - n + (n-1)(n-6) \cdot \left(\frac{1}{2} + H \right) \right) \right. \right. \right. \right. \\ & \left. \left. \left. + 2FF'' \left(2(n-1) \cdot \left(\frac{1}{2} + H \right) - 1 \right) \right) \right) \right) \right] \\
= & \left[R_{\alpha\beta} - \frac{2}{(n-2)(f+C_f)} \cdot \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \right. \right. \\ & \left. \left. - g_{\alpha\beta} \frac{2}{(n-2)(f+C_f)} \cdot \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \right. \right. \\ & \left. \left. + g_{\alpha\beta} \left(\frac{2}{(n-2)(f+C_f)} \cdot \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac, b} \right) - R \right) \cdot \left(\frac{1}{2} + H \right) \right. \right. \\ & \left. \left. + f_{,\alpha} \cdot f_{,\beta} \frac{2n}{(n-2)(f+C_f)^2} - g_{\alpha\beta} \frac{2f_{,a} f_{,b} g^{ab}}{(n-2)(f+C_f)^2} \right) \right] \cdot (68)
\end{aligned}
\right.
\end{aligned}$$

The direct limit $n \rightarrow \infty$ performed in the last equation yields:

$$0 = \left(\begin{aligned} & R_{\alpha\beta} - \frac{1}{(f+C_f)} \cdot \left(2f_{,\alpha\beta} - f_{,d} g^{cd} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha\beta, c}) \right) + f_{,\alpha} \cdot f_{,\beta} \frac{2}{(f+C_f)^2} \\ & + g_{\alpha\beta} \left(\frac{2}{(f+C_f)} \cdot \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b} \right) - R \right) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \cdot (69)$$

7.1.3 F[f] Fixed Such That the Scalar Nonlinear Terms (Times Metric Tensor) in the Whole Einstein Tensor Vanish

Now our setting for F(f) shall be:

$$F[f] = \begin{cases} C_F \cdot (f+C_f)^{\frac{4(2H(n-1)+n-2)}{(n-2)(2H(n-1)+n-3)}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \cup n = \frac{3+2H}{1+2H} \end{cases} \quad (70)$$

In this case (2) yields the following field equations:

$$\begin{aligned}
0 &= \left(R_{\alpha\beta}^* - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right. \right. \\
&\quad \left. \left. - \left(R - \frac{F'}{2F} \left(\begin{aligned} &(n-1) (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \right. \\
&\quad \left. \left. + \frac{1}{4F^2} f_{,\alpha} \cdot f_{,\beta} (n-2) (3(F')^2 - 2FF'') \right) \right) \delta G^{\alpha\beta} \\
&= \left(\left(R_{\alpha\beta} - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right. \right. \\
&\quad \times \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \\ &+ \frac{1}{2} n g_{\alpha \beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right. \\
&\quad \left. \left. - \left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right) \delta G^{\alpha\beta} \right. \\
&\quad \left. + f_{,\alpha} \cdot f_{,\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \delta G^{\alpha\beta} . \quad (71)
\end{aligned}$$

Assuming non-n-dependent Ricci terms R and $R_{\alpha\beta}$ and performing the limiting procedure for $n \rightarrow \infty$, this gives the following result:

$$\begin{aligned}
0 &= \left(\left(R_{\alpha\beta} - \frac{2}{(f+C_f)} \left(f_{,\alpha\beta} - \frac{f_{,d}}{2} g^{cd} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha \beta, c}) \right) \right) + 2 \frac{f_{,\alpha} \cdot f_{,\beta}}{(f+C_f)^2} \right) \delta G^{\alpha\beta} \\
&\quad - \left(R - \frac{2}{(f+C_f)} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b}) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \delta G^{\alpha\beta} . \quad (72)
\end{aligned}$$

Comparison with (69) shows us that, even though the scaling functions are different and we also obtain different results for the kernel with low numbers of dimensions, we obtain the same field equations in the case of $n \rightarrow \infty$, which is no big surprise, because the n-to-infinity-limits of F are giving the same results.

7.1.4 F[f] Fixed Such That the Tensorial Nonlinear Term in the Ricci Tensor Vanishes

Applying the function:

$$F[f] = C_{f1} (f + C_{f0})^{-2} \quad (73)$$

results in:

$$0 = \lim_{n \rightarrow \infty} \left((f + C_{f0}) \cdot \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right) \\ & - \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} & g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} \\ & - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \end{aligned} \right) \end{aligned} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & + g_{\alpha\beta} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \\ & - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \end{aligned} \right) \right. \\ & \left. + (f + C_{f0}) \cdot \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \\ & - \left(\begin{aligned} & -2f_{,\alpha\beta} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} g_{\alpha c, \beta} \end{aligned} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & - g_{\alpha\beta} \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \right. \\ & \left. + (f + C_{f0}) \cdot n \cdot \left(\begin{aligned} & - \left(f_{,\alpha\beta} - \frac{f_{,d} g^{cd}}{2} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha\beta, c}) \right) \\ & + g_{\alpha\beta} \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b} \right) \cdot \left(\frac{1}{2} + H \right) \\ & + \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n (3+2H) - \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} n^2 (1+2H) \end{aligned} \right) \right) \right), \quad (74)$$

and leaves us with just the following nonlinear term in the case of $n \rightarrow \infty$:

$$0 = \frac{g_{\alpha\beta} f_{,a} f_{,b} g^{ab}}{2} (1+2H). \quad (75)$$

7.2 Whole Variational Integral

In contrast to the previous subsection, where we considered only the kernel of the Hilbert variation (26) after the variation, thereby applying the classical conjecture that it suffices to make this kernel to zero in order to fulfill the complete integral equation, we now take the whole integral into account. Thereby we assume the following “partial” or “degenerate” variational process:

$$\begin{aligned}
 \delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \quad \text{with : } \delta g^{\alpha\beta} \equiv 0 \\
 \Rightarrow 0 &= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
 &\xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
 &= \int_V d^n x \cdot \sqrt{-G} \cdot \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta}^{ab} - F_{,\beta} g_{\alpha b, a}^{ab} \\ &+ F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - F_{,\alpha} g^{ab} g_{\beta b, a} \\ &+ F_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \right) \right. \right. \\ &\quad \left. \left. + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \right) \right. \\ &\quad \left. + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) \\ &\quad - \left(R - \frac{1}{2F} \left(\begin{aligned} &(n-1) (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- n F_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right) g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \quad .(76)
 \end{aligned}$$

7.2.1 F[f] Not Fixed, but Independent on n

Leaving F open and assuming that it does not depend on n, results in the kernel given in (65). Taking the integral environment into account, we result in the following for very high n:

$$\begin{aligned}
 0 &= \lim_{n \rightarrow \infty} \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) , \\
 0 &= \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \quad , \quad (77)
 \end{aligned}$$

whereby the process of derivation can be extracted from here:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
K &= \left(\begin{array}{c} R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ n \cdot \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \\ -n^2 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \end{array} \right), \tag{78}
\end{aligned}$$

$$\begin{aligned}
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left(\begin{array}{c} R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ n \cdot \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \\ - \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \left(\frac{1}{2} + H \right) \\ -n^2 \left(\left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \left(\frac{1}{2} + H \right) \right. \\ \left. \left. - \frac{1}{2F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \right) \left(\frac{1}{2} + H \right) \\ \left. + n^3 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \cdot \delta \left(\frac{1}{F} \right) \end{array} \right). \tag{79}
\end{aligned}$$

7.2.2 F[f] Fixed Such That the Nonlinear F-Term in the Ricci Scalar Vanishes

Our setting for F[f] shall be:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases}. \tag{80}$$

In this case the complete variational integral (31) looks as follows:

$$\begin{aligned}
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot K \cdot \left(1 - \left(\frac{1}{2} + H \right) n \right) \cdot \delta \left(\frac{1}{F} \right) \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left[-n \cdot \left(R + \frac{2}{(n-2)(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \right) \right. \\
&\quad \left. - \left(\frac{2}{(n-2)(f+C_f)} \left(\begin{aligned} &2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ &-f_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) \right. \right. \\
&\quad \left. \left. + \left(R + \frac{2}{(n-2)(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \right) \right. \\
&\quad \left. + n^2 \left(\left(\frac{2}{(n-2)(f+C_f)} \left(\begin{aligned} &2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ &-f_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) \right) \left(\frac{1}{2} + H \right) \right) \right] \cdot \delta \left(\frac{1}{F} \right) \quad (81)
\end{aligned}$$

and could be brought into the following form:

$$\begin{aligned}
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot \left[-n \cdot \left(\frac{2}{(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) - 2R \right) \right. \\
&\quad \left(\frac{2}{(f+C_f)} \left(\begin{aligned} &2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ &-f_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) - R \right) \\
&\quad \left. + \left(\frac{2}{(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) - 2R \right) \left(\frac{1}{2} + H \right) \right) \\
&\quad \left. + n^2 \left(\left(\frac{2}{(f+C_f)} \left(\begin{aligned} &2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ &-f_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) - R \right) \left(\frac{1}{2} + H \right) \right) \right] \cdot \delta \left(\frac{1}{F} \right). \quad (82)
\end{aligned}$$

We see that the nonlinearity in f has vanished. With $n \rightarrow \infty$ we obtain:

$$0 = \int_V d^n x \cdot \sqrt{-G} \cdot \left(\frac{2}{(f+C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - f_{,d}g^{cd}g^{ab}g_{ac,b}) - R \right) \left(\frac{1}{2} + H \right) \cdot \delta \left(\frac{1}{F} \right). \quad (83)$$

7.2.3 $F[f]$ Fixed Such That the Scalar Nonlinear Terms (Times Metric Tensor) in the Whole Einstein Tensor Vanish

We set:

$$F[f] = \begin{cases} C_F \cdot (f+C_f)^{\frac{4(2H(n-1)+n-2)}{(n-2)(2H(n-1)+n-3)}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \cup n = \frac{3+2H}{1+2H} \end{cases} \quad (84)$$

We assume that the metric shall be fixed during the variation so that only the volume remains flexible and can be varied, leading us to:

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \int_V d^n x \cdot \sqrt{-G} \cdot \left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \left(1 - \left(\frac{1}{2} + H \right) n \right) \right. \\ \times \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \cdot \delta \left(\frac{1}{F} \right). \quad (85)$$

In the case of $n \rightarrow \infty$ we would result in something like:

$$0 = \lim_{n \rightarrow \infty} \left(R - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \left(1 - \left(\frac{1}{2} + H \right) n \right) \right. \\ \times \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \\ = \lim_{n \rightarrow \infty} \left(R \left(1 - \left(\frac{1}{2} + H \right) n \right) - \frac{2(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))(f+C_f)} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right. \\ \left. + 2n \frac{\left(\frac{1}{2} + H \right)}{(f+C_f)} \frac{(n-2+2H(n-1))}{(n-2)(n-3+2H(n-1))} \right. \\ \times \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right) \\ = \lim_{n \rightarrow \infty} \left(R \left(1 - \left(\frac{1}{2} + H \right) n \right) + 2n \frac{\left(\frac{1}{2} + H \right)}{(f+C_f)} \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - f_{,d} g^{cd} g^{ab} g_{ac,b} \right. \\ \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{2(n-1)(n-2+2H(n-1))(n-2+2Hn)}{(n-2)(n-3+2H(n-1))^2 (f+C_f)^2} \right. \\ \left. + 2 \frac{\left(\frac{1}{2} + H \right)}{(f+C_f)} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - n f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right), \quad (86)$$

$$\begin{aligned}
&= \left(-R \left(\frac{1}{2} + H \right) + 2 \frac{\left(\frac{1}{2} + H \right)}{(f + C_f)} \left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - f_{,d}g^{cd}g^{ab}g_{ac,b} \right) \right) \\
&\Rightarrow \boxed{0 = \frac{2}{f + C_f} \left(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} - f_{,d}g^{cd}g^{ab}g_{ac,b} \right) - R} \\
&\quad = \frac{2}{f + C_f} \left(2\Delta f - 2f_{,d}g^{cd}_{,c} - f_{,d}g^{cd}g^{ab}g_{ac,b} \right) - R
\end{aligned} \tag{87}$$

The result is equal to the one from (83).

Interestingly, for metrics without shear elements:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \tag{88}$$

(87) results in the metric Klein-Gordon equation with the Ricci scalar acting as the mass term:

$$0 = \frac{4}{f + C_f} \Delta f - R \xrightarrow{f = \Psi - C_f} 0 = \Delta \Psi - \frac{1}{4} \cdot R \cdot \Psi. \tag{89}$$

This agrees perfectly with our result from above (see (83) in subsection “F[f] Fixed Such That the Nonlinear F-Term in the Ricci Scalar Vanishes”) for $n \rightarrow \infty$.

Not only have we obtained the most important specially relativistic quantum equation for both settings of F, (80) and (84), but we also saw the disappearance of the Hilbert variation parameter H during the derivation for systems with many (“infinitely many”) dimensions. We could therefore conclude at this point that the classical extremal principle where H is set to zero, could just accidentally be correct because it is the result that has been obtained anyway for high numbers of n . In other words, an otherwise quite volatile variation gets stabilized and rather arbitrary when the number of degrees of freedom is huge. Hence, when applying the classical concept one might just find agreement with reality not necessarily because the concept is correct, but simply because the systems in question fulfill the condition of $n \rightarrow \infty$ and are therefore rather inert against the setting for the variational outcome, which is to say, it does not matter what results one demands for δW as the resulting field equations after the variation are the same anyway. The same result is obtained for the assumption of a dominating volume variation (see section ““Natural” Linearity and the Classical Extremal Principle by Just Choosing the Right F(f)”).

7.2.4 F[f] Fixed Such That the Tensorial Nonlinear Term in the Ricci Tensor Vanishes

As we know that the tensorial nonlinear term in the general field equations (65) for F, reading $F_{,\alpha} \cdot F_{,\beta}$, vanishes for F[f]:

$$F[f] = C_{f1} (f + C_{f0})^{-2}, \tag{90}$$

and as this function does not contain any dependency on the dimensional parameter n , we can directly extract the result for the whole integral from (65) due to:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left[\left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &-2F_{,\alpha\beta} + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ &+ F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \\ &+ F_{,d} g^{cd} \left(g_{\alpha c,\beta} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \end{aligned} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2F^2} (-3F_{,\alpha} \cdot F_{,\beta} + 2g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \right) \right. \\
&\quad - \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\
&\quad \left. - \left(\begin{aligned} &-\frac{1}{2F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2} g^{cd} (g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right) \\ &+ \frac{1}{4F^2} (3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \end{aligned} \right) \right] g^{\alpha\beta} \delta \left(\frac{1}{F} \right) \\
&\quad + n \left[\left(\begin{aligned} &\frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \right) \\ &- F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\
&\quad \left. - \left(\begin{aligned} &7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ &- \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \end{aligned} \right) \right] \\
&\quad \left. + n^2 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right] \cdot \delta \left(\frac{1}{F} \right) \quad (91)
\end{aligned}$$

Contraction yields:

$$\begin{aligned}
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot \left[\left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \right. \\
&\quad \left. - \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \right) \right. \\
&\quad \left. + n \cdot \left(\begin{aligned} &R - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ &+ \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. - n^2 \left(\begin{aligned} &7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ &- \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. \left. + n^3 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \right] \cdot \delta \left(\frac{1}{F} \right) \quad (92)
\end{aligned}$$

For the limit of $n \rightarrow \infty$ the dominant term is:

$$0 = \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right). \quad (93)$$

In difference to the limiting procedure for the corresponding field equations (75), we see that we lost the metric tensor $g_{\alpha\beta}$. Some readers may find this fact unimportant, but we are going to see further below in this paper that this is anything but.

8 About the Dominant Term in Systems of “Infinite” Dimensionality

In this subsection we are going to connect the classical Dirac theory and the Dirac equation with the field equations in systems with huge numbers of dimensions.

8.1 The Nonlinear Term Prevails in Infinity

As shown elsewhere [7, 8], the subsequent or limiting-residual from (92) and extremely simple equation:

$$0 = f_{,a} f_{,b} g^{ab} \quad (94)$$

can be factorized:

$$0 = f_{,a} f_{,b} \mathbf{e}^a \cdot \mathbf{e}^b \Rightarrow 0 = \begin{cases} f_{,a} \mathbf{e}^a \\ f_{,b} \mathbf{e}^b \end{cases}. \quad (95)$$

Now some may say that this does not even remotely look like a Dirac equation, but the connection becomes clear when we apply the well-known relation (defining condition) of the Dirac matrices, which reads:

$$g^{\alpha\beta} \cdot \mathbf{I} = \frac{\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha}{2}; \quad g_{\alpha\beta} \cdot \mathbf{I} = \frac{\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha}{2}, \quad (96)$$

with the Dirac matrices (here in $n=4$ and for Cartesian coordinates):

$$\gamma^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}; \quad \gamma^1 = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}, \quad (97)$$

$$\gamma^2 = \begin{pmatrix} & & -i & \\ & i & & \\ i & & & \\ -i & & & \end{pmatrix}; \quad \gamma^3 = \begin{pmatrix} & 1 & & \\ & & -1 & \\ -1 & & & \\ & 1 & & \end{pmatrix}; \quad \mathbf{I} = \delta_{ij} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

and subsequently perform the factorization as follows:

$$0 = 2f_{,a} f_{,b} g^{ab} \cdot \mathbf{I} = (\gamma^a \gamma^b + \gamma^b \gamma^a) f_{,a} f_{,b} = \gamma^a f_{,a} f_{,b} \gamma^b + \gamma^b f_{,b} f_{,a} \gamma^a$$

$$\Rightarrow 0 = \begin{cases} \gamma^a f_{,a} \\ f_{,b} \gamma^b \end{cases}. \quad (98)$$

The apparent catch with the missing mass M —in comparison with the original Dirac equation [12]—can be easily cured. Assuming a cartesian coordinate system, we define f as a function $f=h[\dots]*p[xx]$ and demand $p=p[xx]$ with:

$$p_{,xx}p_{,xx} = M^2 \cdot p^2, \quad (99)$$

while h does not depend on xx and thus:

$$f_{,xx}f_{,xx} = h^2 \cdot M^2 \cdot p^2. \quad (100)$$

Consequently (94) would look as follows:

$$0 = h_{,a}h_{,b}g^{ab} + M^2h^2. \quad (101)$$

Thereby xx is just to be considered as an additional degree of freedom or dimension, which here codes the masses M . Assuming n to be the number of classical Dirac dimensions, then our dimension xx increases the total number of dimensions to $n+1$. Applying (96) again, this can be expanded and we obtain³:

$$\begin{aligned} 0 &= 2(h_{,a}h_{,b}g^{ab} \cdot I + M^2h^2) = (\gamma^a\gamma^b + \gamma^b\gamma^a)h_{,a}h_{,b} + M^2h^2 \\ &= \gamma^ah_{,a}h_{,b}\gamma^b + M^2h^2 + \gamma^bh_{,b}h_{,a}\gamma^a + M^2h^2 \\ &= \left[\begin{aligned} &(\gamma^ah_{,a} + i \cdot M \cdot h)(h_{,b}\gamma^b - i \cdot M \cdot h) \\ &+ (\gamma^bh_{,b} + i \cdot M \cdot h)(h_{,a}\gamma^a - i \cdot M \cdot h) \end{aligned} \right] \cdot \end{aligned} \quad (102)$$

$$\Rightarrow 0 = \begin{cases} \gamma^ah_{,a} - i \cdot M \cdot h \\ h_{,b}\gamma^b + i \cdot M \cdot h \end{cases}$$

Now, the remaining task is—just as Dirac did—to recognize the need for a list of functions $h \rightarrow h_i$ in order to make the matrix equation above reasonable, so that in the end, we have reproduced the original Dirac equation:

$$0 = \begin{cases} \gamma^ah_{i,a} - i \cdot M \cdot h_i \\ h_{i,b}\gamma^b + i \cdot M \cdot h_i \end{cases} \quad (103)$$

Thereby we want to point out the fact that the introduction of the list of functions or spinor h_i is not the only way to solve the rank problem from (102). One might also consider a solution of the kind:

$$0 = \begin{cases} \gamma_A^{Ba}h_{,a} - i \cdot M_A^B \cdot h \\ h_{,b}\gamma_A^{Bb} + i \cdot M_A^B \cdot h \end{cases}; \quad \gamma^{\{a,b\}} \equiv \gamma_A^{B\{a,b\}}, \quad (104)$$

where the gamma matrices are “rank-balanced” by matrices of masses M_A^B instead of the classical scalar mass M .

8.2 The “Flaw” in the Dirac Plan

When moving through the evaluation above, we see that the step (101) should rather look as follows:

$$0 = p^2 \cdot (h_{,a}h_{,b}g^{ab} + M^2h^2g^{xxxx}). \quad (105)$$

Consequently this also changes the factorization with the gamma matrices, where we now—should—have to write:

³ Please note that the summation in the expression $h_{,a}h_{,b}g^{ab}$ below only runs from 0 to $n-1$.

$$\begin{aligned}
0 &= 2f_{,\alpha}f_{,\beta}g^{\alpha\beta} \cdot I = (\gamma^{\alpha}\gamma^{\beta} + \gamma^{\beta}\gamma^{\alpha})f_{,\alpha}f_{,\beta}; \quad \alpha, \beta = 0, \dots, n \\
&= p^2 \cdot \gamma^a h_{,a} h_{,b} \gamma^b + h^2 \cdot \gamma^{xx} p_{,xx} p_{,xx} \gamma^{xx} + p^2 \cdot \gamma^b h_{,b} h_{,a} \gamma^a + h^2 \cdot \gamma^{xx} p_{,xx} p_{,xx} \gamma^{xx} \\
&= p^2 \cdot (\gamma^a h_{,a} h_{,b} \gamma^b + h^2 \cdot \gamma^{xx} M^2 \gamma^{xx} + \gamma^b h_{,b} h_{,a} \gamma^a + h^2 \cdot \gamma^{xx} M^2 \gamma^{xx}); \quad a, b = 0, \dots, n-1. \quad (106) \\
&= p^2 \cdot \left[\begin{aligned} &(\gamma^a h_{,a} + \gamma^{xx} \cdot M \cdot h)(h_{,b} \gamma^b + \gamma^{xx} \cdot M \cdot h) \\ &+ (\gamma^b h_{,b} + \gamma^{xx} \cdot M \cdot h)(h_{,a} \gamma^a + \gamma^{xx} \cdot M \cdot h) \end{aligned} \right] \\
\Rightarrow 0 &= p \cdot \left\{ \begin{aligned} &\gamma^a h_{,a} + \gamma^{xx} \cdot M \cdot h \\ &h_{,b} \gamma^b + \gamma^{xx} \cdot M \cdot h \end{aligned} \right\} = \gamma^{\alpha} (p \cdot h)_{,\alpha} = \gamma^{\alpha} f_{,\alpha}
\end{aligned}$$

We find that in contrast to the classical Dirac approach, the rank problem has disappeared, but the question is whether we would be able to find a solution for the equation in the last line of (106) with just a scalar function f , respectively h^*p . We will see, in connection with our investigations on base vectors, that the answer is no.

This is not surprising because our factorization corresponds to a spectral decomposition and as elaborated in [7] section “Towards Fermionic “Coherent Domains”, Part I: Linearity via Spectral-Operator Decomposition” for this we require a set of eigenvalues or eigen solutions. Nothing else are the lists of functions being of need to make the Dirac factorization work. We are going to investigate this issue at the end of this chapter in subsection “Apparently, Dirac Is Not What Everyone Thought It Was” further below.

8.3 Do We Really Need the Dirac Matrices?

It should be noted that, in applying the mass creation via additional dimensions as demonstrated above, this can also be used to produce masses in the original—metrically obtained—equations (95). Again we would use (99) together with a function $p_{[xx]}$ but a general metric where only the coordinate xx appears as Minkowski like. In other words, we have something like:

$$g^{\alpha\beta} = \begin{pmatrix} g^{ab} & 0 \\ 0 & 1 \end{pmatrix}. \quad (107)$$

Now we can repeat the evaluation (102) above with the tetrads and have to write:

$$\begin{aligned}
0 &= h_{,a} h_{,b} g^{ab} + M^2 h^2 = e^a \cdot e^b h_{,a} h_{,b} + M^2 h^2 \\
&= e^a \cdot e^b h_{,a} h_{,b} + M^2 h^2 + i \cdot M \cdot h \overbrace{(e^b h_{,b} - e^a h_{,a})}^{=0} \\
&= (e^a h_{,a} + i \cdot M \cdot h)(h_{,b} e^b - i \cdot M \cdot h) \quad . \\
\Rightarrow 0 &= \begin{cases} e^a h_{,a} - i \cdot M \cdot h \\ h_{,b} e^b + i \cdot M \cdot h \end{cases}
\end{aligned} \quad (108)$$

This time it is the vector character of the e^a, e^b , which forces us to the following options:

$$a) \quad 0 = \begin{cases} e^a h_{,a} - i \cdot M \cdot h \\ h_{,b} e^b + i \cdot M \cdot h \end{cases}; \quad b) \quad 0 = \begin{cases} e^a h_{,a} - i \cdot M \cdot h \\ h_{,b} e^b + i \cdot M \cdot h \end{cases}. \quad (109)$$

It should be pointed out, however, that—perhaps—our vector problem only occurred because we threw away parts of the full field equations (9) or (94) when performing the limiting procedure. When sticking to the full form, namely:

$$0 = \lim_{n \rightarrow \infty} \frac{(f + C_{f0})}{(n-2)} \cdot \left(\begin{aligned} & \frac{(f + C_{f0})^2}{(n-2)} \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) \\ & - \left(\begin{aligned} & f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \begin{pmatrix} g_{ac, \beta} - \frac{1}{2} n g_{ac, \beta} \\ -\frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \end{pmatrix} \end{aligned} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & + g_{\alpha\beta} \left((n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \\ & - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right) \end{aligned} \right) \\ \Rightarrow \boxed{0 = g_{\alpha\beta} f_{,a} f_{,b} g^{ab}} \end{aligned} \quad , \quad (110)$$

we might not ignore the covariant metric tensor as we did in (94) (or (9)). Together with our h*p[xx]-approach this gives:

$$\begin{aligned} 0 &= g_{\alpha\beta} h_{,a} h_{,b} g^{ab} + g_{\alpha\beta} M^2 h^2 = \mathbf{e}_\alpha \cdot \mathbf{e}_\beta \cdot \mathbf{e}^a \cdot \mathbf{e}^b h_{,a} h_{,b} + \mathbf{e}_\alpha \cdot \mathbf{e}_\beta M^2 h^2 \\ &= \mathbf{e}_\alpha \cdot \mathbf{e}_\beta \cdot \mathbf{e}^a \cdot \mathbf{e}^b h_{,a} h_{,b} + \mathbf{e}_\alpha \cdot \mathbf{e}_\beta \cdot M^2 h^2 + \mathbf{e}_\alpha \cdot \mathbf{e}_\beta \cdot i \cdot M \cdot h \cdot \overbrace{(\mathbf{e}^b h_{,b} - \mathbf{e}^a h_{,a})}^{=0} \\ &= \mathbf{e}_\alpha \cdot (\mathbf{e}^a h_{,a} + i \cdot M \cdot h) (h_{,b} \mathbf{e}^b - i \cdot M \cdot h) \cdot \mathbf{e}_\beta \quad . \quad (111) \\ \Rightarrow 0 &= \begin{cases} \mathbf{e}_\alpha \cdot (\mathbf{e}^a h_{,a} + i \cdot M \cdot h) \\ (h_{,b} \mathbf{e}^b - i \cdot M \cdot h) \cdot \mathbf{e}_\beta \end{cases} \end{aligned}$$

Realizing that this does not solve the rank problem, we revisit our derivation from (9) and see that, while it is perfectly ok to consider M^2 as a scalar in (99) and (101), we cannot do so in the factorized version of this equation. Here the correct derivation of (108) should go as follows:

$$\begin{aligned} 0 &= h_{,a} h_{,b} g^{ab} + M^2 h^2 = \mathbf{e}^a \cdot \mathbf{e}^b h_{,a} h_{,b} + \mathbf{e}^{xx} \cdot \mathbf{e}^{xx} M^2 h^2 \\ &= \mathbf{e}^a \cdot \mathbf{e}^b h_{,a} h_{,b} + \overbrace{\mathbf{e}^{xx} \cdot \mathbf{e}^{xx}}^{=1} M^2 h^2 + i \cdot \mathbf{e}^{xx} M \cdot h \cdot \overbrace{(\mathbf{e}^b h_{,b} - \mathbf{e}^a h_{,a})}^{=0} \\ &= (\mathbf{e}^a h_{,a} + i \cdot \mathbf{e}^{xx} M \cdot h) (h_{,b} \mathbf{e}^b - i \cdot \mathbf{e}^{xx} M \cdot h) \quad . \quad (112) \\ \Rightarrow 0 &= \begin{cases} \mathbf{e}^a h_{,a} - i \cdot M \cdot h \cdot \mathbf{e}^{xx} \\ h_{,b} \mathbf{e}^b + i \cdot M \cdot h \cdot \mathbf{e}^{xx} \end{cases} \end{aligned}$$

Now the rank problem has vanished.

But can these equations be solved? Well, the answer—in general—is no, but this we best discuss by the means of a suitable, which in this case means “simple”, example (see subsection below).

The “NO”, however, is not surprising, because our factorization corresponds to a spectral decomposition and as elaborated in [7], section “Towards Fermionic “Coherent Domains”, Part I: Linearity via Spectral-Operator Decomposition”, for this we require a set of eigenvalues or eigen solutions. Nothing else are the lists of functions being of need to make the Dirac factorization work. A more comprehensive discussion of this problem, showing that a solution can be found by the means of spectral decomposition, is given in [6].

8.4 Example in 2D+1

Now we want to consider an example for a massive object in three dimensions with one dimension providing the mass. Assuming the following metric tensor:

$$g_{ab} = g^{ab} = \mathbf{e}^a \cdot \mathbf{e}^b = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (113)$$

with time-like coordinate t , positioned at $a, b=0$ and the last coordinate $a, b=2$ giving the mass producing xx -coordinate, we define the base vectors:

$$\mathbf{e}^a = \begin{pmatrix} \mathbf{e}^0 \\ \mathbf{e}^1 \\ \mathbf{e}^2 \end{pmatrix} = \begin{pmatrix} e_{00} & e_{01} & e_{02} \\ e_{10} & e_{11} & e_{12} \\ e_{20} & e_{21} & e_{22} \end{pmatrix}. \quad (114)$$

The particle shall not be “at rest”. In order to solve the metrically derived Dirac equation in the last line from (112), we have to demand the following conditions for our base vector components:

$$\begin{aligned} e_{00}^2 + e_{01}^2 + e_{02}^2 &= -1, e_{00} \cdot e_{10} + e_{01} \cdot e_{11} + e_{02} \cdot e_{12} = 0, \\ e_{00} \cdot e_{20} + e_{01} \cdot e_{21} + e_{02} \cdot e_{22} &= 0, \\ e_{10}^2 + e_{11}^2 + e_{12}^2 &= 1, e_{10} \cdot e_{20} + e_{11} \cdot e_{21} + e_{12} \cdot e_{22} = 0, \\ e_{20}^2 + e_{21}^2 + e_{22}^2 &= 1, e_{00} + e_{01} = e_{20}, e_{01} + e_{11} = e_{21}, e_{02} + e_{12} = e_{22} \end{aligned} \quad (115)$$

Unfortunately, this system has no solutions, which is, as we elaborated above, because our factorization corresponds to a spectral decomposition, and for this we require a set of eigenvalues or eigen solutions. As already said above, nothing else are the lists of functions or spinors being of need to make the Dirac factorization work.

That being said, however, it should just be mentioned that building in tiny twists or perturbations into the set of equations like the following one, for instance (see the boxed term which contains the “twist”):

$$\begin{aligned} e_{00}^2 + e_{01}^2 + e_{02}^2 &= -1, e_{00} \cdot e_{10} + e_{01} \cdot e_{11} + e_{02} \cdot e_{12} = 0, \\ e_{00} \cdot e_{20} + e_{01} \cdot e_{21} + e_{02} \cdot e_{22} &= 0, \\ e_{10}^2 + e_{11}^2 + e_{12}^2 &= 1, e_{10} \cdot e_{20} + e_{11} \cdot e_{21} + e_{12} \cdot e_{22} = 0, \\ e_{20}^2 + e_{21}^2 + e_{22}^2 &= 1, e_{00} + e_{01} = e_{20}, \boxed{e_{10}} + e_{11} = e_{21}, e_{02} + e_{12} = e_{22} \end{aligned} \quad (116)$$

one can find “solutions”. This time, the system can be solved and we find 8 sets of base vector components:

$$\begin{aligned}
e_{00} &\rightarrow \frac{1}{4}\sqrt{27-7\sqrt{5}}, e_{01} \rightarrow -\frac{1}{4}\sqrt{3+\sqrt{5}}, e_{02} \rightarrow -\frac{1}{4}i(-1+3\sqrt{5}), \\
e_{10} &\rightarrow -\frac{1}{4}\sqrt{5(3+\sqrt{5})}, e_{11} \rightarrow \frac{1}{4}\sqrt{7-3\sqrt{5}}, e_{12} \rightarrow \frac{1}{4}i(1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow \frac{1}{2}\sqrt{7-3\sqrt{5}}, e_{21} \rightarrow -\frac{1}{2}\sqrt{3+\sqrt{5}}, e_{22} \rightarrow -\frac{1}{2}i(-1+\sqrt{5})
\end{aligned} \tag{117}$$

$$\begin{aligned}
e_{00} &\rightarrow \frac{-7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{4}, e_{02} \rightarrow -\frac{1}{4}i(-1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{1}{4}\sqrt{5(3+\sqrt{5})}, e_{11} \rightarrow \frac{-3+\sqrt{5}}{4\sqrt{2}}, e_{12} \rightarrow \frac{1}{4}i(1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow \frac{-3+\sqrt{5}}{2\sqrt{2}}, e_{21} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{2}, e_{22} \rightarrow -\frac{1}{2}i(-1+\sqrt{5})
\end{aligned} \tag{118}$$

$$\begin{aligned}
e_{00} &\rightarrow \frac{1}{4}\sqrt{27-7\sqrt{5}}, e_{01} \rightarrow -\frac{1}{4}\sqrt{3+\sqrt{5}}, e_{02} \rightarrow \frac{1}{4}i(-1+3\sqrt{5}), \\
e_{10} &\rightarrow -\frac{1}{4}\sqrt{5(3+\sqrt{5})}, e_{11} \rightarrow \frac{1}{4}\sqrt{7-3\sqrt{5}}, e_{12} \rightarrow -\frac{1}{4}i(1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow \frac{1}{2}\sqrt{7-3\sqrt{5}}, e_{21} \rightarrow -\frac{1}{2}\sqrt{3+\sqrt{5}}, e_{22} \rightarrow \frac{1}{2}i(-1+\sqrt{5})
\end{aligned} \tag{119}$$

$$\begin{aligned}
e_{00} &\rightarrow \frac{-7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{4}, e_{02} \rightarrow \frac{1}{4}i(-1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{1}{4}\sqrt{5(3+\sqrt{5})}, e_{11} \rightarrow \frac{-3+\sqrt{5}}{4\sqrt{2}}, e_{12} \rightarrow -\frac{1}{4}i(1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow \frac{-3+\sqrt{5}}{2\sqrt{2}}, e_{21} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{2}, e_{22} \rightarrow \frac{1}{2}i(-1+\sqrt{5})
\end{aligned} \tag{120}$$

$$\begin{aligned}
e_{00} &\rightarrow -\frac{7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow -\frac{1}{4}\sqrt{3-\sqrt{5}}, e_{02} \rightarrow -\frac{1}{4}i(1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{1}{4}\sqrt{15-5\sqrt{5}}, e_{11} \rightarrow -\frac{3+\sqrt{5}}{4\sqrt{2}}, e_{12} \rightarrow \frac{1}{4}i(-1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow -\frac{3+\sqrt{5}}{2\sqrt{2}}, e_{21} \rightarrow \text{Root}[1-6\#1^2+4\#1^4\&,2], e_{22} \rightarrow -\frac{1}{2}i(1+\sqrt{5})
\end{aligned} \tag{121}$$

$$\begin{aligned}
e_{00} &\rightarrow \frac{7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow \frac{\sqrt{3-\sqrt{5}}}{4}, e_{02} \rightarrow -\frac{1}{4}i(1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{-5+\sqrt{5}}{4\sqrt{2}}, e_{11} \rightarrow \frac{1}{4}\sqrt{7+3\sqrt{5}}, e_{12} \rightarrow \frac{1}{4}i(-1+\sqrt{5}), \quad , \\
e_{20} &\rightarrow \frac{1}{2}\sqrt{7+3\sqrt{5}}, e_{21} \rightarrow \frac{\sqrt{3-\sqrt{5}}}{2}, e_{22} \rightarrow -\frac{1}{2}i(1+\sqrt{5})
\end{aligned} \tag{122}$$

$$\begin{aligned}
e_{00} &\rightarrow -\frac{7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow -\frac{1}{4}\sqrt{3-\sqrt{5}}, e_{02} \rightarrow \frac{1}{4}i(1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{1}{4}\sqrt{15-5\sqrt{5}}, e_{11} \rightarrow -\frac{3+\sqrt{5}}{4\sqrt{2}}, e_{12} \rightarrow -\frac{1}{4}i(-1+\sqrt{5}), \\
e_{20} &\rightarrow -\frac{3+\sqrt{5}}{2\sqrt{2}}, e_{21} \rightarrow \text{Root}\left[1-6\#1^2+4\#1^4\&,2\right], e_{22} \rightarrow \frac{1}{2}i(1+\sqrt{5})
\end{aligned} \tag{123}$$

$$\begin{aligned}
e_{00} &\rightarrow \frac{7+\sqrt{5}}{4\sqrt{2}}, e_{01} \rightarrow \frac{\sqrt{3-\sqrt{5}}}{4}, e_{02} \rightarrow \frac{1}{4}i(1+3\sqrt{5}), \\
e_{10} &\rightarrow \frac{-5+\sqrt{5}}{4\sqrt{2}}, e_{11} \rightarrow \frac{1}{4}\sqrt{7+3\sqrt{5}}, e_{12} \rightarrow -\frac{1}{4}i(-1+\sqrt{5}), \\
e_{20} &\rightarrow \frac{1}{2}\sqrt{7+3\sqrt{5}}, e_{21} \rightarrow \frac{\sqrt{3-\sqrt{5}}}{2}, e_{22} \rightarrow \frac{1}{2}i(1+\sqrt{5})
\end{aligned} \tag{124}$$

With these base vectors, the last line from (112) can be written as follows:

$$\begin{aligned}
0 &= h_{,a} h_{,b} g^{ab} + M^2 h^2 = e^a \cdot e^b h_{,a} h_{,b} + e^{xx} \cdot e^{xx} M^2 h^2 \\
&= e^a \cdot e^b h_{,a} h_{,b} + \overbrace{e^{xx} \cdot e^{xx}}^{=1} M^2 h^2 + i \cdot e^{xx} M \cdot h \left(\overbrace{e^b h_{,b} - e^a h_{,a}}^{=0} \right) \\
&= (e^a h_{,a} + i \cdot e^{xx} M \cdot h) (h_{,b} e^b - i \cdot e^{xx} M \cdot h) \\
&\Rightarrow 0 = \begin{cases} e^0 h_{,0} + e^1 h_{,1} - i \cdot M \cdot h \cdot e^{xx} \\ e^0 h_{,0} + e^1 h_{,1} + i \cdot M \cdot h \cdot e^{xx} \end{cases}
\end{aligned} \tag{125}$$

The situation simplifies when considering a “particle at rest” with $f=f[t]$, where we would still have to solve the following set of equations when considering the full metric tensor including the mass-creating coordinate xx :

$$\begin{aligned}
e_{00}^2 + e_{01}^2 + e_{02}^2 &= -1, e_{00} \cdot e_{10} + e_{01} \cdot e_{11} + e_{02} \cdot e_{12} = 0, \\
e_{00} \cdot e_{20} + e_{01} \cdot e_{21} + e_{02} \cdot e_{22} &= 0, \\
e_{10}^2 + e_{11}^2 + e_{12}^2 &= 1, e_{10} \cdot e_{20} + e_{11} \cdot e_{21} + e_{12} \cdot e_{22} = 0, \\
e_{20}^2 + e_{21}^2 + e_{22}^2 &= 1, e_{00} = e_{20}, e_{01} = e_{21}, e_{02} = e_{22}
\end{aligned} \tag{126}$$

Again, no solution can be found.

When, however, only considering the metric part without the xx -coordinate and treating the mass in the classical Dirac way, namely just as a scalar parameter “falling from the sky”, (112) can be written as follows:

$$\begin{aligned}
0 &= h_{,a} h_{,b} g^{ab} + M^2 h^2 = e^a \cdot e^b h_{,a} h_{,b} + e^{xx} \cdot e^{xx} M^2 h^2 \\
&= e^a \cdot e^b h_{,a} h_{,b} + e^{xx} \cdot e^{xx} M^2 h^2 + i \cdot e^{xx} M \cdot h (e^b h_{,b} - e^a h_{,a}) \\
&= e^a \cdot e^b h_{,a} h_{,b} + e^{xx} \cdot e^{xx} M^2 h^2 + i \cdot e^{xx} M \cdot h \overbrace{(e^0 h_{,0} + e^1 h_{,1} - (e^0 h_{,0} + e^1 h_{,1}))}^{=0} \\
&= (e^a h_{,a} + i \cdot e^{xx} M \cdot h) (h_{,b} e^b - i \cdot e^{xx} M \cdot h) \\
\Rightarrow 0 &= \begin{cases} e^0 h_{,0} + e^1 h_{,1} - i \cdot M \cdot h \cdot e^{xx} \\ e^0 h_{,0} + e^1 h_{,1} + i \cdot M \cdot h \cdot e^{xx} \end{cases} \xrightarrow{h[t]_i=0} = \begin{cases} e^0 h_{,0} - i \cdot M \cdot h \cdot e^{xx} \\ e^0 h_{,0} + i \cdot M \cdot h \cdot e^{xx} \end{cases}
\end{aligned} \tag{127}$$

As said, we do not need to take care about the properties of the vector e^{xx} other than that it satisfies the condition:

$$e^0 = e^{xx} \tag{128}$$

or

$$e^0 = \text{const} \cdot e^{xx}. \tag{129}$$

Interestingly, with the resulting boundary conditions as:

$$\begin{aligned}
e^{00^2} + e^{01^2} + e^{02^2} &= -1, e^{00} \cdot e^{10} + e^{01} \cdot e^{11} + e^{02} \cdot e^{12} = 0, \\
e^{10^2} + e^{11^2} + e^{12^2} &= 1, e^{00} = e^{20}, e^{01} = e^{21}, e^{02} = e^{22}
\end{aligned} \tag{130}$$

we now can find solutions. The only problem would be that these solutions are creating metrics with zero determinants $g=0$, which means they are coding spaces with no volume. A way to overcome the problematic issue is to incorporate other dimensions/coordinates on which the function f does not depend. For our example of the “particle at rest” in 2 dimensions plus a dimension for the mass and another one to keep the space determinant non-zero, the following approach would do:

$$e^\alpha = \begin{pmatrix} e^0 \\ e^1 \\ e^2 \\ e^3 \end{pmatrix} = \begin{pmatrix} e^{00} & e^{01} & e^{02} & 0 \\ e^{10} & e^{11} & e^{12} & 0 \\ e^{20} & e^{21} & e^{22} & e^{23} \\ 0 & 0 & e^{32} & e^{33} \end{pmatrix}. \tag{131}$$

For a non-resting particle we would need one dimension more together with the following boundary conditions:

$$\begin{aligned}
e^{00^2} + e^{01^2} + e^{02^2} + e^{03^2} &= -1, e^{00} \cdot e^{10} + e^{01} \cdot e^{11} + e^{02} \cdot e^{12} + e^{03} \cdot e^{13} = 0, \\
e^{10^2} + e^{11^2} + e^{12^2} + e^{13^2} &= 1, e^{00} = e^{20}, e^{01} = e^{21}, e^{02} = e^{22}, e^{03} = e^{23}, \\
e^{10} &= e^{13}, e^{11} = e^{31}, e^{12} = e^{32}, e^{13} = e^{33}
\end{aligned} \tag{132}$$

The corresponding base vectors could be:

$$e^\alpha = \begin{pmatrix} e^0 \\ e^1 \\ e^2 \\ e^3 \\ e^4 \end{pmatrix} = \begin{pmatrix} e^{00} & e^{01} & e^{02} & e^{03} & 0 \\ e^{10} & e^{11} & e^{12} & e^{13} & 0 \\ e^{20} & e^{21} & e^{22} & e^{23} & e^{24} \\ e^{30} & e^{31} & e^{32} & e^{33} & 0 \\ 0 & 0 & e^{42} & 0 & e^{44} \end{pmatrix}. \tag{133}$$

8.5 Apparently, Dirac Is Not What Everyone Thought It Was

For convenience and as the following is very important, if not to say quite stunning, we here repeat the derivations above from the very beginning, which is to say from the extended Hilbert-Action integral (e.g., appendix (198)), reading:

$$\begin{aligned}
 0 &= \int_V d^n x \sqrt{-g} \left(R_{\kappa\lambda} \delta g^{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} \delta g^{\kappa\lambda} \right) \\
 &= \int_V d^n x \sqrt{-g} \left(R_{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} \right) \delta g^{\kappa\lambda} \quad . \quad (134) \\
 &\Rightarrow R_{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} = 0
 \end{aligned}$$

We saw that the complete equation, when assuming a scaled metric tensor of the form (1) and setting

$$F[f] = C_{f1} (f + C_{f0})^{-2}, \quad (135)$$

would read:

$$0 = \frac{(f + C_{f0})}{(n-2)} \cdot \left(\begin{aligned} &\frac{(f + C_{f0})^2}{(n-2)} \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) \\ &- \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,a} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \end{aligned} \right) \\ &- g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ &+ g_{\alpha\beta} \left((n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \\ &- g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right) \end{aligned} \right) \quad . \quad (136)
 \end{aligned}$$

When investigating the situation for bigger n , we realize that for systems with high dimensionality the terms:

$$0 = \frac{(f + C_{f0})}{(n-2)} \cdot \left(\begin{aligned} &\frac{(f + C_{f0})^2}{(n-2)} \cdot \left(g_{\alpha\beta} \mathbf{R} \cdot \left(\frac{1}{2} + H \right) - \mathbf{R}_{\alpha\beta} \right) \\ &- \left(\begin{aligned} &f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,a} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} \right) \end{aligned} \right) \\ &- g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ &+ g_{\alpha\beta} \left((n-1) \left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \\ &- g_{\alpha\beta} f_{,c} f_{,d} g^{cd} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right) \end{aligned} \right) \quad (137)
 \end{aligned}$$

become recessive, while the nonlinear term:

$$-g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right) \quad (138)$$

dominates the field (equations).

In other words, our field equations (136) (or (6)) converge to the following extremely simple but still nonlinear result in a limiting procedure for high numbers of n:

$$0 = \lim_{n \rightarrow \infty} \left(\frac{(f + C_{f0})^2}{(n-2)} \cdot \left(g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right) \right. \\ \left. - \left(f_{,\alpha\beta} (n-2) + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \right) \right. \\ \left. - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} \right) \right. \\ \left. - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha \beta, c} \right) \\ \left. - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \right. \\ \left. + g_{\alpha\beta} \left((n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} \right) \right) \cdot \left(\frac{1}{2} + H \right) \right. \\ \left. - 2g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{4(n-2)} \right) \right) \\ \Rightarrow \boxed{0 = 2g_{\alpha\beta} f_{,a} f_{,b} g^{ab}} \quad (139)$$

It should be pointed out that this is the same evaluation as before, but this time we did not ignore the metric tensor $g_{\alpha\beta}$ in the term $-g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right)$. Now we remember what was said in [7], sections “Towards Fermionic “Coherent Domains”, Part I: Linearity via Spectral-Operator Decomposition” and “Towards Fermionic “Coherent Domains”, Part II:...”, where we observed the product $g_{\alpha\beta} g^{ab}$ and realized that this is already very close to the metric definition of the classical Dirac matrices usually given as:

$$g^{\alpha\beta} \cdot I = \frac{\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha}{2}; \quad g_{\alpha\beta} \cdot I = \frac{\gamma_\alpha \gamma_\beta + \gamma_\beta \gamma_\alpha}{2}, \quad (140)$$

only that here, we do not need to artificially weave in the unit matrix I, but only have to adjust the Dirac matrices a bit, so that they could reproduce something like:

$$g_{\mu\nu} g^{ab} = \left[\frac{\gamma^a \gamma^b + \gamma^b \gamma^a}{2} \right]_{\mu\nu} = \frac{\gamma^a \gamma^b + \gamma^b \gamma^a}{2}. \quad (141)$$

Now the last line in (139) can be transformed to:

$$0 = 2g_{\alpha\beta} f_{,a} f_{,b} g^{ab} = g_{\alpha\beta} g^{ab} f_{,a} f_{,b} + g_{\alpha\beta} g^{ab} f_{,a} f_{,b} = (\gamma^a \gamma^b + \gamma^b \gamma^a) f_{,a} f_{,b} = \gamma^a f_{,a} f_{,b} \gamma^b + \gamma^b f_{,a} f_{,b} \gamma^a. \quad (142)$$

From there, Dirac equations can be extracted by direct factorization, leading to:

$$0 = \gamma^a f_{,a} = f_{,b} \gamma^b . \quad (143)$$

The reader wondering about the missing mass term from the complete classical Dirac equation should be reminded that masses and other scalar parameters can always be added via additional dimensions (c.f. subsection “The Nonlinear Term Prevails in Infinity” above). In the classical case with 4 dimensions with mass we then have to apply 5D-Dirac matrices, but could also apply the usual binomial separation as used in the original Dirac work [12].

The other and more important difference to the origin of the classical Dirac equation is that, while we have to decompose (factorize) a Klein-Gordon equation in the classical case, namely:

$$0 = \Delta f \xrightarrow{\text{or here}} 0 = \Delta f \cdot g_{\alpha\beta} , \quad (144)$$

which in metrics with constant components can be written as:

$$0 = g_{\alpha\beta} f_{,ab} g^{ab} , \quad (145)$$

we find something completely different in the limit $n \rightarrow \infty$ of our quantum gravity equations (136). Here the origin reads (c.f. (139)):

$$0 = 2g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \quad (146)$$

and this can be factorized by using the Dirac matrices without bothering about the difference between (144) and (145) in case with non-constant metrics. However, the matrices—of course—still change with the coordinate systems also in the (146) case.

The difference of the Dirac versions being extracted from (145) and (146) become visible in curvilinear coordinates. As an example we here consider spherical coordinates in 3 spatial dimensions plus time with f only depending on the time and the radius coordinate. While (144) could be factorized via:

$$\begin{aligned} 0 = 2g_{\alpha\beta} \Delta f &= g_{\alpha\beta} \left(\partial_t^2 + \partial_r^2 + 2 \frac{\partial_r}{r} \right) f + g_{\alpha\beta} \left(\partial_t^2 + \partial_r^2 + 2 \frac{\partial_r}{r} \right) f \\ &= \left(\left(\gamma^t \partial_t + \gamma^r \left(\partial_r + \frac{1}{r} \right) \right) \left(\gamma^t \partial_t + \gamma^r \left(\partial_r + \frac{1}{r} \right) \right) f \right) , \\ &= \left(\left(\gamma^r \left(\partial_r + \frac{1}{r} \right) + \gamma^t \partial_t \right) \left(\gamma^r \left(\partial_r + \frac{1}{r} \right) + \gamma^t \partial_t \right) f \right) \end{aligned} \quad (147)$$

we have the much simpler expression for (146):

$$\begin{aligned} 0 = 2g_{\alpha\beta} f_{,a} f_{,b} g^{ab} &= g_{\alpha\beta} \left((\partial_t f)^2 + (\partial_r f)^2 \right) + g_{\alpha\beta} \left((\partial_t f)^2 + (\partial_r f)^2 \right) \\ &= (\gamma^t \partial_t f + \gamma^r \partial_r f) (\gamma^t \partial_t f + \gamma^r \partial_r f) + (\gamma^r \partial_r f + \gamma^t \partial_t f) (\gamma^r \partial_r f + \gamma^t \partial_t f) \end{aligned} \quad (148)$$

We see that in (146) the typical radial symmetry term $1/r$ has disappeared and that the resulting equation has become Cartesian or—rather—Cartesian-like, as the underlying metric—of course—still is a spherical one. This means that in systems with huge numbers of dimensions, certain geometrical aspects, namely the ones showing in the second derivative, simply disappear because there is no second derivative for the scaling function f in (146). A more comprehensive consideration to the “Cartesian Dirac” situation in systems with high numbers of dimensions is given in [6].

8.6 Asymmetries

Taking (139) and assuming a situation just before n being big enough to truly make the term (138) dominant, we have an interesting constellation. For simplicity, we consider metrics of constants and can write (139) as follows:

$$0 = \lim_{n \rightarrow \infty} \left(\frac{(f + C_{f0})}{(n-2)} \cdot (g_{\alpha\beta} f_{,ab} g^{ab} ((n-1) \cdot (1+2H) - 1) - f_{,\alpha\beta} (n-2)) \right. \\ \left. - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2(n-2)} \right) \right) \quad (149)$$

$$= \lim_{n \rightarrow \infty} \left((f + C_{f0}) \cdot (g_{\alpha\beta} f_{,ab} g^{ab} ((n-1) \cdot (1+2H) - 1) - f_{,\alpha\beta} (n-2)) \right. \\ \left. - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \left(\frac{(n-1)(n+2nH-2)}{2} \right) \right)$$

This, however, adds something potentially asymmetric to the otherwise completely symmetric term (138) and could potentially explain the matter-antimatter asymmetry of the universe. We are going to further investigate this aspect in [7].

9 The Mass-Formation Induced Reduction of Dimensionality and the Subsequent Dominance of Gravity

So far we went from the general quantum gravity field equations via a limiting procedure towards huge numbers of dimensions to our classical quantum field equations without, respectively, only a weak gravity. In many constellations (e.g., (9)) we thereby saw all Ricci- and thus, gravitational terms become recessive against Dirac-like quantum equations.

The opposite process, meaning where gravity comes back into the game and perhaps becomes dominant or even the only recognizable force, consequently requires a dimensional reduction. These are definitively the characteristics of the formation of black holes, where the external observer (meaning the one for whom the object increases its gravitational dominance in comparison to other forces) realizes the phenomenological collapse of a complex ensemble of intricately entangled degrees of freedom to only three parameters, which are mass m or Schwarzschild radius r_s , electric charge q or characteristic Reissner-Nordstrom length r_0 , and angular momentum J ⁴. The internal observer may not even see such a reduction in dimensionality and consequently would still observe a relatively weak gravity. To him or her, the environment might even just appear like a whole universe of its own.

9.1 Field Equations in 6 Dimensions

In the sections above, we have seen that due to the fundamental degree of freedom that we have with the functional wrapper of the metric scaling or volume function $F[f]$, we end up with a great variety of options also with respect to the behavior in systems with a fixed number of dimensions. Taking the 3 parameters of a black hole, Schwarzschild radius r_s , characteristic Reissner-Nordstrom length r_0 , and angular momentum J , and adding the 3 spatial coordinates (assuming that the black hole has no time dimension), we end up with parameter spaces of 6 dimensions. As before in the section “Summing up a Few Infinity Options” we here now want to present the various field options for this case.

⁴ Thereby we ignore any movement of the black hole as a whole, because this could always be transformed away.

9.1.1 Kernel Only

We start with the consideration of only the kernel of the Hilbert variation (26) after the variation, thereby applying the classical conjecture that it suffices to make this kernel to zero in order to fulfill the complete integral equation.

9.1.1.1 F[f] Not fixed, but Independent on n

Taking (3), we have seen that we have a variety of options to obtain various n-dependencies. Leaving F unfixed and assuming that it does not depend on the number of dimensions, n=6 results in:

$$0 = \left(\begin{aligned} & \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} & -2F_{,\alpha\beta} + F_{,ab}g_{\alpha\beta}g^{ab} - F_{,\beta}g^{ab}g_{\alpha b,a} \\ & + F_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha}g^{ab}g_{\beta b,a} \\ & + F_{,d}g^{cd} \left(g_{\alpha c,\beta} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \right) \end{aligned} \right) \right. \\ & \quad \left. + \frac{1}{2F^2}(-3F_{,\alpha} \cdot F_{,\beta} + 2g_{\alpha\beta}F_{,c}F_{,d}g^{cd}) \right) \\ & - \left(R + \frac{1}{2F}(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}) - 3\frac{g^{ab}F_{,a} \cdot F_{,b}}{2F^2} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ & + 3 \left(\begin{aligned} & \left(-\frac{1}{F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2}g^{cd}(g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right. \\ & \quad \left. + \frac{1}{2F^2}(3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta}F_{,c}F_{,d}g^{cd}) \right) \\ & + \left(\frac{1}{F}(2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c} - F_{,d}g^{cd}g^{ab}g_{ac,b}) \right) \\ & \quad \left. + \frac{11}{2F^2} \frac{g^{ab}F_{,a} \cdot F_{,b}}{2F^2} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \end{aligned} \right) \end{aligned} \right). \quad (150)$$

9.1.1.2 F[f] Fixed Such That the Nonlinear F-Term in the Ricci Scalar Vanishes

Setting F[f] as:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases} \xrightarrow{n=6} F[f] = C_F \cdot (f + C_f), \quad (151)$$

(2) evolves to:

$$0 = \left(\begin{aligned} & R_{\alpha\beta} - \frac{1}{2(f + C_f)} \cdot \left(\begin{aligned} & 4f_{,\alpha\beta} + f_{,a}g^{ab}(g_{\beta b,\alpha} - g_{\beta\alpha,b}) - f_{,\alpha}g^{ab}g_{\beta b,a} \\ & - f_{,\beta}g^{ab}g_{\alpha b,a} + f_{,d}g^{cd}(g_{\alpha c,\beta} + 3(g_{\alpha\beta,c} - g_{\alpha c,\beta} - g_{\beta c,\alpha})) \end{aligned} \right) \\ & - g_{\alpha\beta} \frac{1}{2(f + C_f)} \cdot \left(f_{,ab}g^{ab} + f_{,d}g^{cd} \frac{1}{2}g_{ab,c}g^{ab} \right) \\ & + g_{\alpha\beta} \left(\frac{1}{2(f + C_f)} \cdot \left(\begin{aligned} & 5(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ & - 6f_{,d}g^{cd}g^{ab}g_{ac,b} \end{aligned} \right) - R \right) \cdot \left(\frac{1}{2} + H \right) \\ & + f_{,\alpha} \cdot f_{,\beta} \frac{3}{(f + C_f)^2} - g_{\alpha\beta} \frac{f_{,a}f_{,b}g^{ab}}{2(f + C_f)^2} \end{aligned} \right). \quad (152)$$

9.1.1.3 F[f] Fixed Such That the Scalar Nonlinear Terms (Times Metric Tensor) in the Whole Einstein Tensor Vanish

Now our setting for F(f) shall be:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4(2H(n-1)+n-2)}{(n-2)(2H(n-1)+n-3)}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \cup n = \frac{3+2H}{1+2H} \end{cases} \xrightarrow{n=6} \cdot \quad (153)$$

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{2(5H+2)}{10H+3}} & 6 \neq \frac{3+2H}{1+2H} \Rightarrow H \neq -\frac{3}{10} \\ C_F \cdot e^{f \cdot C_f} & 6 = \frac{3+2H}{1+2H} \Rightarrow H = -\frac{3}{10} \end{cases}$$

In this case (2) yields the following field equations:

$$0 = \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}}$$

$$= \left(\left(R_{\alpha\beta} - \frac{F'}{2F} \left(\begin{aligned} &4f_{,\alpha\beta} + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} + 3(g_{\alpha \beta, c} - g_{\alpha c, \beta} - g_{\beta c, \alpha}) \\ &+ \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right) \right. \\ \left. - \left(R - \frac{F'}{2F} \left(\begin{aligned} &5(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- 6f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\ \left. + \frac{f_{,\alpha} \cdot f_{,\beta}}{F^2} (3(F')^2 - 2FF'') \right) \delta G^{\alpha\beta}$$

$$= \left(\left(R_{\alpha\beta} - \frac{4+10H}{2(3+10H)(f+C_f)} \right. \right. \\ \left. \times \left(\begin{aligned} &4f_{,\alpha\beta} + f_{,ab} g_{\alpha\beta} g^{ab} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ &- f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} \left(\begin{aligned} &g_{\alpha c, \beta} + 3(g_{\alpha \beta, c} - g_{\alpha c, \beta} - g_{\beta c, \alpha}) \\ &+ \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \end{aligned} \right) \end{aligned} \right) \right) \\ \left. - \left(R - \frac{(2+5H)}{(3+10H)(f+C_f)} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\ \left. \times \left(\begin{aligned} &5(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ &- 6f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \right. \\ \left. + f_{,\alpha} \cdot f_{,\beta} \frac{20(2+5H)(1+3H)}{(3+10H)^2 (f+C_f)^2} \right) \delta G^{\alpha\beta} \quad (154)$$

9.1.1.4 F[f] Fixed Such That the Tensorial Nonlinear Term in the Ricci Tensor Vanishes

Applying the function:

$$F[f] = C_{f1} (f + C_{f0})^{-2}, \quad (155)$$

results in:

$$0 = (f + C_{f0}) \cdot \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right) \\ & - \left(\begin{aligned} & 4f_{,\alpha\beta} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} (g_{\alpha c, \beta} + 3(g_{\alpha\beta, c} - g_{\alpha c, \beta} - g_{\beta c, \alpha})) \end{aligned} \right) \\ & - g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & + g_{\alpha\beta} \left(\begin{aligned} & 5(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \\ & - 6f_{,d} g^{cd} g^{ab} g_{ac, b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \\ & - 5g_{\alpha\beta} f_{,a} f_{,b} g^{ab} (3 + 6H - 1) \end{aligned} \right) \\ = \left(\begin{aligned} & (f + C_{f0})^2 \cdot \left(g_{\alpha\beta} R \cdot \left(\frac{1}{2} + H \right) - R_{\alpha\beta} \right) - g_{\alpha\beta} f_{,a} f_{,b} g^{ab} \\ & - \left(\begin{aligned} & -2f_{,\alpha\beta} + f_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - f_{,\alpha} g^{ab} g_{\beta b, a} \\ & - f_{,\beta} g^{ab} g_{\alpha b, a} + f_{,d} g^{cd} g_{\alpha c, \beta} \end{aligned} \right) \\ & + (f + C_{f0}) \cdot \left(\begin{aligned} & -g_{\alpha\beta} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab, c} g^{ab} \right) \\ & - g_{\alpha\beta} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c}) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right) \\ & + (f + C_{f0}) \cdot 6 \cdot \left(\begin{aligned} & - \left(f_{,\alpha\beta} - \frac{f_{,d} g^{cd}}{2} (g_{\alpha c, \beta} + g_{\beta c, \alpha} - g_{\alpha\beta, c}) \right) \\ & + g_{\alpha\beta} (2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab, c} - f_{,d} g^{cd} g^{ab} g_{ac, b}) \cdot \left(\frac{1}{2} + H \right) \\ & - 3g_{\alpha\beta} f_{,a} f_{,b} g^{ab} (3 + 10H) \end{aligned} \right) \end{aligned} \right). \quad (156)$$

9.1.2 Whole Variational Integral

In contrast to the previous subsection, where we considered only the kernel of the Hilbert variation (26) after the variation, thereby applying the classical conjecture that it suffices to make this kernel zero in order to fulfill the complete scalar integral equation, we now take the whole integral into account and thus keep the scalar character of the whole. Unfortunately, in order to do so, we would require some knowledge about the variational term $\delta G^{\alpha\beta}$, which we usually do not have. In order to help us out we assume the following “partial” or “degenerate” variational process:

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \quad \text{with : } \delta g^{\alpha\beta} \equiv 0$$

$$\Rightarrow 0 = \int_V d^n x \cdot \sqrt{-G} \cdot \left(R^*_{\alpha\beta} - R^* \left(\frac{1}{2} + H \right) G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}}. \quad (157)$$

$\xrightarrow{\delta g^{\alpha\beta} \equiv 0}$

This yields:

$$0 = \int_V d^n x \cdot \sqrt{-G} \cdot \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ &+ F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \\ &+ F_{,d} g^{cd} \left(g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} \right) \right. \right. \\ &\left. \left. + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \right) + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right). \quad (158) \\ &\left(\begin{aligned} &\frac{1-n}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \\ &+ \frac{nF_{,d}}{2F} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\ &\left(R - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \end{aligned} \right)$$

For n=6 we obtain:

$$0 = \int_V d^6 x \cdot \sqrt{-G} \cdot \left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &4F_{,\alpha\beta} + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ &+ F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \\ &+ F_{,d} g^{cd} \left(g_{\alpha c,\beta} + 3(g_{\alpha\beta,c} - g_{\alpha c,\beta} - g_{\beta c,\alpha}) \right) \right. \right. \\ &\left. \left. + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \right) + \frac{1}{2F^2} (6F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \right) g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right). \quad (159) \\ &\left(\begin{aligned} &R - \frac{1}{2F} \left(5(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \right. \right. \\ &\left. \left. - 6F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \end{aligned} \right)$$

9.1.2.1 F[f] Not Fixed, but Independent on n

Leaving F open and assuming that it does not depend on n results in the kernel given in (159) for n=6. Taking the integral environment into account, we result in the following derivation:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^6 x \cdot \sqrt{-G} \cdot R^* \\
&= \int_V d^6 x \cdot \sqrt{-G} \cdot K \cdot \left(1 - 6 \left(\frac{1}{2} + H \right) \right) \cdot \delta \left(\frac{1}{F} \right) \\
K &= \left(R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right. \\
&\quad \left. 3 \cdot \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} - \frac{1}{F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \right. \\
&\quad \left. - 9 \frac{g^{ab} F_{,a} \cdot F_{,b}}{F^2} \right), \tag{160}
\end{aligned}$$

$$\begin{aligned}
0 &= \int_V d^6 x \cdot \sqrt{-G} \cdot K \cdot \left(1 - 6 \left(\frac{1}{2} + H \right) \right) \cdot \delta \left(\frac{1}{F} \right) \\
&= \int_V d^6 x \cdot \sqrt{-G} \cdot \left(\begin{array}{c} R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \\ 6 \cdot \left(\begin{array}{c} R + \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \\ - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \end{array} \right) \left(\frac{1}{2} + H \right) \\ -18 \left(\begin{array}{c} \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} - \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} \right) \\ \left(-\frac{1}{F} \left(\begin{array}{c} 2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \\ -F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \right) \end{array} \right) \left(\frac{1}{2} + H \right) \\ +54 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{F^2} \end{array} \right) \cdot \delta \left(\frac{1}{F} \right) \tag{161}
\end{aligned}$$

9.1.2.2 F[f] Fixed Such That the Nonlinear F-Term in the Ricci Scalar Vanishes

Our setting for F[f] shall be:

$$F[f] = \begin{cases} C_F \cdot (f + C_f)^{\frac{4}{n-2}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \end{cases} \xrightarrow{n=6} F[f] = C_F \cdot (f + C_f). \tag{162}$$

We see it as kind of interesting that for n=6 F takes on total linearity in f. In this case the complete variational integral (31) looks as follows:

$$\begin{aligned}
0 &= \int_V d^6x \cdot \sqrt{-G} \cdot K \cdot \left(1 - 6\left(\frac{1}{2} + H\right)\right) \cdot \delta\left(\frac{1}{F}\right) \\
&= \int_V d^6x \cdot \sqrt{-G} \cdot \left[-6 \cdot \left(R + \frac{2}{(f + C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \right) \right. \\
&\quad \left. \left(\frac{1}{2(f + C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \right) \right. \\
&\quad \left. + \left(R + \frac{1}{2(f + C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. + 18 \left(\left(\frac{1}{(f + C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \right) \left(\frac{1}{2} + H \right) \right) \right] \cdot \delta\left(\frac{1}{F}\right) \quad (163)
\end{aligned}$$

and could be brought into the following form:

$$\begin{aligned}
0 &= \int_V d^6x \cdot \sqrt{-G} \cdot \left[-6 \cdot \left(\frac{2}{(f + C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) - 2R \right) \right. \\
&\quad \left(\frac{2}{(f + C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} - R \right) \\
&\quad \left. + \left(\frac{2}{(f + C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) - 2R \right) \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. + 36 \left(\left(\frac{2}{(f + C_f)} \begin{pmatrix} 2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c} \\ -f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} - R \right) \left(\frac{1}{2} + H \right) \right) \right] \cdot \delta\left(\frac{1}{F}\right). \quad (164)
\end{aligned}$$

Further simplification yields:

$$\begin{aligned}
0 &= \left(\begin{aligned} &4R \left(1 - 6\left(\frac{1}{2} + H\right)\right) \\ &+ \frac{10}{(f + C_f)} (2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \left(1 + 6\left(\frac{1}{2} + H\right)\right) \\ &+ \frac{12}{(f + C_f)} f_{,d}g^{cd}g^{ab}g_{ac,b} \left(1 - 6\left(\frac{1}{2} + H\right)\right) \end{aligned} \right) \cdot \delta\left(\frac{1}{F}\right). \quad (165)
\end{aligned}$$

For metrics of the type:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \quad (166)$$

this simplifies to:

$$\begin{aligned}
0 &= 4 \left(R \left(1 - 6 \left(\frac{1}{2} + H \right) \right) + \frac{5}{(f + C_f)} \Delta f \left(1 + 6 \left(\frac{1}{2} + H \right) \right) \right) \\
\Rightarrow 0 &= R \left(1 - 6 \left(\frac{1}{2} + H \right) \right) + \frac{5}{(f + C_f)} \Delta f \left(1 + 6 \left(\frac{1}{2} + H \right) \right)
\end{aligned} \tag{167}$$

where we recognize the classical Klein-Gordon quantum equation.

9.1.2.3 F[f] Fixed Such That the Scalar Nonlinear Terms (Times Metric Tensor) in the Whole Einstein Tensor Vanish

We set:

$$\begin{aligned}
F[f] &= \begin{cases} C_F \cdot (f + C_f)^{\frac{4(2H(n-1)+n-2)}{(n-2)(2H(n-1)+n-3)}} & n \neq 2 \\ C_F \cdot e^{f \cdot C_f} & n = 2 \cup n = \frac{3+2H}{1+2H} \end{cases} \\
&\xrightarrow{n=6} \\
F[f] &= \begin{cases} C_F \cdot (f + C_f)^{\frac{2(5H+2)}{10H+3}} & 6 \neq \frac{3+2H}{1+2H} \Rightarrow H \neq -\frac{3}{10} \\ C_F \cdot e^{f \cdot C_f} & 6 = \frac{3+2H}{1+2H} \Rightarrow H = -\frac{3}{10} \end{cases}
\end{aligned} \tag{168}$$

We assume that the metric shall be fixed during the variation so that only the volume remains flexible and can be varied, leading us to:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^6 x \cdot \sqrt{-G} \cdot R \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^6 x \cdot \sqrt{-G} \cdot \left(\left(R - \frac{2+5H}{(3+10H)(f+C_f)} \right) \times \left(5 \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) - 6 f_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right. \\
&\quad \left. + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{20(2+5H)(1+3H)}{(3+10H)^2 (f+C_f)^2} \right) \cdot \delta \left(\frac{1}{F} \right)
\end{aligned} \tag{169}$$

Only demanding the kernel to give zero, we would result in:

$$\begin{aligned}
0 &= \left(\left(R - \frac{2+5H}{(3+10H)(f+C_f)} \right) \times \begin{pmatrix} 5(2g^{ab}f_{,ab} + f_{,d}g^{cd}g^{ab}g_{ab,c}) \\ -6f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \right) \left(1 - 6\left(\frac{1}{2} + H\right) \right) \\
&\quad + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{20(2+5H)(1+3H)}{(3+10H)^2 (f+C_f)^2} \\
&= \left(\left(R - \frac{2+5H}{(3+10H)(f+C_f)} \right) \begin{pmatrix} 5(2\Delta f - 2f_{,d}g^{cd}{}_{,c}) \\ -6f_{,d}g^{cd}g^{ab}g_{ac,b} \end{pmatrix} \right) \left(1 - 6\left(\frac{1}{2} + H\right) \right) \\
&\quad + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{20(2+5H)(1+3H)}{(3+10H)^2 (f+C_f)^2}
\end{aligned} \tag{170}$$

For metrics without shear elements with:

$$g_{ij} = \begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1} \end{pmatrix}; \quad g_{ii,i} = 0, \tag{171}$$

(170) results in the metric Klein-Gordon equation with the Ricci scalar acting as the mass term:

$$\begin{aligned}
&\left(\left(R - \frac{2+5H}{(3+10H)(f+C_f)} 10\Delta f \right) \left(1 - 6\left(\frac{1}{2} + H\right) \right) \right) \\
&\quad + f_{,\alpha} \cdot f_{,\beta} g^{\alpha\beta} \frac{20(2+5H)(1+3H)}{(3+10H)^2 (f+C_f)^2} \\
&\quad \xrightarrow{f=\Psi-C_f} \\
0 &= \Delta\Psi \cdot \Psi - \frac{3+10H}{2+5H} \cdot R \cdot \Psi^2 + \frac{10}{3+10H} \cdot \Psi_{,\alpha} \cdot \Psi_{,\beta} g^{\alpha\beta}
\end{aligned} \tag{172}$$

9.1.2.4 F[f] Fixed Such That the Tensorial Nonlinear Term in the Ricci Tensor Vanishes

As we know that the tensorial nonlinear term in the general field equations (65) for F, reading $F_{,\alpha} \cdot F_{,\beta}$, vanishes for F[f]:

$$F[f] = C_{f1} (f + C_{f0})^{-2}, \tag{173}$$

and as this function does not contain any dependency on the dimensional parameter n, we can directly extract the result for the whole integral from (65) due to:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \cdot \sqrt{-G} \cdot R^* \xrightarrow{\delta g^{\alpha\beta} \equiv 0} \\
&= \int_V d^n x \cdot \sqrt{-G} \cdot \left[\left(R_{\alpha\beta} - \frac{1}{2F} \left(\begin{aligned} &-2F_{,\alpha\beta} + F_{,ab} g_{\alpha\beta} g^{ab} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ &+ F_{,a} g^{ab} (g_{\beta b,\alpha} - g_{\beta\alpha,b}) - F_{,\alpha} g^{ab} g_{\beta b,a} \\ &+ F_{,d} g^{cd} \left(g_{\alpha c,\beta} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \end{aligned} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2F^2} (-3F_{,\alpha} \cdot F_{,\beta} + 2g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \right) \right. \\
&\quad - \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \\
&\quad \left. - \left(\begin{aligned} &-\frac{1}{2F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2} g^{cd} (g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right) \\ &+ \frac{1}{4F^2} (3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \end{aligned} \right) \right] g^{\alpha\beta} \delta \left(\frac{1}{F} \right) \\
&\quad + n \left[\left(\begin{aligned} &\frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \right) \\ &- F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right. \\
&\quad \left. - \left(\begin{aligned} &-\frac{1}{2F} \left(F_{,\alpha\beta} - \frac{F_{,d}}{2} g^{cd} (g_{\alpha c,\beta} + g_{\beta c,\alpha} - g_{\alpha\beta,c}) \right) \right) \\ &+ \frac{1}{4F^2} (3F_{,\alpha} \cdot F_{,\beta} - g_{\alpha\beta} F_{,c} F_{,d} g^{cd}) \end{aligned} \right) \right] \\
&\quad \left. + n^2 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right] \quad . \quad (174)
\end{aligned}$$

Contraction yields:

$$\begin{aligned}
0 &= \int_V d^n x \cdot \sqrt{-G} \cdot \left[\left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \right. \\
&\quad \left. - \left(\begin{aligned} &7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \right) \\ &- F_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \right] \cdot \delta \left(\frac{1}{F} \right) \\
&\quad + n \cdot \left[\left(R - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right) \cdot \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \right] \cdot \delta \left(\frac{1}{F} \right) \\
&\quad - n^2 \left[\left(\begin{aligned} &7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \\ &- \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \right. \\
&\quad \left. + n^3 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right] \quad . \quad (175)
\end{aligned}$$

For n=6 and only demanding the kernel to result in zero, this gives:

$$0 = \int_V d^n x \cdot \sqrt{-G} \cdot \left(R + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} \right. \\ \left. + 6 \cdot \left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} - \frac{1}{2F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \right. \\ \left. - \left(R - 6 \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} + \frac{1}{2F} (2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}) \right) \left(\frac{1}{2} + H \right) \right) \cdot \delta \left(\frac{1}{F} \right) \\ \left. - 18 \left(\left(7 \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} - \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F^2} \right) - \frac{1}{F} \left(2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c} \right) - F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \left(\frac{1}{2} + H \right) \right) \right. \\ \left. + 54 \left(\frac{1}{2} + H \right) \frac{g^{ab} F_{,a} \cdot F_{,b}}{F^2} \right) . \quad (176)$$

10 Conclusions

It was shown that the quantum gravity field equations, which one obtains by evaluating the Einstein-Hilbert action variational problem with a scaled metric tensor, converge to rather simple and—depending on the setting of the scaling function—partially even completely classical quantum equations in the case of systems with high numbers of dimensions.

Gravity partially is rendered recessive and only becomes dominant again when there is a phenomenological dimensional reduction due to its own influence. The collapse of a star to a black hole might be seen as such a process.

The Dirac equation evolves into an extremely simple shape with an almost completely coordinate-independent general Cartesian appearance.

11 References

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12 Appendix

From Wikipedia, the free encyclopedia (https://en.wikipedia.org/wiki/Hamilton's_principle):

In physics, Hamilton's principle is William Rowan Hamilton's formulation of the principle of stationary action. It states that the dynamics of a physical system are determined by a variational problem for a functional based on a single function, the Lagrangian, which may contain all physical information concerning the system and the forces acting on it. The variational problem is equivalent to and allows for the derivation of the differential equations of motion of the physical system. Although formulated originally for classical mechanics, Hamilton's principle also applies to classical fields such as the electromagnetic and gravitational fields, and plays an important role in quantum mechanics, quantum field theory and criticality theories.

So, the definition of the Hamilton principle is based on its “formulation of the principle of stationary action”. In simpler words, the variation of such an action should be zero or, mathematically formulated, should be put as follows:

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L. \quad (177)$$

Here L stands for the Lagrangian, W the action, and g gives the determinant of the metric tensor, which describes the system in question within an arbitrary Riemann space-time with the coordinates x. Thereby, we used the Hilbert formulation of the Hamilton principle [1] in a slightly more general form. We were able to show in [2] that the original Hilbert variation does not only produce the Einstein field equations [3] but also contains the Quantum Theory [2, 4, 5]. It should be noted that, while the original Hilbert paper [1] started with the Ricci scalar R as the integral kernel, which is to say $L=R$, we here used a general Lagrangian, because—as we will show later in this appendix—this generality—in principle—is already contained inside the original Hilbert formulation. Even, as strange as it may sound at this point, general kernels with functions of the Ricci scalar f(R) [6] are already included (see [14]) in the Hilbert approach.

But what if we lived in a universe where the only thing that was certain was uncertainty?

One of the authors in [7] Dr. David Martin always used the analogy of a moving fulcrum to demonstrate his uneasiness with the formulation (177) [13].

In [7] we were able to show that the Hamilton principle itself hinders us to localize any system or object at a certain position. We also see that this contradicts the concept of particles. Everything seems to be permanently on the move or—rather—ever-jittering.

But if this ever-jittering fulcrum was one of the fundamental properties of our universe, should we then not take this into account when formulating the laws of this very universe? Shouldn't we better be a bit more cautious and write (177) as follows:

$$\delta W \rightarrow 0 \cong \delta \int_V d^n x \cdot \sqrt{-g} \cdot L ? \quad (178)$$

And while we are at it, should we not start to investigate an even more general principle like:

$$\delta W \rightarrow f(W, x, g_{\alpha\beta}) = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L ? \quad (179)$$

The interesting aspect about this is that this investigation was already—partially—done by (surprise, surprise) e.g., Hilbert and Einstein. But instead of explaining it in this way, they have “hidden” their generalization inside other concepts like the introduction of a cosmological constant or—oh yes and even more significant—the postulation of matter and its introduction via an ominous and purely postulated parameter L_M , evolving to the energy momentum tensor during the variational process, which is to say, a Lagrange matter term.

12.1 The Classical Hamilton Extremal Principle and How to Obtain Einstein's General Theory of Relativity with Matter (!) and Quantum Theory... Also with Matter (!)

The famous German mathematician David Hilbert [1], even though applying his technique only to derive the Einstein field equations for the General Theory of Relativity [3] in four dimensions,—in principle—extended the classical Hamilton principle to an arbitrary Riemann space-time with a very general variation by not only—as Hamilton and others had done—concentrating on the evolution of the given problem or system in time, but with respect to all its dimensions. His formulation of the Hamilton extremal principle looked as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot (R - 2\Lambda + L_M) \right). \quad (180)$$

There we have the Ricci scalar of curvature R , the cosmological constant Λ , the Lagrange density of matter L_M , and the determinant g of the metric tensor of the Riemann space-time $g_{\alpha\beta}$. For historical reasons, it should be mentioned that Hilbert's original work [1] did not contain the cosmological constant, because it was added later by Einstein in order to obtain a static universe, but this is not of any importance here. The evaluation of the so-called Einstein-Hilbert action (180) brought indeed the Einstein General Theory of Relativity [3], but it did not produce the other great theory physicists have found, which is the Quantum Theory. It was not before this author, about one hundred years after the publication of Hilbert's paper [1], extended Hilbert's approach by considering scaling factors to the metric tensor and showed that Quantum Theory already resides inside the sufficiently general General Theory of Relativity [2, 4, 7, 8, 9, 10]. We will not discuss the reason why this simple idea has not been tried out by other scientists before, but we may still express our amazement about the fact that a simple extension of the type:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \quad (181)$$

solves one of the greatest problems in science⁵, namely the unification of physics and that it took science more than 100 years to come up with the idea. Using the symbol G for the determinant of the scaled metric tensor $G_{\alpha\beta}$ from (181) of the Riemann space-time, we can rewrite the Einstein-Hilbert action from (180) as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot (R^* - 2\Lambda + L_M) \right). \quad (182)$$

It should be noted that we also investigated more general actions of the type:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot (R^* - 2\Lambda + L_M) \right), \quad (183)$$

which still converge to the classical form for $F \rightarrow 1$. Here, which is to say in this paper, we will only consider examples with $q=0$, but for completeness and later investigation we shall mention that a comprehensive consideration of variational integrals for the cases of general q are to be found in [4]. Performing the variation in (182) with respect to the metric $G_{\alpha\beta}$ and remembering that the Ricci curvature of such a metric (e.g., [7], appendix D) changes the whole variation to:

$$\begin{aligned} \delta W = 0 &= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot (R^* - 2\Lambda + L_M) \right) \\ &= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(\left(\frac{R}{F} - \frac{1}{2F^2} \left((n-1) \left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}_{,c}} \right) \right) \right) - 2\Lambda + L_M \right) \right), \end{aligned} \quad (184)$$

results in:

$$\begin{aligned} 0 &= \left(R^*_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\ &= \left(\left(R_{\alpha\beta} - \frac{1}{2F} \left(F_{,\alpha} g^{ab} g_{\beta b,a} - F_{,\beta} g^{ab} g_{\alpha b,a} + F_{,d} g^{cd} \left(g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} \right) \right. \right. \right. \\ &\quad \left. \left. \left. + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \right) \right) \right) \delta G^{\alpha\beta} \\ &\quad + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \\ &\quad + (n-1) \left(\frac{1}{2F} \left(\left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}_{,c}} \right) - \frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac,b} \right) + \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2} (n-6) - \frac{R}{(n-1)} \cdot \frac{g_{\alpha\beta}}{2} \right) \delta G^{\alpha\beta} \end{aligned} \quad (185)$$

when setting $q=0$ and assuming a vanishing cosmological constant. With a cosmological constant we have to write:

⁵ This does not mean, of course, that we should not also look out for generalizations of the scaled metric and investigate those as we did in [10].

$$\begin{aligned}
0 &= \left(R^*_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta \left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\boxed{\boxed{R_{\alpha\beta} - R \frac{g_{\alpha\beta}}{2}}} + \boxed{\Lambda \cdot g_{\alpha\beta}} \right. \\
&\quad \left. - \frac{1}{2F} \left(F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} + F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta \alpha, b}) - \right. \right. \\
&\quad \left. \left. F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} + F_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} \right) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \right) \right. \\
&\quad \left. + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right. \\
&\quad \left. + (n-1) \left(\frac{1}{2F} \left(-\frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac, b} \right) + \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \frac{g_{\alpha\beta}}{2} \right) \delta G^{\alpha\beta} . \quad (186)
\end{aligned}$$

For better recognition of the classical terms, we have reordered a bit and boxed the classical vacuum part of the Einstein field equations (double lines) and the cosmological constant term (single line). Everything else can be—no, represents (!)—matter or quantum effects or both.

Thus, we also—quite boldly—have set the matter density L_M equal to zero, because we see that already our simple metric scaling brings in quite some options for the construction of matter. It will be shown elsewhere [10] that there is much more which is based on the same technique.

12.2 The Principle of the Ever-Jittering Fulcrum and the Alternate Hamilton Principle

We might bring forward two reasons why we could doubt the fundamentality of the Hamilton principle even in its most general form of the generalized Einstein-Hilbert action:

- The principle was postulated and never fundamentally derived.
- Even the formulation of this principle in its classical form (180) results in a variety of options where factors, constants, kernel adaptations, etc. could be added, so that the rigid setting of the integral to zero offers some doubt in itself. A calculation process which offers a variety of add-ons and options should not contain such a dogma. The result should be kept open and general. Dr. David Martin proposed this as the “tragedy of the jittering fulcrum” and we therefore named this principle “David’s principle of the ever-jittering fulcrum” [13]. It demands:

$$\begin{aligned}
\delta_{g_{\alpha\beta}} W &\simeq ? \simeq \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R \\
\delta_{G_{\alpha\beta}} W &\simeq ? \simeq \delta_{G_{\alpha\beta}} \int_V d^n x \sqrt{-G} \times R^* . \quad (187)
\end{aligned}$$

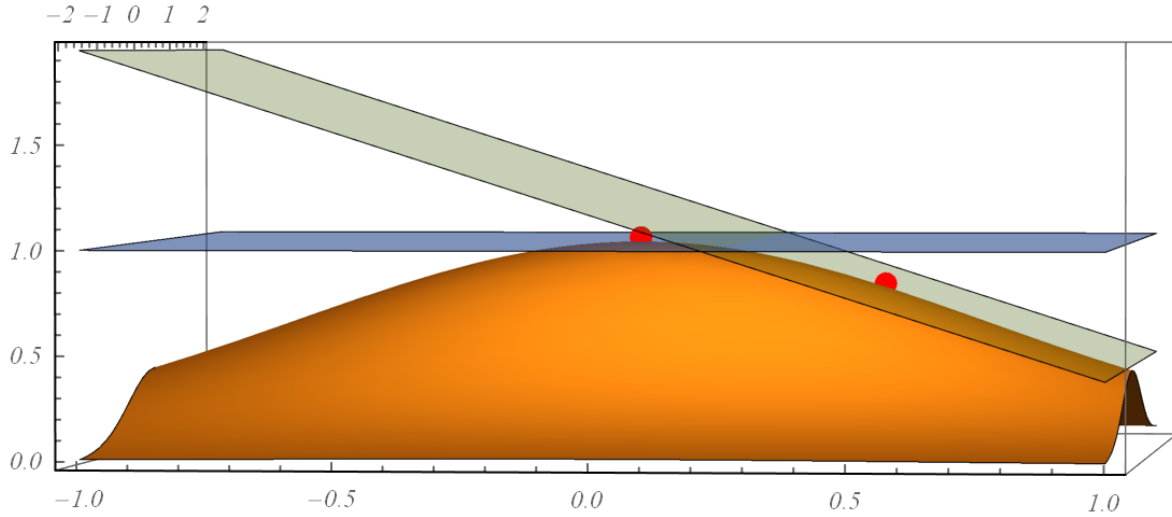


Fig. A1: David's principle of the ever-jittering fulcrum cannot accept a dogmatic insistence on a zero outcome of the Einstein-Hilbert action (180) or (generalized and also bringing about the Quantum Theory) (182). Instead it should allow for all states and not just the extremal position (see the two red dots and the corresponding tangent planes in the picture).

One of the simplest generalizations of the classical principle could be the linear one, which is illustrated in figure A1. It could be constructed as follows:

$$\int_V d^n x \sqrt{-g} \times \chi^{\alpha\beta} \cdot g_{\alpha\beta} = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R. \quad (188)$$

Thereby we have used the classical form with the unscaled metric tensor, respectively without setting the factor apart from the rest of the metric. Performing of the variation on the right-hand side and setting

$$\chi^{\alpha\beta} = H \cdot \delta g^{\alpha\beta} \quad (189)$$

or—for the reason of—maximum generality even:

$$\chi^{\alpha\beta} = H_{ab}^{\alpha\beta} \cdot \delta \gamma^{ab} = H \cdot \delta g^{\alpha\beta} \quad (190)$$

just gives us the same result as we would obtain it when assuming a non-zero cosmological constant, because evaluation yields:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H \cdot \delta g^{\alpha\beta} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta}, \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned} \quad (191)$$

respectively:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H_{ab}^{\alpha\beta} \cdot \delta \gamma^{ab} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned} \quad (192)$$

Simply setting $H = -\Lambda$ (c.f. single-line boxed term in equation (186)) demonstrates this.

Nothing else is the usage of a general functional term T , being considered a function of the coordinates of the system (perhaps even the metric tensor) in a general manner, as follows:

$$\int_V d^n x \sqrt{-g} \times T = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R. \quad (193)$$

As before, performing of the variation on the right-hand side and setting

$$T = T_{\alpha\beta} \cdot \delta g^{\alpha\beta} \quad (194)$$

gives us something which was classically postulated under the variational integral, namely the classical energy-matter tensor. This time, however, it simply pops up as a result of David's principle of the jittering fulcrum and is equivalent to the introduction of the term L_M under the variational integral. Evaluation yields:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \cdot T_{\alpha\beta} \cdot \delta g^{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - T_{\alpha\beta} \right) \delta g^{\alpha\beta}. \end{aligned} \quad (195)$$

So, we see that in introducing a cosmological constant and in postulating a matter term, even Einstein and Hilbert already—in principle—"experimented" with a non-extremal setting for the Hamilton extremal principle.

Apart from linear dependencies and other functions or functional terms, we could just assume a general outcome like:

$$f(W) = f\left(\int_V d^n x \sqrt{-g} \times R\right) = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R. \quad (196)$$

This, however, would not give us any substantial hint where to move on, respectively, which of the many possible paths to follow. We therefore here start our investigation with the assumption of an eigen result for the variation as follows:

$$\mathbb{X} \cdot W = \mathbb{X} \cdot \int_V d^n x \sqrt{-g} \times R = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R. \quad (197)$$

This leads to:

$$\int_V d^n x \sqrt{-g} \left(R_{\kappa\lambda} \delta g^{\kappa\lambda} - R \cdot \left(\frac{1}{2} \cdot g_{\kappa\lambda} \delta g^{\kappa\lambda} + \mathbb{X} \right) \right) = 0. \quad (198)$$

As the term $\mathbb{X}$ could always be expanded into an expression like:

$$\mathbb{X} = H \cdot g_{\kappa\lambda} \delta g^{\kappa\lambda}, \quad (199)$$

we obtain from (198):

$$\begin{aligned} 0 &= \int_V d^n x \sqrt{-g} \left(R_{\kappa\lambda} \delta g^{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} \delta g^{\kappa\lambda} \right) \\ &= \int_V d^n x \sqrt{-g} \left(R_{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} \right) \delta g^{\kappa\lambda} \\ &\Rightarrow R_{\kappa\lambda} - R \cdot \left(\frac{1}{2} + H \right) g_{\kappa\lambda} = 0 \end{aligned} \quad (200)$$

We realize that the term H can be a general scalar even if we would demand the term χ to be a constant.

The complete equation when assuming a scaled metric tensor of the form (181) would read:

$$\left(\left(R_{\alpha\beta} - \frac{1}{2F} \left(F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab} + F_{,a} g^{ab} (g_{\beta b, \alpha} - g_{\beta\alpha, b}) - F_{,\alpha} g^{ab} g_{\beta b, a} - F_{,\beta} g^{ab} g_{\alpha b, a} + F_{,d} g^{cd} \left(g_{\alpha c, \beta} - \frac{1}{2} n g_{\alpha c, \beta} - \frac{1}{2} n g_{\beta c, \alpha} + \frac{1}{2} n g_{\alpha\beta, c} + \frac{1}{2} g_{\alpha\beta} g_{ab, c} g^{ab} \right) + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) \right) - \left(R - \frac{1}{2F} \left((n-1) \left(\overbrace{2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab, c}}^{=2\Delta F - 2F_{,d} g^{cd}_{,c}} \right) - n F_{,d} g^{cd} g^{ab} g_{ac, b} \right) - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2} (n-6) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right) = 0, \quad (201)$$

and in the case of metrics with constant components this equation simplifies to:

$$\left(\left(R_{\alpha\beta} - \frac{1}{2F} (F_{,\alpha\beta} (n-2) + F_{,ab} g_{\alpha\beta} g^{ab}) + \frac{1}{4F^2} (F_{,\alpha} \cdot F_{,\beta} (3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd} (4-n)) \right) - \left(R - \frac{(n-1)}{2F} \left(2g^{ab} F_{,ab} + \frac{g^{ab} F_{,a} \cdot F_{,b}}{2F} (n-6) \right) \right) \cdot \left(\frac{1}{2} + H \right) g_{\alpha\beta} \right) = 0. \quad (202)$$

12.2.1 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (187) correspond to more stable states than non-extremal solutions, and in fact we will find this in connection with the 3-generations problem, which we will now derive in the next subsection of this appendix.

The interested reader finds further discussion in our publications [11, 12]. There, among other things, we also considered a path to solve the so-called three generations principle of elementary particles via alternative concepts, like a dimensional dependency of mass [12].

12.3 The 3-Generations Problem as a Polynomial of Third-Order

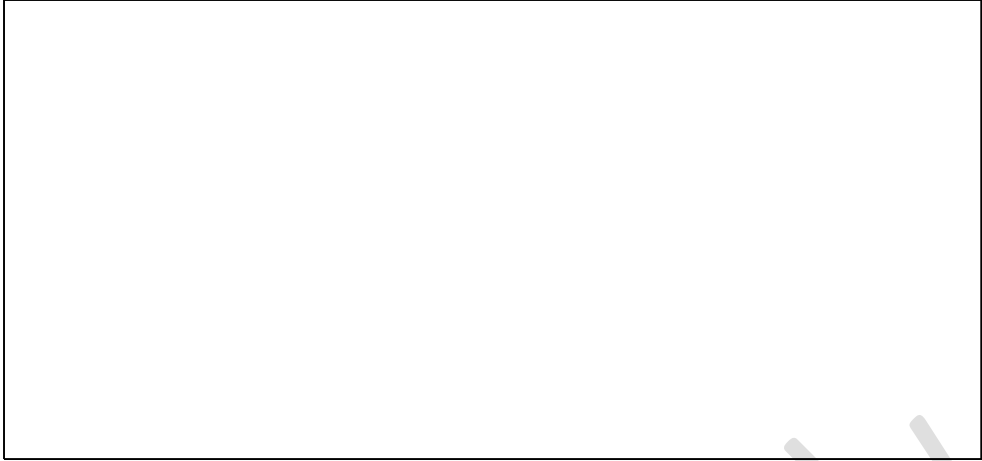
When investigating the complete field equations (201) under covariant derivation (with respect to the unscaled metric tensor g_{ab}), which is to say:

$$\Rightarrow 0 = \left(\begin{aligned} & -\frac{F'}{2F} g^{\beta\nu} \left(\begin{aligned} & f_{,\alpha\nu} (n-2) + f_{,a} g^{ab} (g_{vb,\alpha} - g_{v\alpha,b}) - f_{,\alpha} g^{ab} g_{vb,a} \\ & -f_{,\nu} g^{ab} g_{ab,a} + f_{,d} g^{cd} \left(g_{ac,\nu} - \frac{1}{2} n g_{ac,\nu} - \frac{1}{2} n g_{vc,\alpha} + \frac{1}{2} n g_{\alpha\nu,c} \right) \end{aligned} \right) \\ & -\delta_{\alpha}^{\beta} \frac{F'}{2F} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab,c} g^{ab} \right) - \delta_{\alpha}^{\beta} R \cdot H \\ & + \delta_{\alpha}^{\beta} \frac{F'}{2F} \left(\begin{aligned} & (n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) \\ & - n f_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \\ & + \frac{g^{\beta\nu}}{4F^2} f_{,\alpha} \cdot f_{,\nu} (n-2) (3(F')^2 - 2FF'') \\ & + \delta_{\alpha}^{\beta} \frac{1}{4F^2} f_{,c} f_{,d} g^{cd} \left(((F')^2 (4-n) - 2FF'') \right) \\ & + \delta_{\alpha}^{\beta} \frac{1}{4F^2} \left(f_{,c} f_{,d} g^{cd} (n-1) (4FF'' + (F')^2 (n-6)) \right) \cdot \left(\frac{1}{2} + H \right) \end{aligned} \right)_{;\beta} \quad (203)$$

in the slightly compactified form:

$$0 = \left(\begin{aligned} & -\frac{F'}{2F} g^{\beta\nu} \left(\begin{aligned} & f_{,\alpha\nu} (n-2) + f_{,a} g^{ab} (g_{vb,\alpha} - g_{v\alpha,b}) - f_{,\alpha} g^{ab} g_{vb,a} \\ & -f_{,\nu} g^{ab} g_{ab,a} + f_{,d} g^{cd} \left(g_{ac,\nu} - \frac{1}{2} n g_{ac,\nu} - \frac{1}{2} n g_{vc,\alpha} + \frac{1}{2} n g_{\alpha\nu,c} \right) \end{aligned} \right) \\ & -\delta_{\alpha}^{\beta} R \cdot H - \delta_{\alpha}^{\beta} \frac{F'}{2F} \left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab,c} g^{ab} \right) \\ & + \delta_{\alpha}^{\beta} \frac{F'}{2F} \left(\begin{aligned} & (n-1) \left(2g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c} \right) \\ & - n f_{,d} g^{cd} g^{ab} g_{ac,b} \end{aligned} \right) \cdot \left(\frac{1}{2} + H \right) \\ & + \frac{1}{4F^2} \left(\begin{aligned} & (3(F')^2 - 2FF'') (n-2) g^{\beta\nu} f_{,\alpha} \cdot f_{,\nu} \\ & + \delta_{\alpha}^{\beta} f_{,c} f_{,d} g^{cd} \left(\overbrace{(F')^2 \left(4-n + (n-1)(n-6) \cdot \left(\frac{1}{2} + H \right) \right)}^{E^2} \right) \\ & + 2FF'' \left(2(n-1) \cdot \left(\frac{1}{2} + H \right) - 1 \right) \end{aligned} \right) \end{aligned} \right)_{;\beta} \quad (204)$$

under the assumption of a true particle at rest, which means that everything, also the metric, only depends on the time coordinate $t=x_0$, we obtain:



(205)

For demonstration and because one may consider the “particle at rest” situation as a Hamilton equilibrium state of some extremal character, we set $H=0$:



(206)

We are going to see that this will not allow us to obtain a polynomial of third-order with respect to the parameter mass but that in fact we require $H \neq 0$ for the 3rd generation. On the other hand, our

reasoning for the assumption of a vanishing Hamilton parameter was a stable equilibrium or extremal state for the particle being coded by that state. We know, however, that all particles of the higher generations are unstable and thus, are probably not coded by an extremal state and hence, the setting for a $H \neq 0$ is more reasonable. We will see!

When assuming a typical particle at rest behavior, we should be able to demand the following:

$$\boxed{}. \quad (207)$$

This gives us:

$$\boxed{}. \quad (208)$$

This may be considered as a polynomial in m . We realize that, even already without the covariant derivative being performed, we are missing the constant term with respect to m and that, thus, we cannot have three solution for this parameter.

So, we bring back in the parameter H and have to write (208) as follows:

$$\boxed{}. \quad (209)$$

and can short-write the covariant derivative:

$$\boxed{}. \quad (210)$$

Together with the affine connection expressed in terms of the metric tensor, we obtain:

$$\boxed{\phantom{\nabla_X Y = \partial_X Y + \Gamma^k_{ij} Y^i \partial^j}}, \quad (211)$$

where we see with:

$$\boxed{\phantom{\Gamma^k_{ij} = \frac{1}{2} g^{kl} (\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij})}}. \quad (212)$$

and subsequently:



(213)

that we have now obtained a set of n polynomials with at least one polynomial of third-order in m , and are “just” left with the task of finding:

- a) solutions to all such n equations (indexed $\alpha=0,1,\dots,n-1$) and
- b) suitable metrics for the coding of elementary particles.

As a hint, it shall just be said that metrics with time dependencies, completely solving the quantum gravity field equations, are given in the appendix N of our book [9].

We realize that without the generalized Hamilton principle [13] the problem of the 3 generations of elementary particles cannot be solved as we require the parameter H to be non-zero. This, however, as elaborated above, could also deliver a simple explanation for the instability of the higher-generation particles.

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