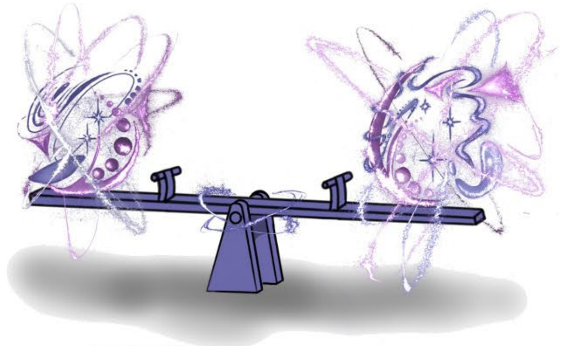
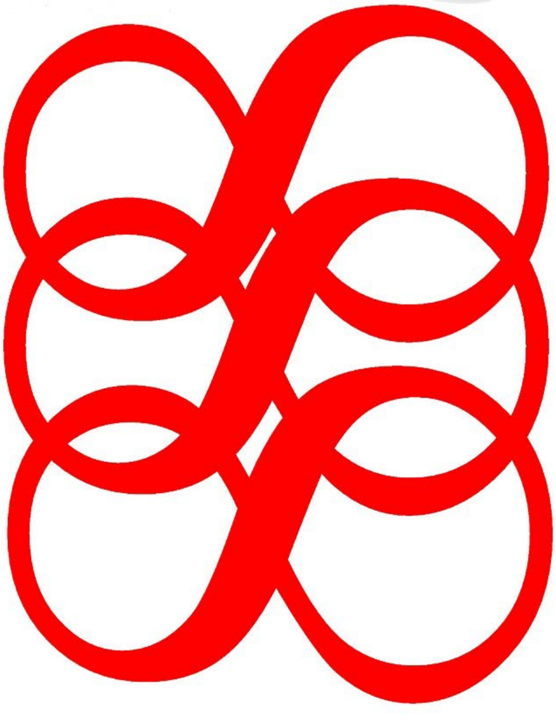
The Principle of the Ever-Jittering Fulcrum and the 3-Generations Problem of Elementary Particles





by

Dr. Norbert Schwarzer

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The Principle of the Ever-Jittering Fulcrum and the 3-Generations Problem of Elementary Particles

By Dr. Norbert Schwarzer

1 Abstract

In this brief paper we address the so-called "three generations problem" of elementary particles and suggest a solution via an extended Hamilton "extremal" principle in Hilbert variational form for a scaled metric tensor followed by an application of the contracted Bianchi identity with respect to the unscaled metric tensor. The resulting equations are of third order and consequently have the potential to produce three solutions in mass with otherwise identical properties for the corresponding objects described by these solutions. Assuming these objects to be particles because they have the Dirac structure regarding their time-coordinate dependency, we have subsequently obtained three particle-solutions of different masses, hence, generations.

We therefore consider this a suitable path to the solution of the so-called three generations problem of elementary particles.

2 Introduction

There are three mass versions of fermions, which are called generations. They are given in the following table, which we extracted from [1]:

Generations of matter				
Fermion categories		**Elementary particle generation**		
Type	***Subtype***	***First***	***Second***	***Third***
Quarks (colored)	**down-type**	down	strange	bottom
	up-type	up	charm	top
Leptons (color-free)	**charged**	electron	muon	tauon
	neutral	electron neutrino	muon neutrino	tau neutrino

The existence of these generations—so far—is an unsolved problem in physics.

We will not comment here on the many attempts to solve the problem why there are exactly three generations and why there are such great differences regarding their individual particle masses[1].

[1] In [1], we find the following information to the topic: The origin of multiple generations of fermions, and the particular count of 3, is an unsolved problem of physics. String theory provides a cause for multiple generations, but the particular number depends on the details of the compactification of the D-brane intersections. Additionally, E_8 grand unified theories in 10 dimensions compactified on certain orbifolds down to 4 D naturally contain 3 generations of matter [A]. This includes many heterotic string theory models.

Instead, we just want to point out that—obviously—the 3 generations problem is a quantum gravity problem as it contains mass as just THE parameter and that, thus, the solutions of this problem requires a Quantum Gravity Theory. So, without the latter, any attempt to satisfactory solve this problem is—probably—futile.

Hence, the decisive question here is:

Do we have a Quantum Gravity Theory?

…and here the answer is:

Yes, we think so!

3 Theory

3.1 The Quantum Gravity Properties of Scaled Metric Tensor

We start with the following scaled metric tensor:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \tag{1}$$

and force it into the Einstein-Hilbert action [2] variational problem as follows:

$$\delta W = 0 = \delta \int_V d^n x \sqrt{-G} \cdot R^* . \tag{2}$$

Here G denotes the determinant of the metric tensor from (1) and R^* gives the corresponding Ricci scalar. The complete variational task would then read (e.g., [3] appendix D):

$$\begin{aligned} \delta W = 0 &= \delta \int_V d^n x \sqrt{-G} \cdot R^* \\ &= \delta \int_V d^n x \left(\sqrt{-G} \cdot \left(\begin{array}{c} \frac{R}{F} - \frac{1}{2F^2}\left((n-1)\left(2\Delta F - 2F_{,d} g^{cd}{}_{,c}\right) - nF_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ -(n-1)\frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^3}(n-6) \end{array} \right) \right) . \end{aligned} \tag{3}$$

Performing the variation with respect to the metric $G_{\alpha\beta}$ results in:

In standard quantum field theory, under certain assumptions, a single fermion field can give rise to multiple fermion poles with mass ratios of around $e^{\pi} \approx 23$ and $e^{2\pi} \approx 535$ potentially explaining the large ratios of fermion masses between successive generations and their origin [B].

The existence of precisely three generations with the correct structure was at least tentatively derived from first principles through a connection with gravity [C]. The result implies a unification of gauge forces into SU(5). The question regarding the masses is unsolved, but this is a logically separate question, related to the Higgs sector of the theory.

[A] Luboš Motl, (13 July 2021). "The "pure joy" E8 SUSY toroidal orbifold TOE". The Reference Frame (blog). Retrieved 23 August 2021 – via motls.blogspot.com.

[B] A. Blumhofer, M. Hutter, (1997). "Family structure from periodic solutions of an improved gap equation". Nuclear Physics B. 484 (1): 80–96. Bibcode:1997NuPhB.484...80B. CiteSeerX 10.1.1.343.783. doi:10.1016/S0550-3213(96)00644-X. (Erratum: doi:10.1016/S0550-3213(97)00228-9)

[C] J.J. van der Bij, (28 December 2007). "Cosmotopological relation for a unified field theory". Physical Review D. 76 (12): 121702. arXiv:0708.4179. doi:10.1103/PhysRevD.76.121702

$$0=\left(\begin{array}{c}\left(\begin{array}{c}R_{\alpha\beta}-\frac{1}{2F}\left(\begin{array}{c}F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)-\\ F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\ +\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\end{array}\right)\end{array}\right)\\ +\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\end{array}\right)\\ +\left(\frac{(n-1)}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}(n-6)-\frac{R}{(n-1)}\right)\cdot\frac{g_{\alpha\beta}}{2}\end{array}\right)\delta G^{\alpha\beta}. \quad (4)$$

A bit of reordering gives us:

$$0=\left(\begin{array}{c}\boxed{R_{\alpha\beta}-R\frac{g_{\alpha\beta}}{2}}\\ -\frac{1}{2F}\left(\begin{array}{c}F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)-\\ F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\ +\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\end{array}\right)\end{array}\right)\\ +\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\\ +\left(\frac{(n-1)}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}(n-6)\right)\cdot\frac{g_{\alpha\beta}}{2}\end{array}\right)\delta G^{\alpha\beta} \quad (5)$$

and shows us that we have not only obtained the classical Einstein Theory of Relativity [4] (see boxed terms exactly giving the Einstein field equations in vacuum), but also a set of field equations for the scaling function F.

It was shown in our previous publications [3, 5, 6, 7, 8, 9, 10] that these additional terms are clearly quantum equations fully covering the main aspects of relativistic classical Quantum Theory. We can briefly demonstrate this by just assuming weak gravity conditions of the following kind:

$$\delta G^{\alpha\beta}=G^{\alpha\beta}\cdot\delta_{0}+\overbrace{G^{ab}\delta_{ab}^{\alpha\beta}}^{\text{Gravity}}\xrightarrow{\forall\,\delta_{ab}^{\alpha\beta}\ll\delta_{0}}=\frac{g^{\alpha\beta}}{F}\cdot\delta_{0}\,. \quad (6)$$

This reduces (5) to:

$$\left(\begin{array}{c}\frac{R}{F}-\frac{1}{2F^{2}}\left((n-1)\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}\right)-nF_{,d}g^{cd}g^{ab}g_{ac,b}\right)\\ -(n-1)\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{3}}(n-6)\end{array}\right)=0, \quad (7)$$

where, demanding the condition:

$$0=4FF''+(F')^{2}(n-6), \quad (8)$$

and satisfying it with the following wrapping approach for F=F[f] for the volume scaling of the metric tensor:

$$F[f]=\begin{cases} C_F\cdot\left(f+C_f\right)^{\frac{4}{n-2}} & n-2\neq 0 \\ C_F\cdot e^{f\cdot C_f} & n-2=0 \end{cases}, \tag{9}$$

we are able to completely linearize the remaining—now scalar—field equations via:

$$\begin{aligned}
0&=\left(R^{*}{}_{\alpha\beta}-\frac{1}{2}R^{*}G_{\alpha\beta}\right)\overbrace{\left(\frac{1}{F}\cdot\delta g^{\alpha\beta}+g^{\alpha\beta}\cdot\delta\left(\frac{1}{F}\right)\right)}^{\delta G^{\alpha\beta}} \\
&\xrightarrow{\delta G^{\alpha\beta}=G^{\alpha\beta}\cdot\delta_0+\overbrace{G^{ab}\delta_{ab}^{\alpha\beta}}^{\text{Gravity}}\xrightarrow{\forall\,\delta_{ab}^{\alpha\beta}\ll\delta_0}=\frac{g^{\alpha\beta}}{F}\cdot\delta_0} \\
&=\left(R^{*}-\frac{1}{2}R^{*}G_{\alpha\beta}\right)g^{\alpha\beta}\cdot\delta\left(\frac{1}{F}\right)=R^{*}\left(1-\frac{n}{2}F\right)\cdot\delta\left(\frac{1}{F}\right) \qquad , \\
&=\left(\begin{array}{c} R-\dfrac{F'}{2F}\left(\begin{array}{c}(n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right) \\ -nf_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right) \\ -(n-1)\dfrac{g^{ab}f_{,a}\cdot f_{,b}}{4F^{2}}\left(4FF''+(F')^{2}(n-6)\right)\end{array}\right)\cdot\left(1-\frac{n}{2}\right)\delta\left(\frac{1}{F}\right)
\end{aligned} \tag{10}$$

and finally obtain:

$$0=R-\frac{F'}{2F}\left((n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right)-nf_{,d}g^{cd}g^{ab}g_{ac,b}\right). \tag{11}$$

This equation is completely linear in f, which not only has the characteristics of a quantum function, but—for a change—gives us the opportunity to metrically see what QUANTUM actually means, namely, a volume jitter to the metric of the system in question… at least this is one quantum option, because we have already seen others (e.g., see [3, 5, 6, 7, 8, 9, 10]).

Interestingly, for metrics without shear elements like (which regards many metrics, like cartesian, spherical, elliptical, cylindrical, and so on, used in standard Quantum Theory):

$$g_{ij}=\begin{pmatrix} g_{00} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{n-1n-1}\end{pmatrix};\quad g_{ii,i}=0, \tag{12}$$

and applying the solution for F[f] from (9), the derivative terms in (11), which is to say:

$$(n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right)-nf_{,d}g^{cd}g^{ab}g_{ac,b}, \tag{13}$$

converge to the ordinary Laplace operator, namely:

$$\begin{aligned}
&R^{*}=0\quad\rightarrow\quad 0=F\cdot R+F'\cdot(1-n)\cdot\Delta f \\
&\Rightarrow\quad 0=\begin{cases}\left(f-C_f\right)^{\frac{4}{n-2}}\cdot C_F\left(R+\dfrac{4}{n-2}\cdot\dfrac{(1-n)}{\left(f-C_f\right)}\cdot\Delta f\right) & n>2 \\ e^{C_f\cdot f}\cdot C_F\left(R+C_f\cdot(1-n)\cdot\Delta f\right) & n=2\end{cases}.
\end{aligned} \tag{14}$$

We recognize just the most important specially relativistic quantum equation, namely, the Klein-Gordon equation.

From there it only requires text book knowledge to obtain the Schrödinger and the Dirac equation in the usual way (e.g., see [3, 5, 6, 7, 8, 9, 10]).

As a side note, we need to point out that, in the case of n>2, we always also have the option for a constant (broken symmetry) solution of the kind:

$$0 = \mathrm{f} - C_{\mathrm{f}0} \quad \Rightarrow \quad \mathrm{f} = C_{\mathrm{f}0}. \tag{15}$$

In all other cases, meaning where $\mathrm{f} \neq C_{\mathrm{f}0}$, we have the simple equations:

$$0 = \begin{cases} \left(\mathrm{f} - C_{\mathrm{f}0}\right)\cdot \mathrm{R} + \left(1-\mathrm{n}\right)\cdot \dfrac{4}{\mathrm{n}-2}\cdot \Delta \mathrm{f} & \mathrm{n} > 2 \\ \mathrm{R} + C_{\mathrm{f}0}\cdot\left(1-\mathrm{n}\right)\cdot \Delta \mathrm{f} & \mathrm{n} = 2 \end{cases}. \tag{16}$$

A critical argument should now be that this equation is not truly of Klein-Gordon character, as it does neither contain any potential nor mass, but we have already shown that this problem is easily solved by adding additional dimensions carrying the right properties to produce masses and potentials (e.g., [3, 5, 6, 7, 8, 9, 10]).

So, we conclude that we indeed have a Quantum Gravity Theory at hand.

However, before we can address the three generations problem of elementary particles, we first have to dive a bit deeper into the fundamentals of the classical Hamilton principle on which the Hilbert variation (2) is based.

3.2 Principle of the Ever-Jittering Fulcrum

Let us assume that it is usually impossible in this universe to define a balance point, because fundamental uncertainties are making the attempt of the definition of an absolute frame an unsolvable task [33]. In other words: There simply is no way to absolutely fix a fulcrum (figure 1).

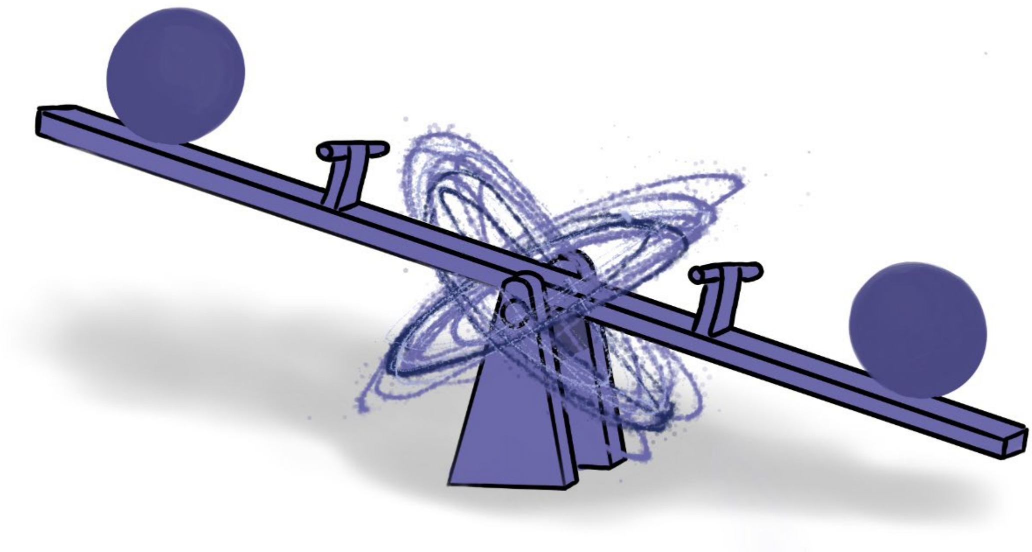

Fig. 1: Symbolizing the "Principle of the Ever-Jittering Fulcrum" (artist: Livia Schwarzer)

In consequence, this would also influence our variational principle (2), which we would be forced to substitute by something more general, namely:

$$\begin{aligned}\delta_{g_{\alpha\beta}} W \simeq ? \simeq \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R \\ \delta_{G_{\alpha\beta}} W \simeq ? \simeq \delta_{G_{\alpha\beta}} \int_V d^n x \sqrt{-G} \times R^* \end{aligned} . \tag{17}$$

The corresponding consequences on the resulting Quantum Gravity field equations is evaluated in the appendix of this paper.

3.3 The 3-Generations Problem of Elementary Particles

From Wikipedia, the free encyclopedia (https://en.wikipedia.org/wiki/Generation_(particle_physics)):

In particle physics, a generation or family is a division of the elementary particles. Between generations, particles differ by their flavour quantum number and mass, but their electric and strong interactions are identical.

There are three generations according to the Standard Model of particle physics. Each generation contains two types of leptons and two types of quarks. The two leptons may be classified into one with electric charge −1 (electron-like) and neutral (neutrino); the two quarks may be classified into one with charge −1⁄3 (down-type) and one with charge +2⁄3 (up-type). The basic features of quark–lepton generation or families, such as their masses and mixings etc., can be described by some of the proposed family symmetries.

…

Each member of a higher generation has greater mass than the corresponding particle of the previous generation, with the possible exception of the neutrinos (whose small but non-zero masses have not been accurately determined). For example, the first-generation electron has a mass of only 0.511 MeV/c^2, the second-generation muon has a mass of 106 MeV/c2, and the third-generation tau has a mass of 1777 MeV/c^2 (almost twice as heavy as a proton). This mass hierarchy [11] causes particles of higher generations to decay to the first generation, which explains why everyday matter (atoms) is made of particles from the first generation only. Electrons surround a nucleus made of protons and neutrons, which contain up and down quarks. The second and third generations of charged particles do not occur in normal matter and are only seen in extremely high-energy environments such as cosmic rays or particle accelerators. The term generation was first introduced by Haim Harari in Les Houches Summer School, 1976 [12, 13].

Neutrinos of all generations stream throughout the universe but rarely interact with other matter [14]. It is hoped that a comprehensive understanding of the relationship between the generations of the leptons may eventually explain the ratio of masses of the fundamental particles, and shed further light on the nature of mass generally, from a quantum perspective [15].

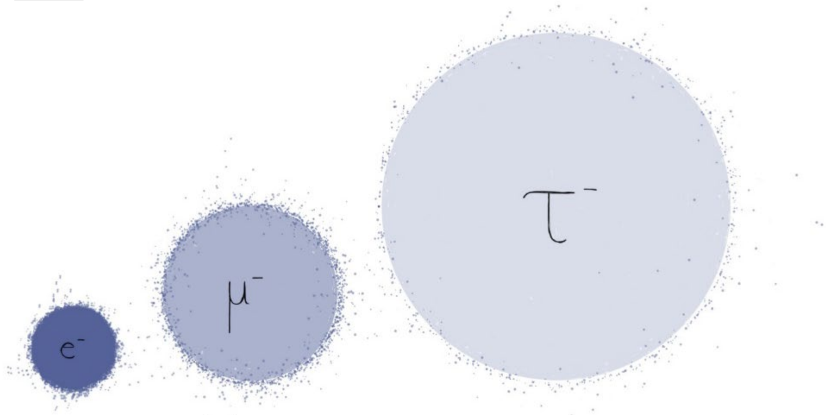

Fig. 2: Symbolizing the 3 generations of particles in the case of charged leptons: electron, muon, and tauon (artist: Livia Schwarzer)

In essence, the 3-generations problem of elementary particles has not been solved so far. In this paper, we are now applying our extended Hamilton extremal or Hilbert variation principle to this problem. As the derivation is lengthy, we place the mathematical main part in the appendix, while in the next subsection only discussing its connection with the concept of the ever-jittering fulcrum.

3.4 How Does the Principle of the Ever-Jittering Fulcrum Help Us to Solve the 3-Generations Problem of Elementary Particles

Evaluation of the second equation of (17) gives us the following Quantum Gravity field equations:

$$\Omega \equiv \left(\left(R_{\alpha\beta} - \frac{1}{2F} \begin{pmatrix} F_{,\alpha\beta}(n-2) + F_{,ab} g_{\alpha\beta} g^{ab} \\ +F_{,a} g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b}\right) - F_{,\alpha} g^{ab} g_{\beta b,a} - F_{,\beta} g^{ab} g_{\alpha b,a} \\ +F_{,d} g^{cd}\left(g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab}\right) \end{pmatrix} + \frac{1}{4F^2}\left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd}(4-n)\right) \right) - \left(R - \frac{1}{2F}\left((n-1)\Big(\overbrace{2g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}}^{=2\Delta F - 2F_{,d} g^{cd}{}_{,c}} \Big) - nF_{,d} g^{cd} g^{ab} g_{ac,b} \right) - (n-1)\frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^2}(n-6) \right) \cdot \left(\frac{1}{2} + H\right) g_{\alpha\beta} \right). \tag{18}$$

According to the variational task, we would have to solve the equation under the condition:

$$\Omega \equiv 0\,. \tag{19}$$

But even without this condition, performing the covariant derivative of Ω results in 0, because of the following symmetric properties of the Riemann tensor:

$$0 = R_{\alpha\beta\gamma\delta;\varepsilon} + R_{\alpha\beta\varepsilon\gamma;\delta} + R_{\alpha\beta\delta\varepsilon;\gamma}\,, \tag{20}$$

via contraction with the metric tensor as follows:

$$0 = \left(R_{\alpha\beta\gamma\delta;\varepsilon} + R_{\alpha\beta\varepsilon\gamma;\delta} + R_{\alpha\beta\delta\varepsilon;\gamma}\right) g^{\alpha\delta} g^{\beta\gamma} = \xrightarrow{g^{\alpha\delta}{}_{;\lambda}=0} = 2 \cdot R^{\alpha}{}_{\varepsilon;\alpha} - R_{;\varepsilon}\,. \tag{21}$$

The latter is called the contracted Bianchi identity.

These equations—based on the same identity—are just different forms of the condition for the vacuum Einstein field equations where they give a generalized conservation law:

$$0 = \left(R^{\alpha\beta} - \frac{R}{2} g^{\alpha\beta}\right)_{;\beta} = \left(R^{\beta}_{\alpha} - \frac{R}{2}\delta^{\beta}_{\alpha}\right)_{;\beta}. \tag{22}$$

Rewriting (20) for the Riemann tensor of the scaled metric:

$$0 = R^{*}{}_{\alpha\beta\gamma\delta;\varepsilon} + R^{*}{}_{\alpha\beta\varepsilon\gamma;\delta} + R^{*}{}_{\alpha\beta\delta\varepsilon;\gamma}\,, \tag{23}$$

gives the same result, meaning:

$$0 = \left(R^{*\alpha\beta} - \frac{R^{*}}{2} G^{\alpha\beta}\right)_{;\beta} = \left(R^{*\beta}_{\alpha} - \frac{R^{*}}{2}\delta^{\beta}_{\alpha}\right)_{;\beta}, \tag{24}$$

only that we have substituted the metric tensor by the scaled metric tensor everywhere.
Thus, we find that:

$$\Omega_{\alpha\beta}{}^{;\beta} = 0\,. \tag{25}$$

This, however, only holds when the covariant derivative is performed with respect to the scaled metric tensor $G_{\alpha\beta}$. When instead performing the derivation with respect to $g_{\alpha\beta}$, we obtain—lo and behold—equations of third order for our quantum function f, reading as follows:

$$\Rightarrow 0 = \left(\begin{array}{c} -\frac{F'}{2F} g^{\beta\nu} \left(\begin{array}{c} f_{,\alpha\nu}(n-2) + f_{,a} g^{ab}\left(g_{\nu b,\alpha} - g_{\nu\alpha,b}\right) - f_{,\alpha} g^{ab} g_{\nu b,a} \\ -f_{,\nu} g^{ab} g_{\alpha b,a} + f_{,d} g^{cd}\left(g_{\alpha c,\nu} - \frac{1}{2} n g_{\alpha c,\nu} - \frac{1}{2} n g_{\nu c,\alpha} + \frac{1}{2} n g_{\alpha\nu,c}\right) \end{array} \right) \\ -\delta_{\alpha}^{\beta} \frac{F'}{2F}\left(f_{,ab} g^{ab} + f_{,d} g^{cd} \frac{1}{2} g_{ab,c} g^{ab} \right) - \delta_{\alpha}^{\beta} R \cdot H \\ +\delta_{\alpha}^{\beta} \frac{F'}{2F} \left(\begin{array}{c} (n-1)\left(2 g^{ab} f_{,ab} + f_{,d} g^{cd} g^{ab} g_{ab,c}\right) \\ -n f_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \cdot \left(\frac{1}{2} + H \right) \\ +\frac{g^{\beta\nu}}{4F^2} f_{,\alpha} \cdot f_{,\nu} (n-2)\left(3(F')^2 - 2FF''\right) \\ +\delta_{\alpha}^{\beta} \frac{1}{4F^2} f_{,c} f_{,d} g^{cd} \left(\left((F')^2 (4-n) - 2FF'' \right) \right) \\ +\delta_{\alpha}^{\beta} \frac{1}{4F^2} \left(f_{,c} f_{,d} g^{cd} (n-1)\left(4FF'' + (F')^2 (n-6)\right)\right) \cdot \left(\frac{1}{2} + H \right) \end{array} \right)_{;\beta} . \tag{26}$$

The Einstein terms $R^{\alpha\beta} - \frac{R}{2} g^{\alpha\beta}$—on the other hand—have disappeared due to the contracted Bianchi identity (22) for the unscaled metric. However, as the term $R^{\alpha\beta} - \frac{R}{2} g^{\alpha\beta}$ is also considered the vacuum term in the classical General Theory of Relativity [4], the rest in the field equations (18) must consequently be matter and this makes (26) a conservation law for this very matter. These conservation equations are of third order and when inserting a Dirac-like particle at rest ansatz, we obtain three solutions in mass (see appendix and below).

The practical evaluation of real problems (like the three generations problem) with the fundamental equations that we have derived, however, is pretty complicated… and not very "illustrative" either. This especially results from the vector character of the third order conservation law. Aiming for a suitable way to scalarize the problem, we apply the "weak gravity condition" in covariant:

$$\delta G^{\alpha\beta} = G^{\alpha\beta} \cdot \delta_0 + \overbrace{G^{ab} \delta_{ab}^{\alpha\beta}}^{\text{Gravity}} = \frac{1}{F} \cdot \left(g^{\alpha\beta} \cdot \delta_0 + \overbrace{g^{ab} \delta_{ab}^{\alpha\beta}}^{\text{Gravity}} \right) \xrightarrow{\forall\, \delta_{ab}^{\alpha\beta} \ll \delta_0} = \frac{g^{\alpha\beta}}{F} \cdot \delta_0 \tag{27}$$

or in mixed form:

$$\delta G_{\beta}^{\alpha} = G_{\beta}^{\alpha} \cdot \delta_0 + \overbrace{G_{a}^{b} \delta_{\alpha\beta}^{b\alpha}}^{\text{Gravity}} \xrightarrow{\forall\, \delta_{\alpha\beta}^{b\alpha} \ll \delta_0} = G_{\beta}^{\alpha} \cdot \delta_0 = g_{\beta}^{\alpha} \cdot \delta_0 = \delta_{\beta}^{\alpha} \cdot \delta_0 \tag{28}$$

and define:

$$I_{\alpha}^{\beta} \equiv R_{\alpha}^{\beta} - \frac{\delta_{\alpha}^{\beta}}{2} R - \delta_{\alpha}^{\beta} R \cdot H + \frac{1}{2F} \not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta}; F\right], \tag{29}$$

with:

$$\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\equiv\frac{1}{2F}\left(\begin{array}{c}
-F'g^{\beta\nu}\left(\begin{array}{c} f_{,\alpha\nu}(n-2)+f_{,a}g^{ab}\left(g_{\nu b,\alpha}-g_{\nu\alpha,b}\right)-f_{,\alpha}g^{ab}g_{\nu b,a}\\ -f_{,\nu}g^{ab}g_{\alpha b,a}\\ +f_{,d}g^{cd}\left(g_{\alpha c,\nu}-\frac{1}{2}ng_{\alpha c,\nu}-\frac{1}{2}ng_{\nu c,\alpha}+\frac{1}{2}ng_{\alpha\nu,c}\right)\end{array}\right)\\
-\delta_{\alpha}^{\beta}F'\left(f_{,ab}g^{ab}+f_{,d}g^{cd}\frac{1}{2}g_{ab,c}g^{ab}\right)\\
+\delta_{\alpha}^{\beta}F'\left(\begin{array}{c}(n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right)\\ -nf_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)\cdot\left(\frac{1}{2}+H\right)\\
+\frac{g^{\beta\nu}}{2F}f_{,\alpha}\cdot f_{,\nu}(n-2)\left(3(F')^{2}-2FF''\right)\\
+\delta_{\alpha}^{\beta}\frac{1}{2F}f_{,c}f_{,d}g^{cd}\left(\left((F')^{2}(4-n)-2FF''\right)\right)\\
+\delta_{\alpha}^{\beta}\frac{1}{2F}\left(f_{,c}f_{,d}g^{cd}(n-1)\left(4FF''+(F')^{2}(n-6)\right)\right)\cdot\left(\frac{1}{2}+H\right)
\end{array}\right). \tag{30}$$

From there we can evaluate the covariant derivative of the integrand of (17) as follows:

$$\begin{aligned}\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}&=\left(I_{\alpha}^{\beta}\left(G_{\beta}^{\alpha}\cdot\delta_{0}+\overbrace{G_{a}^{b}\delta_{a\beta}^{b\alpha}}^{\text{Gravity}}\right)\right)_{;\gamma}\xrightarrow{\forall\,\delta_{a\beta}^{b\alpha}\ll\delta_{0}}=\left(I_{\alpha}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0}\right)_{;\gamma}\\
&=I_{\alpha;\gamma}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0}+\overbrace{I_{\alpha}^{\beta}G_{\beta;\gamma}^{\alpha}\cdot\delta_{0}}^{=0}+I_{\alpha}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0;\gamma}\\
&=I_{\alpha;\gamma}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0}+I_{\alpha}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0;\gamma}\end{aligned}. \tag{31}$$

Assuming that the second term in the last line is small compared to the first, this simplifies to:

$$\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}\simeq I_{\alpha;\gamma}^{\beta}G_{\beta}^{\alpha}\cdot\delta_{0}+\overbrace{I_{\alpha}^{\beta}G_{\beta;\gamma}^{\alpha}\cdot\delta_{0}}^{=0}=\left(I_{\alpha}^{\beta}G_{\beta}^{\alpha}\right)_{;\gamma}\cdot\delta_{0}. \tag{32}$$

In consequence, the covariant derivative of the integrand (17) reads:

$$\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}\simeq\left(\begin{array}{c}\left(\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)G_{\beta}^{\alpha}\right)_{;\gamma}\cdot\delta_{0}\\ +\left(\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)G_{\beta}^{\alpha}\right)\cdot\delta_{0;\gamma}\end{array}\right). \tag{33}$$

In the classical theory, only the integrand is being considered, which means we would only have to consider (full derivation is being given in the appendix):

$$\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}\simeq\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)_{;\gamma}\cdot G_{\beta}^{\alpha}\delta_{0}. \tag{34}$$

Now we have two ways to perform the derivation in (33). At first, we evaluate the usual covariant derivation of the kernel, giving us:

$$\begin{aligned}\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma} &\simeq \left(\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)G_{\beta}^{\alpha}\right)_{;\gamma}\cdot\delta_0\\
&=\left(\begin{array}{l}\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)_{;\gamma}G_{\beta}^{\alpha}\\ +\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)G_{\beta;\gamma}^{\alpha}\end{array}\right)\cdot\delta_0\\
&\xrightarrow{\gamma\to\beta}\\
&=\left(\left(-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)_{;\beta}G_{\beta}^{\alpha}+0\right)\cdot\delta_0\end{aligned} \qquad (35)$$

As in (34), we thereby ignored the derivation of the variation term and applied the contracted Bianchi identity from (21).

The other option is to first process the contraction with the metric tensor and then derivate, giving us:

$$\begin{aligned}\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma} &\simeq \left(\begin{array}{l}\left(1-\frac{n}{2}\right)\left(R-\left(\frac{n-1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{F_{,i}\cdot F_{,j}}{4F^2}g^{ij}\left((n-6)(n-1)\right)\right)\right)\\ -n\cdot\left(R-\left(\frac{n-1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{F_{,i}\cdot F_{,j}}{4F^2}g^{ij}\left((n-6)(n-1)\right)\right)\right)\cdot H\end{array}\right)_{;\gamma}\cdot\delta_0\\
&=\left(\left(1-\frac{n}{2}-n\cdot H\right)\left(R-\left(\frac{n-1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{F_{,i}\cdot F_{,j}}{4F^2}g^{ij}\left((n-6)(n-1)\right)\right)\right)\right)_{;\gamma}\cdot\delta_0\end{aligned} \qquad (36)$$

As shown before (c.f. (8)), we can get rid of the non-linear differential operator with the approach for F[f], via:

$$4FF''+F'\cdot F'(n-6)=0 \quad\Rightarrow\quad F[f]=\begin{cases}C_F\cdot\left(f+C_f\right)^{\frac{4}{n-2}} & n\neq 2\\ C_F\cdot e^{f\cdot C_f} & n=2\end{cases}. \qquad (37)$$

Making the last part of (36) to:

$$\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}\simeq\left(\left(1-\frac{n}{2}-n\cdot H\right)\left(R-\left(\frac{n-1}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\ -\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)\right)\right)\right)_{;\gamma}\cdot\delta_0, \qquad (38)$$

we realize that when simplifying this to:

$$\left(I^{\beta}_{\alpha}\delta G^{\alpha}_{\beta}\right)_{;\gamma} \simeq \left(R-\left(\frac{n-1}{2F}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\right)\right)_{;\gamma}\cdot\delta_{0}, \tag{39}$$

and intending it to evolve it into a conservation law with:

$$0=\left(R-\left(\frac{n-1}{2F}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\right)\right)_{;\gamma}, \tag{40}$$

we will have no dependency with respect to H, because this was lost during the various simplifications and compactifications. Nevertheless, we still have an equation of third order in F, which we can write as follows:

$$\begin{aligned}
0&=\left(\begin{array}{l} R_{;\gamma}-\left(\frac{n-1}{2F}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\right)_{;\gamma} \\ +\left(\frac{n-1}{2F^{2}}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\cdot F_{;\gamma}\right) \end{array}\right)\\
&=\left(\begin{array}{l} R_{,\gamma}-\left(\frac{n-1}{2F}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\right)_{;\gamma} \\ +\left(\frac{n-1}{2F^{2}}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\cdot F_{;\gamma}\right) \end{array}\right).\\
0&=\left(\begin{array}{l} R_{,\gamma}F^{2}-\left(\frac{n-1}{2}F\left(2\Delta F_{,\gamma}-2F_{,d\gamma}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d\gamma}g^{cd}g^{ab}g_{ac,b}\right)\right) \\ +\left(\frac{n-1}{2}\left(2\Delta F-2F_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\right)\cdot F_{;\gamma}\right) \end{array}\right)
\end{aligned} \tag{41}$$

Assuming that the function F depends somehow Dirac-like on the mass m, we might expect results of the kind:

$$\begin{aligned}
0&=\left(\begin{array}{l} R_{,\gamma}F^{2}-\left(\frac{n-1}{2}F\cdot m^{2}\left(m\cdot 2\Delta\tilde{F}_{,\gamma}-2\tilde{F}_{,d\gamma}g^{cd}{}_{,c}-\frac{n}{(n-1)}\tilde{F}_{,d\gamma}g^{cd}g^{ab}g_{ac,b}\right)\right) \\ +\left(\frac{n-1}{2}m^{2}\cdot\left(2m\cdot\Delta\tilde{F}-2\tilde{F}_{,d}g^{cd}{}_{,c}-\frac{n}{(n-1)}\tilde{F}_{,d}g^{cd}g^{ab}g_{ac,b}\right)\cdot\tilde{F}_{,\gamma}\right) \end{array}\right),\\
&\Rightarrow 0=C_{0}+C_{2}\cdot m^{2}+C_{3}\cdot m^{3}
\end{aligned} \tag{42}$$

where we are clearly missing the $C_1\cdot m$-term in order to truly get three solutions for the mass m.

We want to investigate what causes such a reduction.

Could it be the usage of (34)?

Here the answer is “no”, because when going back to (33), we see that the term we have ignored is:

$$\begin{aligned}&\left(\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)G_{\beta}^{\alpha}\right)\cdot\delta_{0;\gamma}\\&=\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]\right)\cdot\left(G_{\beta}^{\alpha}\delta_{0}\right)_{;\gamma}\end{aligned}. \tag{43}$$

This, however, is no big deal, because:

$$R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right] \tag{44}$$

are the terms of the field equations of the scaled metric tensor and we demand them to vanish anyway:

$$R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R-\delta_{\alpha}^{\beta}R\cdot H+\frac{1}{2F}\not{\partial}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]=0. \tag{45}$$

Consequently, it has to be the "weak gravity" condition (27) which "messes up" our attempt to derive the 3-generations from a scalarized version of the field equations.

While we investigate the complete evaluation with *H* (without scalarization) from the path above in the appendix, we here also want to investigate the option of substituting the parameter *H* by a generalized kernel. Thereby, we start by incorporating the "jittering fulcrum" into the variational integrand via a perturbation function of the integration kernel, ending up with:

$$\begin{aligned}&\delta_{G}W=0=\delta_{G}\int_{V}d^{n}x\left(\sqrt{-G}\cdot\Phi\cdot R^{*}\right)\\&\delta W=0=\left[\begin{array}{c}\int_{V}d^{n}x\left(\sqrt{-G}\cdot\left(\begin{array}{c}\Phi\cdot R^{*}{}_{\mu\nu}-\frac{1}{2}\Phi\cdot R^{*}\cdot G_{\mu\nu}\\+\Lambda G_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-G_{\mu\nu}\Delta_{G}\right)\Phi\end{array}\right)\right)\delta G^{\mu\nu}\\-\int_{Surface}d^{n-1}y\left(\sqrt{|h|}\cdot\varepsilon\cdot\Phi\cdot N^{\lambda}h^{\mu\nu}\partial_{\lambda}\left(\delta G_{\mu\nu}\right)\right)\end{array}\right].\end{aligned} \tag{46}$$

Thereby we have assumed:

$$\delta W=0=\delta\int_{V}d^{n}x\left(\sqrt{-G}\cdot\left(\Phi\cdot R^{*}-\overbrace{2\Lambda+L_{M}}^{=0}\right)\right) \tag{47}$$

and used the results from [16] within the derivation.

The usual, and rather tedious, path forward would be the discussion of the last equation and its investigation with respect to all its constituents. In previous papers (e.g., [17-29]), however, we have shown how to avoid such cumbersome equations (especially the surface term). It was derived in [30] that the resulting variational integral would look as follows:

$$\delta W=0=\int_{V}d^{n}x\left(\sqrt{-G}\cdot\left(\begin{array}{c}\Phi\cdot R^{*}{}_{\mu\nu}-\frac{1}{2}\Phi\cdot R^{*}G_{\mu\nu}+\Lambda G_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-G_{\mu\nu}\Delta_{G}\right)\Phi\\+\left\{\begin{array}{ll}0&\ldots\text{"vacuum"}\\\kappa T_{\mu\nu}&\ldots\text{postulated matter}\end{array}\right\}\Phi\end{array}\right)\right)\delta G^{\mu\nu}. \tag{48}$$

Now we apply the "weak gravity condition" in covariant:

$$\delta G^{\alpha\beta}=G^{\alpha\beta}\cdot\delta_{0}+\overbrace{G^{ab}\delta_{ab}^{\alpha\beta}}^{\text{Gravity}}=\frac{1}{F}\cdot\left(g^{\alpha\beta}\cdot\delta_{0}+\overbrace{g^{ab}\delta_{ab}^{\alpha\beta}}^{\text{Gravity}}\right)\xrightarrow{\forall\,\delta_{ab}^{\alpha\beta}\ll\delta_{0}}=\frac{g^{\alpha\beta}}{F}\cdot\delta_{0} \tag{49}$$

or in mixed form:

$$\delta G_\beta^\alpha = G_\beta^\alpha \cdot \delta_0 + \overbrace{G_a^b \delta_{a\beta}^{b\alpha}}^{\text{Gravity}} \xrightarrow{\forall\, \delta_{a\beta}^{b\alpha} \ll \delta_0} = G_\beta^\alpha \cdot \delta_0 = g_\beta^\alpha \cdot \delta_0 = \delta_\beta^\alpha \cdot \delta_0 \,, \tag{50}$$

and define:

$$I_\alpha^\beta \equiv R_\alpha^\beta - \frac{\delta_\alpha^\beta}{2} R + \frac{1}{2F} \not{\delta}_\alpha^\beta \left[g_{\alpha\beta}; F \right] - \frac{\left(G^{\beta\mu} \nabla_\mu \nabla_\alpha - \delta_\alpha^\beta \Delta_G \right) \Phi}{\Phi} \tag{51}$$

with:

$$\frac{1}{2F} \not{\delta}_\alpha^\beta \left[g_{\alpha\beta}; F \right] \equiv \frac{1}{2F} \left(\begin{array}{c} \frac{\delta_\alpha^\beta}{2} \left(\begin{array}{c} (n-1) \left(2\Delta F - 2F_{,d} g^{cd}{}_{,c} - \frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac,b} \right) \\ + \frac{F_{,i} \cdot F_{,j}}{2F} g^{ij} \left((n-6)(n-1) \right) \end{array} \right) \\ -g^{\beta\gamma} \left(\begin{array}{c} F_{,\alpha\gamma} (n-2) + F_{,ab} g_{\alpha\gamma} g^{ab} \\ + F_{,a} g^{ab} \left(g_{\gamma b,\alpha} - g_{\gamma\alpha,b} \right) - F_{,\alpha} g^{ab} g_{\gamma b,a} - F_{,\gamma} g^{ab} g_{\alpha b,a} \\ + F_{,d} g^{cd} \frac{1}{2} n \left(\begin{array}{c} \frac{2}{n} g_{\alpha c,\gamma} - g_{\alpha c,\gamma} - g_{\gamma c,\alpha} \\ + g_{\alpha\gamma,c} + \frac{1}{n} g_{\alpha\gamma} g_{ab,c} g^{ab} \end{array} \right) \\ - \frac{1}{2F} \left(F_{,\alpha} \cdot F_{,\gamma} (3n-6) + g_{\alpha\gamma} F_{,c} F_{,d} g^{cd} (4-n) \right) \end{array} \right) \end{array} \right) . \tag{52}$$

From there we can evaluate the covariant derivative of the integrand of (48) as follows:

$$\begin{aligned} \left(I_\alpha^\beta \delta G_\beta^\alpha \right)_{;\gamma} &= \left(I_\alpha^\beta \left(G_\beta^\alpha \cdot \delta_0 + \overbrace{G_a^b \delta_{a\beta}^{b\alpha}}^{\text{Gravity}} \right) \right)_{;\gamma} \xrightarrow{\forall\, \delta_{a\beta}^{b\alpha} \ll \delta_0} = \left(I_\alpha^\beta G_\beta^\alpha \cdot \delta_0 \right)_{;\gamma} \\ &= I_{\alpha;\gamma}^\beta G_\beta^\alpha \cdot \delta_0 + \overbrace{I_\alpha^\beta G_{\beta;\gamma}^\alpha \cdot \delta_0}^{=0} + I_\alpha^\beta G_\beta^\alpha \cdot \delta_{0;\gamma} \\ &= I_{\alpha;\gamma}^\beta G_\beta^\alpha \cdot \delta_0 + I_\alpha^\beta G_\beta^\alpha \cdot \delta_{0;\gamma} \end{aligned} . \tag{53}$$

Assuming that the second term in the last line is small compared to the first, this simplifies to:

$$\left(I_\alpha^\beta \delta G_\beta^\alpha \right)_{;\gamma} \simeq I_{\alpha;\gamma}^\beta G_\beta^\alpha \cdot \delta_0 + \overbrace{I_\alpha^\beta G_{\beta;\gamma}^\alpha \cdot \delta_0}^{=0} = \left(I_\alpha^\beta G_\beta^\alpha \right)_{;\gamma} \cdot \delta_0 . \tag{54}$$

In consequence, the covariant derivative of the integrand (48) (using (51) and (54)) reads:

$$\begin{aligned} \left(I_\alpha^\beta \delta G_\beta^\alpha \right)_{;\gamma} &\simeq \left(\left(R_\alpha^\beta - \frac{\delta_\alpha^\beta}{2} R + \frac{1}{2F} \not{\delta}_\alpha^\beta \left[g_{\alpha\beta}; F \right] - \frac{\left(G^{\beta\mu} \nabla_\mu \nabla_\alpha - \delta_\alpha^\beta \Delta_G \right) \Phi}{\Phi} \right) G_\beta^\alpha \right)_{;\gamma} \cdot \delta_0 \\ &= \left(\left(1 - \frac{n}{2} \right) \left(R - \left(\begin{array}{c} \frac{n-1}{2F} \left(\begin{array}{c} 2\Delta F - 2F_{,d} g^{cd}{}_{,c} \\ - \frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) \\ + \frac{F_{,i} \cdot F_{,j}}{4F^2} g^{ij} \left((n-6)(n-1) \right) \end{array} \right) \right) + (n-1) \frac{\Delta_G \Phi}{\Phi} \right)_{;\gamma} \cdot \delta_0 \end{aligned} . \tag{55}$$

As shown before (c.f. (8)), we can get rid of the non-linear differential operator with the approach for F[f], via:

$$4FF''+F'\cdot F'(n-6)=0\quad\Rightarrow\quad F[f]=\begin{cases}C_F\cdot\left(f+C_f\right)^{\frac{4}{n-2}} & n\neq 2\\ C_F\cdot e^{f\cdot C_f} & n=2\end{cases},\tag{56}$$

making (55) to:

$$\begin{aligned}\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}&\simeq\left(\left(R_{\alpha}^{\beta}-\frac{\delta_{\alpha}^{\beta}}{2}R+\frac{1}{2F}\not{\delta}_{\alpha}^{\beta}\left[g_{\alpha\beta};F\right]-\frac{\left(G^{\beta\mu}\nabla_{\mu}\nabla_{\alpha}-\delta_{\alpha}^{\beta}\Delta_G\right)\Phi}{\Phi}\right)G_{\beta}^{\alpha}\right)_{;\gamma}\cdot\delta_0\\
&=\left(\begin{array}{c}\left(1-\frac{n}{2}\right)\left(\begin{array}{c}R-\frac{g^{\alpha\beta}}{2F}F'\left(2f_{,\alpha\beta}(n-1)+f_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-g_{\beta\alpha,c}-g_{c\beta,\alpha}\\+\frac{n}{2}\left(2g_{\alpha\beta,c}-g_{\alpha c,\beta}-g_{\beta c,\alpha}\right)\end{array}\right)\right)\\-(n-1)\frac{f_{,\alpha}\cdot f_{,\beta}}{4F^2}g^{\alpha\beta}\overbrace{\left(4FF''+F'\cdot F'(n-6)\right)}^{=0}\end{array}\right)\\+(n-1)\frac{\Delta_G\Phi}{\Phi}\end{array}\right)_{;\gamma}\cdot\delta_0\\
&=\left(\begin{array}{c}\left(1-\frac{n}{2}\right)\left(R-\frac{g^{\alpha\beta}}{2F}F'\left(2f_{,\alpha\beta}(n-1)+f_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-g_{\beta\alpha,c}-g_{c\beta,\alpha}\\+\frac{n}{2}\left(2g_{\alpha\beta,c}-g_{\alpha c,\beta}-g_{\beta c,\alpha}\right)\end{array}\right)\right)\right)\\+(n-1)\frac{\Delta_G\Phi}{\Phi}\end{array}\right)_{;\gamma}\cdot\delta_0\end{aligned}\quad.\tag{57}$$

As this should be a conservation law, the results have to be zero, and thus:

$$\begin{aligned}&\left(I_{\alpha}^{\beta}\delta G_{\beta}^{\alpha}\right)_{;\gamma}=0\\&\Rightarrow\\&\left(\begin{array}{c}\left(1-\frac{n}{2}\right)\left(R-\frac{g^{\alpha\beta}}{2F}F'\left(2f_{,\alpha\beta}(n-1)+f_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-g_{\beta\alpha,c}-g_{c\beta,\alpha}\\+\frac{n}{2}\left(2g_{\alpha\beta,c}-g_{\alpha c,\beta}-g_{\beta c,\alpha}\right)\end{array}\right)\right)\right)\\+(n-1)\frac{\Delta_G\Phi}{\Phi}\end{array}\right)_{;\gamma}=0\end{aligned}\quad.\tag{58}$$

Now we assume that the function f[t,x,y,z]=f[t] shall code a Dirac particle at rest [31], which would be given due to:

$$f[t]=e^{\pm i\cdot\frac{m_R\cdot c^2}{\hbar}\cdot t}\cdot C_f=e^{\pm i\cdot\mu\cdot t}\cdot C_f,\tag{59}$$

with m_R giving the rest mass of the particle, and $\hbar$ denoting the reduced Planck constant.
Further assuming an ordinary space-time in 4 dimensions with the metric tensor:

$$g_{\alpha\beta} = \begin{pmatrix} g_t[t] & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad g_t[t] = e^{\pm i \cdot m \cdot t} \tag{60}$$

and an oscillating perturbation of the kernel of the Einstein-Hilbert action via the kernel scaling function:

$$\Phi[t] = e^{\pm i \cdot M \cdot t} \cdot C_\Phi, \tag{61}$$

and inserting all this into (58), does result in the following algebraic equation:

$$\frac{3mM^2}{8} - \frac{m^2M}{8} - \frac{M^3}{4} + \frac{\left(2mM - m^2\right)}{4} \cdot \mu + \frac{1}{2} M\mu^2 + \mu^3 = 0. \tag{62}$$

Thereby we have assumed only positive signs in the exponents for all functions f, Φ, and g_t. Equation (62) can be written as follows:

$$0 = -C_0 + C_1 \cdot \mu - C_2 \cdot \mu^2 + \mu^3. \tag{63}$$

The general solution to a third-order polynomial could be given via the following product form:

$$\begin{aligned} &(\mu - \mu_1) \cdot (\mu - \mu_2) \cdot (\mu - \mu_3) = \\ &\mu^3 - \mu^2 \cdot (\mu_1 + \mu_2 + \mu_3) + \mu \cdot (\mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3) - \mu_1\mu_2\mu_3 \end{aligned}. \tag{64}$$

Comparing the last line of (64) with (63) gives us:

$$\begin{aligned} C_2 &= \mu_1 + \mu_2 + \mu_3 = -\frac{M}{2} \\ C_1 &= \mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3 = \frac{\left(2mM - m^2\right)}{4} \\ C_0 &= \mu_1\mu_2\mu_3 = \frac{1}{4}\left(M^3 - \frac{3mM^2}{2} + \frac{m^2M}{2}\right) \end{aligned}. \tag{65}$$

Unfortunately, we only have two parameters and thus will not be able to find a metric setting for the explanation of the three generations of elementary particles. The reason for this may be seen in our far too simple metric approach, which we have only chosen to explicitly keep things simple and stick to the classical (Dirac) particle at rest. However, keeping the lazy spirit, how about a slight extension to a 3-parameter approach with still purely time-dependent metric components like:

$$g_{\alpha\beta} = \begin{pmatrix} g_t[t] & 0 & 0 & 0 \\ 0 & g_s[t] & 0 & 0 \\ 0 & 0 & g_s[t] & 0 \\ 0 & 0 & 0 & g_s[t] \end{pmatrix}; \quad g_s[t] = e^{\pm i \cdot \nu \cdot t}\ ? \tag{66}$$

Now we result in the equation:

$$\begin{pmatrix} -\frac{m^2M}{8} + \frac{3mM^2}{8} - \frac{M^3}{4} - \frac{m^2\nu}{8} + \frac{mM\nu}{2} - \frac{3M^2\nu}{8} + \frac{m\nu^2}{4} \\ -\frac{M\nu^2}{4} - \frac{m^2\mu}{4} + \frac{mM\mu}{2} + \frac{m\nu\mu}{2} + \frac{\nu^2\mu}{2} + \frac{M\mu^2}{2} + \frac{3\nu\mu^2}{2} + \mu^3 \end{pmatrix} = 0. \tag{67}$$

3.4.1 The Three Generations of Elementary Particles

The numerical solutions to the third order algebraic equation (67) for the charged leptons (electron μ=0.511 MeV/c^2, muon μ=105.7 MeV/c^2, and tau μ=1777 MeV/c^2) would be (all in MeV/c^2):

$$m \rightarrow 1220.68,\ M \rightarrow 1221.7,\ \nu \rightarrow -548.507,$$

$$m \rightarrow -1054.41,\ M \rightarrow -1053.39,\ \nu \rightarrow 209.857,$$

where we have given only the two real solutions, while there are six complex ones in total. The corresponding results for the neutrinos and the quarks are given in [20]. Thereby we found funny asymmetries of numerical character with respect to matter and antimatter particles [21].

3.5 Surprise: Every System Can Exist in Three Generations

We found a manageable way to extract a weak-gravity scalar from the quantum Einstein field equations. Subjecting it to the covariant derivative operation, we obtained conservation laws. Applying the latter to a simple metric with just time-dependent components directly resulted in a polynomial of third order in the mass-parameter for the object described by the metric. The polynomial gives three solutions for such masses, and by feeding the known masses of the three elementary particle generations into the approach, we were able to evaluate the metric settings for charged and uncharged leptons and all quarks [20]. Interpreting the masses in a more general way for other systems than just elementary particles, namely as inertia, we might even ask whether also intelligent systems, for which the same fundamental (because purely mathematical) law holds, can or even have to exist in such 3 generations. The same applies when sticking to full, which is to say the non-scalarized and non-simplified field equations (see appendix).

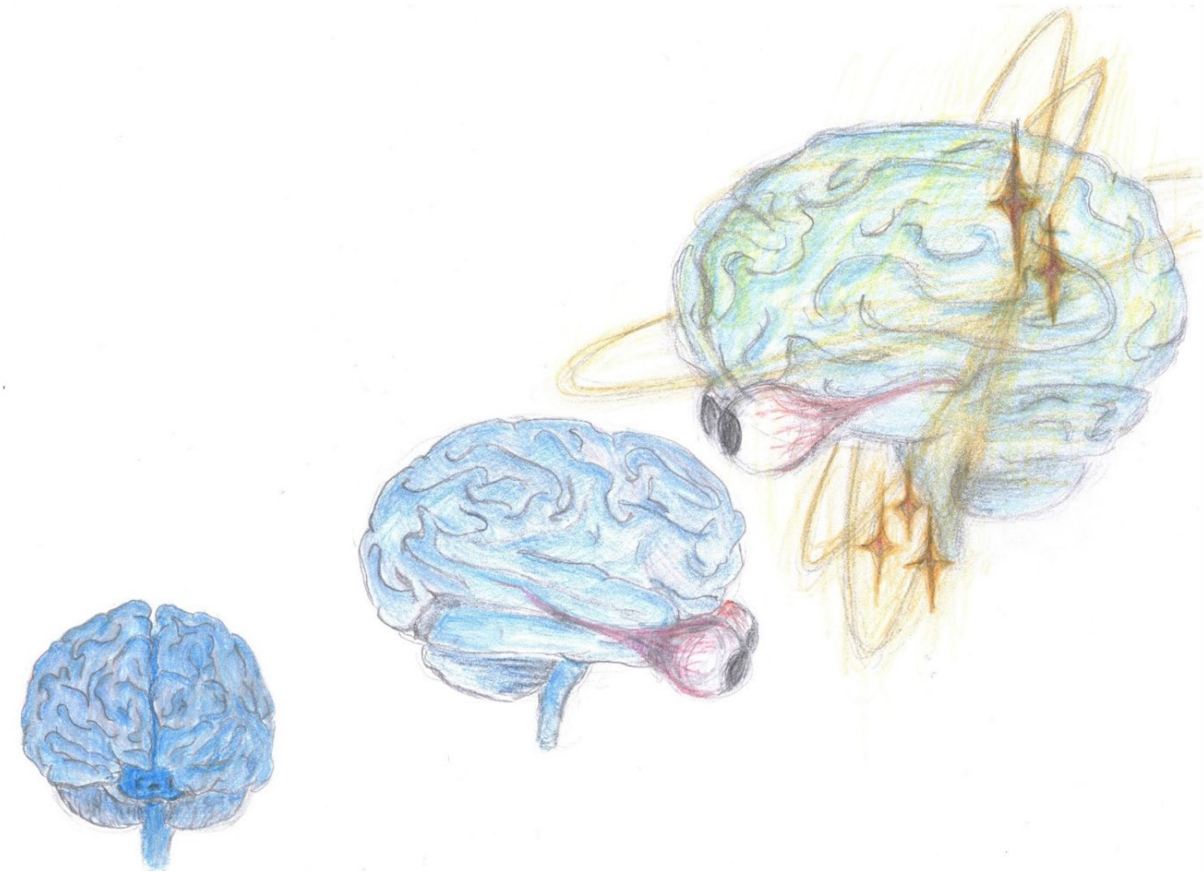

Fig. 3: "The Three Generations of Consciousness" by Livia Schwarzer (see [32] for more)

3.6 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (78) correspond to more stable states than non-extremal solutions, and in fact we see this in connection with the 3-generations problem. There we obtain equations of third order only in cases with non-zero *H*, which is to say for the non-extremal Hamilton situation. This agrees with the reality where only the electron is stable, while muon and tauon have a rather brief life-span.

Thus, we realized mathematically that some matter forms can only be based on the generalized Hamilton or "ever-jittering fulcrum" principle, and this may directly lead to the conjecture that such forms are less stable (e.g., muon and tauon) than the ones resulting from extremal settings (e.g., electron). What is more, it might well be possible that the many possibilities for non-extremal settings all come with different rates of stability, interactivity, and probability to appear at all.

After all, and as already hinted elsewhere ([10] and appendix of this paper), the introduction of a cosmological constant or the postulation of a matter Lagrange density can be mathematically reformulated as non-extremal or perturbated Hamilton principles in Hilbert integral form.

4 Conclusions

Using the contracted Bianchi identity and distinguishing the covariant derivative therein with respect to the scaling function, we were able to obtain equations of third order for the metric scaling factor. Consequently, this leads to three solutions for certain parameters on which the scaling depends. Mass probably is one of these parameters and can therefore occur in three solutions, of which only one is stable.

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6 Appendix

From Wikipedia, the free encyclopedia (https://en.wikipedia.org/wiki/Hamilton's_principle):

> *In physics, Hamilton's principle is William Rowan Hamilton's formulation of the principle of stationary action. It states that the dynamics of a physical system are determined by a variational problem for a functional based on a single function, the Lagrangian, which may contain all physical information concerning the system and the forces acting on it. The variational problem is equivalent to and allows for the derivation of the differential equations of motion of the physical system. Although formulated originally for classical mechanics, Hamilton's principle also applies to classical fields such as the electromagnetic and gravitational fields, and plays an important role in quantum mechanics, quantum field theory and criticality theories.*

So, the definition of the Hamilton principle is based on its "formulation of the principle of stationary action". In simpler words, the variation of such an action should be zero or, mathematically formulated, should be put as follows:

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L . \tag{68}$$

Here L stands for the Lagrangian, W the action, and g gives the determinant of the metric tensor, which describes the system in question within an arbitrary Riemann space-time with the coordinates x. Thereby, we used the Hilbert formulation of the Hamilton principle [1] in a slightly more general form. We were able to show in [2] that the original Hilbert variation does not only produce the Einstein field equations [3] but also contains the Quantum Theory [2, 4, 5]. It should be noted that, while the original Hilbert paper [1] started with the Ricci scalar R as the integral kernel, which is to say L=R, we here used a general Langrangian, because—as we will show later in this appendix—this generality—in principle—is already contained inside the original Hilbert formulation. Even, as strange as it may sound at this point, general kernels with functions of the Ricci scalar f(R) [6] are already included (see [12]) in the Hilbert approach.

But what if we lived in a universe where the only thing that was certain was uncertainty?

One the authors in [7], Dr. David Martin, always used the analogy of a moving fulcrum to demonstrate his uneasiness with the formulation (68) [11].

In [7] we were able to show that the Hamilton principle itself hinders us to localize any system or object at a certain position. We also see that this contradicts the concept of particles. Everything seems to be permanently on the move or—rather—ever-jittering.

But if this ever-jittering fulcrum was one of the fundamental properties of our universe, should we then not take this into account when formulating the laws of this very universe? Shouldn't we better write (68) as follows:

$$\delta W \to 0 \cong \delta \int_V d^n x \cdot \sqrt{-g} \cdot L \; ? \tag{69}$$

And while we are at it, should we not start to investigate an even more general principle like:

$$\delta W \to f\left(W, x, g_{\alpha\beta}\right) = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L\,? \tag{70}$$

The interesting aspect about this is that this investigation was already—partially—done by (surprise, surprise) e.g., Hilbert and Einstein. But instead of explaining it in this way, they have "hidden" their generalization inside other concepts like the introduction of a cosmological constant or—oh yes—the postulation of matter and its introduction via an ominous and purely postulated parameter L_M, which is to say, a Lagrange matter term.

6.1 The Classical Hamilton Extremal Principle and How to Obtain Einstein's General Theory of Relativity with Matter (!) and Quantum Theory… Also with Matter (!)

The famous German mathematician David Hilbert [1], even though applying his technique only to derive the Einstein field equations for the General Theory of Relativity [3] in four dimensions,—in principle—extended the classical Hamilton principle to an arbitrary Riemann space-time with a very general variation by not only—as Hamilton and others had done—concentrating on the evolution of the given problem or system in time, but with respect to all its dimensions. His formulation of the Hamilton extremal principle looked as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot \left(R - 2\Lambda + L_M\right)\right). \tag{71}$$

There we have the Ricci scalar of curvature R, the cosmological constant Λ, the Lagrange density of matter L_M, and the determinant g of the metric tensor of the Riemann space-time $g_{\alpha\beta}$. For historical reasons, it should be mentioned that Hilbert's original work [1] did not contain the cosmological constant, because it was added later by Einstein in order to obtain a static universe, but this is not of any importance here. The evaluation of the so-called Einstein-Hilbert action (71) indeed brought the Einstein General Theory of Relativity [3], but it did not produce the other great theory physicists have found, which is the Quantum Theory. It was not before this author, about one hundred years after the publication of Hilbert's paper [1], extended Hilbert's approach by considering scaling factors to the metric tensor and showed that Quantum Theory already resides inside the sufficiently general General Theory of Relativity [2, 4, 7, 8, 9]. We will not discuss the reason why this simple idea has not been tried out by other scientists before, but we may still express our amazement about the fact that a simple extension of the type:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \tag{72}$$

solves one of the greatest problems in science[2], namely the unification of physics and that it took science more than 100 years to come up with the idea. Using the symbol G for the determinant of the scaled metric tensor $G_{\alpha\beta}$ from (72) of the Riemann space-time, we can rewrite the Einstein-Hilbert action from (71) as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot \left(R^* - 2\Lambda + L_M\right)\right). \tag{73}$$

It should be pointed out that a setting like:

[2] This does not mean, of course, that we should not also look out for generalizations of the scaled metric and investigate those as we did in [10].

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right) \tag{74}$$

could also be possible and still converges to the classical form for F→1. Here, which is to say in this paper, we will only consider examples with q=0, but for completeness and later investigation we shall mention that a comprehensive consideration of variational integrals for the cases of general q are to be found in [4].

Performing the variation in (74) with respect to the metric $G_{\alpha\beta}$ and remembering that the Ricci curvature of such a metric (e.g., [7] appendix D) changes the whole variation to:

$$\begin{aligned}
\delta W = 0 &= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right) \\
&= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(\left(\begin{array}{c} \dfrac{R}{F} - \dfrac{1}{2F^2} \left(\begin{array}{l} (n-1)\left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}} \right) \\ -nF_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) \\ -(n-1)\dfrac{g^{ab}F_{,a} \cdot F_{,b}}{4F^3}(n-6) \end{array} \right) - 2\Lambda + L_M \right) \right),
\end{aligned} \tag{75}$$

results in:

$$\begin{aligned}
0 &= \left(R^*{}_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta\left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\
&= \left(\begin{array}{c} \left(R_{\alpha\beta} - \dfrac{1}{2F} \left(\begin{array}{c} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b} \right) - \\ F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd} \left(\begin{array}{l} g_{\alpha c,\beta} - \dfrac{1}{2}ng_{\alpha c,\beta} - \dfrac{1}{2}ng_{\beta c,\alpha} \\ + \dfrac{1}{2}ng_{\alpha\beta,c} + \dfrac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{array} \right) \end{array} \right) \\ + \dfrac{1}{4F^2}\left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n) \right) \end{array} \right) \delta G^{\alpha\beta} \\ + \left(\dfrac{(n-1)}{2F} \left(\begin{array}{l} \left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}} \right) \\ - \dfrac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) + \dfrac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2}(n-6) - \dfrac{R}{(n-1)} \right) \cdot \dfrac{g_{\alpha\beta}}{2} \end{array} \right),
\end{aligned} \tag{76}$$

when setting q=0 and assuming a vanishing cosmological constant. With a cosmological constant we have to write:

$$
0=\left(R^{*}{}_{\alpha\beta}-\frac{1}{2}R^{*}\cdot G_{\alpha\beta}\right)\overbrace{\left(\frac{1}{F}\cdot\delta g^{\alpha\beta}+g^{\alpha\beta}\cdot\delta\left(\frac{1}{F}\right)\right)}^{\delta G^{\alpha\beta}}
$$

$$
=\left(\begin{array}{c}
\boxed{\boxed{R_{\alpha\beta}-R\frac{g_{\alpha\beta}}{2}}}+\boxed{\Lambda\cdot g_{\alpha\beta}} \\
-\frac{1}{2F}\left(\begin{array}{c}
F_{,\alpha\beta}\left(n-2\right)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)- \\
F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}
g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha} \\
+\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}
\end{array}\right)
\end{array}\right) \\
+\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}\left(3n-6\right)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}\left(4-n\right)\right) \\
+\left(\frac{\left(n-1\right)}{2F}\left(\begin{array}{c}
2\Delta F-2F_{,d}g^{cd}{}_{,c} \\
-\frac{n}{\left(n-1\right)}F_{,d}g^{cd}g^{ab}g_{ac,b}
\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}\left(n-6\right)\right)\cdot\frac{g_{\alpha\beta}}{2}
\end{array}\right)\delta G^{\alpha\beta}\quad. \tag{77}
$$

For better recognition of the classical terms, we have reordered a bit and boxed the classical vacuum part of the Einstein field equations (double lines) and the cosmological constant term (single line). Everything else can be—no, represents (!)—matter or quantum effects or both.

Thus, we also—quite boldly—have set the matter density L_M equal to zero, because we see that already our simple metric scaling brings in quite some options for the construction of matter. It will be shown elsewhere [10] that there is much more which is based on the same technique.

6.2 The Principle of the Ever-Jittering Fulcrum and the Alternate Hamilton Principle

We might bring forward two reasons why we could doubt the fundamentality of the Hamilton principle even in its most general form of the generalized Einstein-Hilbert action:

a) The principle was postulated and never fundamentally derived.
b) Even the formulation of this principle in its classical form (71) results in a variety of options where factors, constants, kernel adaptations, etc. could be added, so that the rigid setting of the integral to zero offers some doubt in itself. A calculation process which offers a variety of add-ons and options should not contain such a dogma. The result should be kept open and general. Dr. David Martin proposed this as the "tragedy of the jittering fulcrum" and we therefore named this principle "David's principle of the ever-jittering fulcrum" [11]. It demands:

$$
\begin{array}{l}
\delta_{g_{\alpha\beta}}W\simeq ?\simeq\delta_{g_{\alpha\beta}}\int\limits_{V}d^{n}x\sqrt{-g}\times R \\
\delta_{G_{\alpha\beta}}W\simeq ?\simeq\delta_{G_{\alpha\beta}}\int\limits_{V}d^{n}x\sqrt{-G}\times R^{*}
\end{array}\,. \tag{78}
$$

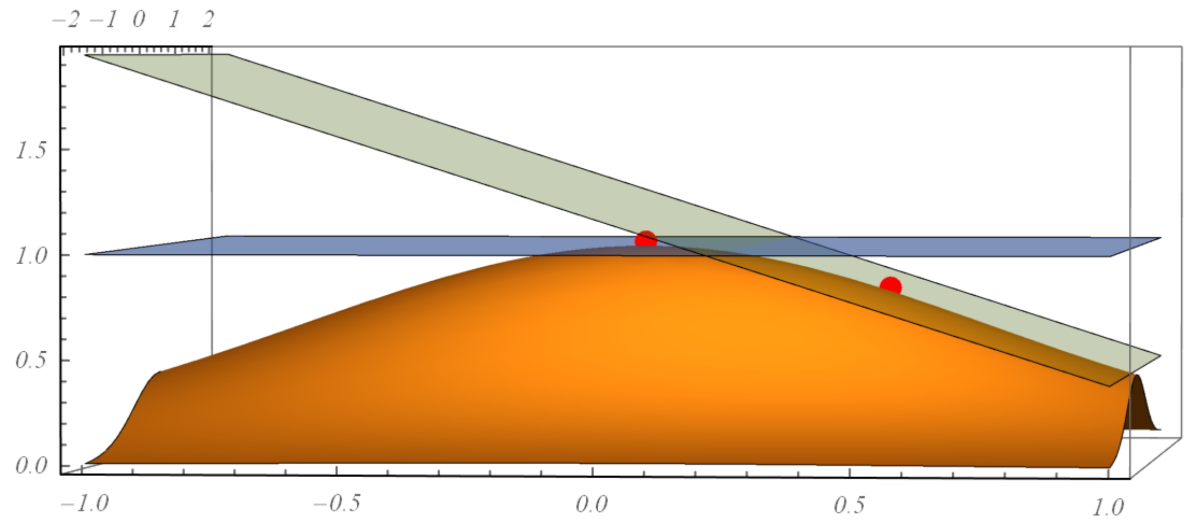

Fig. A1: David's principle of the ever-jittering fulcrum cannot accept a dogmatic insistence on a zero outcome of the Einstein-Hilbert action (71) or (generalized and also bringing about the Quantum Theory) (74). Instead it should allow for all states and not just the extremal position (see the two red dots and the corresponding tangent planes in the picture).

One of the simplest generalizations of the classical principle could be the linear one, which is illustrated in figure A1. It could be constructed as follows:

$$\int_V d^n x \sqrt{-g} \times \not{\lambda}^{\alpha\beta} \cdot g_{\alpha\beta} = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R\,. \tag{79}$$

Thereby we have used the classical form with the unscaled metric tensor, respectively, without setting the factor apart from the rest of the metric. Performing the variation on the right-hand side and setting

$$\not{\lambda}^{\alpha\beta} = H \cdot \delta g^{\alpha\beta} \tag{80}$$

or—for the reason of maximum generality—even:

$$\not{\lambda}^{\alpha\beta} = H^{\alpha\beta}_{ab} \cdot \delta\gamma^{ab} = H \cdot \delta g^{\alpha\beta} \tag{81}$$

just gives us the same result as we would obtain it when assuming a non-zero cosmological constant, because evaluation yields:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H \cdot \delta g^{\alpha\beta} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned}, \tag{82}$$

respectively:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H^{\alpha\beta}_{ab} \cdot \delta\gamma^{ab} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned}. \tag{83}$$

Simply setting H=-Λ (c.f. single-line boxed term in equation (77)) demonstrates this.

Nothing else is the usage of a general functional term T, being considered a function of the coordinates of the system (perhaps even the metric tensor) in a general manner, as follows:

$$\int_V d^n x\sqrt{-g}\times T = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x\sqrt{-g}\times R \,. \tag{84}$$

As before, performing the variation on the right-hand side and setting

$$T = T_{\alpha\beta}\cdot \delta g^{\alpha\beta} \tag{85}$$

gives us something which was classically postulated under the variational integral, namely the classical energy-matter tensor. This time, however, it simply pops up as a result of David's principle of the jittering fulcrum and is equivalent to the introduction of the term L_M under the variational integral. Evaluation yields:

$$\begin{aligned}\int_V d^n x\sqrt{-g}\cdot T_{\alpha\beta}\cdot \delta g^{\alpha\beta} &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}\right)\delta g^{\alpha\beta}\\ \Rightarrow 0 &= \int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}-T_{\alpha\beta}\right)\delta g^{\alpha\beta}\end{aligned}\,. \tag{86}$$

So, we see that in introducing a cosmological constant and in postulating a matter term, even Einstein and Hilbert already—in principle—"experimented" with a non-extremal setting for the Hamilton extremal principle.

Apart from linear dependencies and other functions or functional terms, we could just assume a general outcome like:

$$f(W) = f\left(\int_V d^n x\sqrt{-g}\times R\right) = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x\sqrt{-g}\times R \,. \tag{87}$$

This, however, would not give us any substantial hint where to move on, respectively, which of the many possible paths to follow. We therefore here start our investigation with the assumption of an eigen result for the variation as follows:

$$\xcancel{h}\cdot W = \xcancel{h}\cdot\int_V d^n x\sqrt{-g}\times R = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x\sqrt{-g}\times R \,. \tag{88}$$

This leads to:

$$\int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda} - R\cdot\left(\frac{1}{2}\cdot g_{\kappa\lambda}\delta g^{\kappa\lambda}+\xcancel{h}\right)\right)=0\,. \tag{89}$$

As the term $\xcancel{h}$ could always be expanded into an expression like:

$$\xcancel{h} = H\cdot g_{\kappa\lambda}\delta g^{\kappa\lambda}\,, \tag{90}$$

we obtain from (89):

$$\begin{aligned}0 &= \int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\delta g^{\kappa\lambda}\right)\\ &= \int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\right)\delta g^{\kappa\lambda}\\ &\Rightarrow R_{\kappa\lambda} - R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda} = 0\end{aligned}\,. \tag{91}$$

We realize that the term *H* can be a general scalar even if we would demand the term $\xcancel{h}$ to be a constant.

The complete equation when assuming a scaled metric tensor of the form (72) would read:

$$\left(\left(R_{\alpha\beta} - \frac{1}{2F}\begin{pmatrix} F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab} \\ +F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right) - F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} \\ +F_{,d}g^{cd}\left(g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} + \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\right)\end{pmatrix} + \frac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\right) - \left(R - \frac{1}{2F}\left((n-1)\left(\overbrace{2g^{ab}F_{,ab}+F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F-2F_{,d}g^{cd}{}_{,c}}\right) - nF_{,d}g^{cd}g^{ab}g_{ac,b}\right) - (n-1)\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6)\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\right) = 0, \tag{92}$$

and in the case of metrics with constant components this equation simplifies to:

$$\left(\begin{pmatrix} R_{\alpha\beta} - \frac{1}{2F}\left(F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}\right) \\ +\frac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\end{pmatrix} - \left(R - \frac{(n-1)}{2F}\left(2g^{ab}F_{,ab} + \frac{g^{ab}F_{,a}\cdot F_{,b}}{2F}(n-6)\right)\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\right) = 0\cdot \tag{93}$$

6.2.1 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (78) correspond to more stable states than non-extremal solutions, and in fact we will find this in connection with the 3-generations problem, which we have discussed in the main part of the paper and will now derive in the next subsection of this appendix.

6.3 The 3-Generations Problem as a Polynomial of Third Order

When investigating the complete field equations (92) under covariant derivation (with respect to the unscaled metric tensor g_{ab}), which is to say:

$$\Rightarrow 0=\left(\begin{array}{c}
-\frac{F'}{2F}g^{\beta\nu}\left(\begin{array}{c}
f_{,\alpha\nu}(n-2)+f_{,a}g^{ab}\left(g_{\nu b,\alpha}-g_{\nu\alpha,b}\right)-f_{,\alpha}g^{ab}g_{\nu b,a} \\
-f_{,\nu}g^{ab}g_{\alpha b,a}+f_{,d}g^{cd}\left(g_{\alpha c,\nu}-\frac{1}{2}ng_{\alpha c,\nu}-\frac{1}{2}ng_{\nu c,\alpha}+\frac{1}{2}ng_{\alpha\nu,c}\right)
\end{array}\right) \\
-\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(f_{,ab}g^{ab}+f_{,d}g^{cd}\frac{1}{2}g_{ab,c}g^{ab}\right)-\delta_{\alpha}^{\beta}R\cdot H \\
+\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(\begin{array}{c}
(n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right) \\
-nf_{,d}g^{cd}g^{ab}g_{ac,b}
\end{array}\right)\cdot\left(\frac{1}{2}+H\right) \\
+\frac{g^{\beta\nu}}{4F^{2}}f_{,\alpha}\cdot f_{,\nu}(n-2)\left(3(F')^{2}-2FF''\right) \\
+\delta_{\alpha}^{\beta}\frac{1}{4F^{2}}f_{,c}f_{,d}g^{cd}\left(\left((F')^{2}(4-n)-2FF''\right)\right) \\
+\delta_{\alpha}^{\beta}\frac{1}{4F^{2}}\left(f_{,c}f_{,d}g^{cd}(n-1)\left(4FF''+(F')^{2}(n-6)\right)\right)\cdot\left(\frac{1}{2}+H\right)
\end{array}\right)_{;\beta} \tag{94}$$

in the slightly compactified form:

$$0=\left(\begin{array}{c}
-\frac{F'}{2F}g^{\beta\nu}\left(\begin{array}{c}
f_{,\alpha\nu}(n-2)+f_{,a}g^{ab}\left(g_{\nu b,\alpha}-g_{\nu\alpha,b}\right)-f_{,\alpha}g^{ab}g_{\nu b,a} \\
-f_{,\nu}g^{ab}g_{\alpha b,a}+f_{,d}g^{cd}\left(g_{\alpha c,\nu}-\frac{1}{2}ng_{\alpha c,\nu}-\frac{1}{2}ng_{\nu c,\alpha}+\frac{1}{2}ng_{\alpha\nu,c}\right)
\end{array}\right) \\
-\delta_{\alpha}^{\beta}R\cdot H-\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(f_{,ab}g^{ab}+f_{,d}g^{cd}\frac{1}{2}g_{ab,c}g^{ab}\right) \\
+\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(\begin{array}{c}
(n-1)\left(2g^{ab}f_{,ab}+f_{,d}g^{cd}g^{ab}g_{ab,c}\right) \\
-nf_{,d}g^{cd}g^{ab}g_{ac,b}
\end{array}\right)\cdot\left(\frac{1}{2}+H\right) \\
+\frac{1}{4F^{2}}\left(\begin{array}{c}
\left(3(F')^{2}-2FF''\right)(n-2)g^{\beta\nu}f_{,\alpha}\cdot f_{,\nu} \\
+\delta_{\alpha}^{\beta}f_{,c}f_{,d}g^{cd}\overbrace{\left(\begin{array}{c}
(F')^{2}\left(4-n+(n-1)(n-6)\cdot\left(\frac{1}{2}+H\right)\right) \\
+2FF''\left(2(n-1)\cdot\left(\frac{1}{2}+H\right)-1\right)
\end{array}\right)}^{E^{2}}
\end{array}\right)
\end{array}\right)_{;\beta} \tag{95}$$

under the assumption of a true particle at rest, which means that everything, also the metric, only depends on the time coordinate $t=x_0$, we obtain:

$$
\Rightarrow 0=\left(\begin{array}{c}
-\frac{F'}{2F}g^{\beta\nu}\left(\begin{array}{c}
f_{,\alpha\nu}(n-2)+f_{,0}g^{0b}\left(g_{\nu b,\alpha}-g_{\nu\alpha,b}\right)-f_{,\alpha}\big|_{\alpha=0}g^{ab}g_{\nu b,a}\\
-f_{,\nu}g^{0b}g_{\alpha b,0}+f_{,0}g^{c0}\left(g_{\alpha c,\nu}-\frac{1}{2}ng_{\alpha c,\nu}-\frac{1}{2}ng_{\nu c,\alpha}+\frac{1}{2}ng_{\alpha\nu,c}\right)
\end{array}\right)\\
-\delta_{\alpha}^{\beta}R[t]\cdot H-\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(f_{,ab}g^{ab}+f_{,0}g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\
+\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(\begin{array}{c}(n-1)\left(2g^{00}f_{,00}+f_{,0}g^{00}g^{ab}g_{ab,0}\right)\\-nf_{,0}g^{c0}g^{a0}g_{ac,0}\end{array}\right)\cdot\left(\frac{1}{2}+H\right)\\
+\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)g^{\beta\nu}f_{,\alpha}\big|_{\alpha=0}f_{,\nu}\big|_{\nu=0}+\delta_{\alpha}^{\beta}f_{,0}f_{,0}g^{00}E^{2}\right)
\end{array}\right)_{;\beta}. \qquad (96)
$$

For demonstration and because one may consider the “particle at rest” situation as a Hamilton equilibrium state of some extremal character, we set H=0:

$$
=\left(\begin{array}{c}
-\frac{F'}{2F}\left(\begin{array}{c}
g^{\beta\nu}f_{,\alpha\nu}(n-2)+g^{\beta\nu}f_{,0}g^{0b}\left(g_{\nu b,\alpha}-g_{\nu\alpha,b}\right)-g^{\beta\nu}f_{,\alpha}\big|_{\alpha=0}g^{ab}g_{\nu b,a}\\
-g^{\beta\nu}f_{,\nu}g^{0b}g_{\alpha b,0}+f_{,0}\left(\begin{array}{c}g^{c0}g^{\beta\nu}g_{\alpha c,\nu}-g^{c0}\frac{g^{\beta\nu}}{2}ng_{\alpha c,\nu}\\-g^{c0}\frac{g^{\beta\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{\beta\nu}}{2}ng^{c0}g_{\alpha\nu,c}\end{array}\right)
\end{array}\right)\\
-\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(f_{,ab}g^{ab}+f_{,0}g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\
+\delta_{\alpha}^{\beta}\frac{F'}{4F}\left(\begin{array}{c}(n-1)\left(2g^{00}f_{,00}+f_{,0}g^{00}g^{ab}g_{ab,0}\right)\\-nf_{,0}g^{c0}g^{a0}g_{ac,0}\end{array}\right)\\
+\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)g^{\beta\nu}f_{,\alpha}\big|_{\alpha=0}f_{,\nu}\big|_{\nu=0}+\delta_{\alpha}^{\beta}f_{,0}f_{,0}g^{00}E^{2}\right)
\end{array}\right)_{;\beta}
$$

$$
=\overbrace{\left(\begin{array}{c}
-\frac{F'}{2F}\left(\begin{array}{c}
g^{\beta0}f_{,\alpha0}(n-2)+g^{\beta\nu}f_{,0}\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right)-g^{\beta\nu}f_{,\alpha}\big|_{\alpha=0}g^{0b}g_{\nu b,0}\\
-g^{\beta0}f_{,0}g^{0b}g_{\alpha b,0}+f_{,0}\left(\begin{array}{c}g^{c0}g^{\beta0}g_{\alpha c,0}-g^{c0}\frac{g^{\beta0}}{2}ng_{\alpha c,0}\\-g^{c0}\frac{g^{\beta\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{\beta\nu}}{2}ng^{00}g_{\alpha\nu,0}\end{array}\right)
\end{array}\right)\\
-\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(f_{,00}g^{00}+f_{,0}g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\
+\delta_{\alpha}^{\beta}\frac{F'}{4F}\left(\begin{array}{c}(n-1)\left(2g^{00}f_{,00}+f_{,0}g^{00}g^{ab}g_{ab,0}\right)\\-nf_{,0}g^{c0}g^{a0}g_{ac,0}\end{array}\right)\\
+\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)g^{\beta\nu}f_{,\alpha}\big|_{\alpha=0}f_{,\nu}\big|_{\nu=0}+\delta_{\alpha}^{\beta}f_{,0}f_{,0}g^{00}E^{2}\right)
\end{array}\right)}^{\equiv T_{\alpha}^{\beta}}{}_{;\beta}. \qquad (97)
$$

$$
=T_{\alpha,\beta}^{\beta}+\Gamma_{\beta\gamma}^{\beta}T_{\alpha}^{\gamma}-\Gamma_{\beta\alpha}^{\gamma}T_{\gamma}^{\beta}
$$

We are going to see that this will not allow us to obtain a polynomial of third order with respect to the parameter mass but that in fact we require H≠0 for the 3rd generation. On the other hand, our reasoning for the assumption of a vanishing Hamilton parameter was a stable equilibrium or extremal state for the particle being coded by that state. We know, however, that all particles of the higher generations are unstable and thus, are probably not coded by an extremal state and hence, the setting for $H\neq0$ is more reasonable. We will see!

When assuming a typical particle at rest behavior, we should be able to demand the following:

$$f_{,\alpha}=\{m\}_{\alpha}\,f;\quad f_{,0}=m\cdot f\,. \tag{98}$$

This gives us:

$$0=\left(\begin{array}{c}-\frac{F'}{2F}\left(\begin{array}{c}g^{\beta0}\{m\}_{\alpha}\,m\cdot f\,(n-2)+g^{\beta\nu}m\cdot f\cdot\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right)\\ -g^{\beta\nu}\{m\}_{\alpha}\,f\cdot g^{0b}g_{\nu b,0}\\ -g^{\beta0}m\cdot f\cdot g^{0b}g_{\alpha b,0}+m\cdot f\cdot\left(\begin{array}{c}g^{c0}g^{\beta0}g_{\alpha c,0}-g^{c0}\frac{g^{\beta0}}{2}ng_{\alpha c,0}\\ -g^{c0}\frac{g^{\beta\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{\beta\nu}}{2}ng^{00}g_{\alpha\nu,0}\end{array}\right)\end{array}\right)\\ -\delta_{\alpha}^{\beta}\frac{F'}{2F}\left(m^{2}\cdot f+m\cdot f\cdot g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\ +\delta_{\alpha}^{\beta}\frac{F'}{4F}\left(\begin{array}{c}(n-1)\left(2m^{2}\cdot f+m\cdot f\cdot g^{00}g^{ab}g_{ab,0}\right)\\ -n\cdot m\cdot f\cdot g^{c0}g^{a0}g_{ac,0}\end{array}\right)\\ +\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)\,g^{\beta0}\{m\}_{\alpha}\,m\cdot f^{2}+\delta_{\alpha}^{\beta}m^{2}f^{2}g^{00}E^{2}\right)\end{array}\right)_{;\beta}. \tag{99}$$

This may be considered as a polynomial in m. We realize that, even already without the covariant derivative being performed, we are missing the constant term with respect to m and that, thus, we cannot have three solutions for this parameter.

So, we bring back in the parameter *H* and have to write (99) as follows:

$$0=\overbrace{\left(\begin{array}{c}-\frac{F'}{2F}\left(\begin{array}{c}g^{\beta0}\{m\}_{\alpha}\,m\cdot f\,(n-2)+g^{\beta\nu}m\cdot f\cdot\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right)\\ -g^{\beta\nu}\{m\}_{\alpha}\,f\cdot g^{0b}g_{\nu b,0}\\ -g^{\beta0}m\cdot f\cdot g^{0b}g_{\alpha b,0}+m\cdot f\cdot\left(\begin{array}{c}g^{c0}g^{\beta0}g_{\alpha c,0}-g^{c0}\frac{g^{\beta0}}{2}ng_{\alpha c,0}\\ -g^{c0}\frac{g^{\beta\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{\beta\nu}}{2}ng^{00}g_{\alpha\nu,0}\end{array}\right)\\ +\delta_{\alpha}^{\beta}\left(m^{2}\cdot f+m\cdot f\cdot g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\ -\delta_{\alpha}^{\beta}\left(\begin{array}{c}(n-1)\left(2m^{2}\cdot f+m\cdot f\cdot g^{00}g^{ab}g_{ab,0}\right)\\ -n\cdot m\cdot f\cdot g^{c0}g^{a0}g_{ac,0}\end{array}\right)\cdot\left(\frac{1}{2}+H\right)\end{array}\right)\\ -\delta_{\alpha}^{\beta}R[t]\cdot H\\ +\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)\,g^{\beta0}\{m\}_{\alpha}\,m\cdot f^{2}+\delta_{\alpha}^{\beta}m^{2}f^{2}g^{00}E^{2}\right)\end{array}\right)}^{\equiv T_{\alpha}^{\beta}}{}_{;\beta} \tag{100}$$

and can short-write the covariant derivative:

$$0=T_{\alpha,\beta}^{\beta}+\Gamma_{\beta\gamma}^{\beta}T_{\alpha}^{\gamma}-\Gamma_{\beta\alpha}^{\gamma}T_{\gamma}^{\beta}. \tag{101}$$

Together with the affine connection expressed in terms of the metric tensor, we obtain:

$$\begin{gathered}
\Gamma_{\beta\gamma}^{\beta}=\frac{g^{\beta a}}{2}\left(g_{\beta a,\gamma}+g_{\gamma a,\beta}-g_{\beta\gamma,a}\right)\\
\Gamma_{\beta\alpha}^{\gamma}=\Gamma_{\alpha\beta}^{\gamma}=\frac{g^{\gamma a}}{2}\left(g_{\alpha a,\beta}+g_{\beta a,\alpha}-g_{\alpha\beta,a}\right)\\
\Rightarrow
\end{gathered}$$

$$\begin{aligned}
0&=T_{\alpha,\beta}^{\beta}+\frac{g^{\beta a}}{2}\left(g_{\beta a,\gamma}+g_{\gamma a,\beta}-g_{\beta\gamma,a}\right)T_{\alpha}^{\gamma}-\frac{g^{\gamma a}}{2}\left(g_{\alpha a,\beta}+g_{\beta a,\alpha}-g_{\alpha\beta,a}\right)T_{\gamma}^{\beta}\\
&=\begin{pmatrix}T_{\alpha,0}^{0}+\frac{1}{2}\left(g^{\beta a}g_{\beta a,0}T_{\alpha}^{0}+g^{0a}g_{\gamma a,0}T_{\alpha}^{\gamma}-g^{\beta 0}g_{\beta\gamma,0}T_{\alpha}^{\gamma}\right)\\ -\frac{1}{2}\left(g^{\gamma a}g_{\alpha a,0}T_{\gamma}^{0}+g^{\gamma a}g_{\beta a,\alpha}T_{\gamma}^{\beta}-g^{\gamma 0}g_{\alpha\beta,0}T_{\gamma}^{\beta}\right)\end{pmatrix}\\
&=T_{\alpha,0}^{0}+\frac{g^{\beta a}g_{\beta a,0}T_{\alpha}^{0}+g^{0a}g_{\gamma a,0}T_{\alpha}^{\gamma}-g^{\beta 0}g_{\beta\gamma,0}T_{\alpha}^{\gamma}-g^{\gamma a}g_{\alpha a,0}T_{\gamma}^{0}-g^{\gamma a}g_{\beta a,\alpha}T_{\gamma}^{\beta}+g^{\gamma 0}g_{\alpha\beta,0}T_{\gamma}^{\beta}}{2}
\end{aligned}, \tag{102}$$

where we see with:

$$T_{\alpha,0}^{0}=\begin{pmatrix}
-\frac{F'}{2F}\begin{pmatrix}g^{00}\{m\}_{\alpha}\,m\cdot f\,(n-2)+g^{0\nu}m\cdot f\cdot\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right)\\ -g^{0\nu}\{m\}_{\alpha}\,f\cdot g^{0b}g_{\nu b,0}\\ -g^{00}m\cdot f\cdot g^{0b}g_{\alpha b,0}+m\cdot f\cdot\begin{pmatrix}g^{c0}g^{00}g_{\alpha c,0}-g^{c0}\frac{g^{00}}{2}ng_{\alpha c,0}\\ -g^{c0}\frac{g^{0\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{0\nu}}{2}ng^{00}g_{\alpha\nu,0}\end{pmatrix}\end{pmatrix}\\
-\delta_{\alpha}^{0}R[t]\cdot H-\delta_{\alpha}^{0}\frac{F'}{2F}\left(m^{2}\cdot f+m\cdot f\cdot g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right)\\
+\delta_{\alpha}^{0}\frac{F'}{2F}\begin{pmatrix}(n-1)\left(2m^{2}\cdot f+m\cdot f\cdot g^{00}g^{ab}g_{ab,0}\right)\\ -n\cdot m\cdot f\cdot g^{c0}g^{a0}g_{ac,0}\end{pmatrix}\cdot\left(\frac{1}{2}+H\right)\\
+\frac{1}{4F^{2}}\left(\left(3(F')^{2}-2FF''\right)(n-2)g^{00}\{m\}_{\alpha}\,m\cdot f^{2}+\delta_{\alpha}^{0}m^{2}f^{2}g^{00}E^{2}\right)
\end{pmatrix}_{,0} \tag{103}$$

and subsequently:

$$
T^0_{\alpha,0}=\left(\begin{array}{c}
-\frac{F'}{2F}\left(\begin{array}{c}
g^{00}{}_{,0}\{m\}_\alpha m\cdot f(n-2)+m\cdot f\cdot\left(g^{0\nu}\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right)\right)_{,0} \\
-\{m\}_\alpha f\cdot\left(g^{0\nu}g^{0b}g_{\nu b,0}\right)_{,0} \\
-m\cdot f\cdot\left(g^{00}g^{0b}g_{\alpha b,0}\right)_{,0}+m\cdot f\cdot\left(\begin{array}{c}
g^{c0}g^{00}g_{\alpha c,0}-g^{c0}\frac{g^{00}}{2}ng_{\alpha c,0} \\
-g^{c0}\frac{g^{0\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{0\nu}}{2}ng^{00}g_{\alpha\nu,0}
\end{array}\right)_{,0}
\end{array}\right) \\
-\delta^0_\alpha R[t]_{,0}\cdot H-\delta^0_\alpha\frac{F'}{2F}\left(m^2\cdot f+m\cdot f\cdot\frac{1}{2}\left(g^{00}g_{ab,0}g^{ab}\right)_{,0}\right) \\
+\delta^0_\alpha\frac{F'}{2F}\left(\begin{array}{c}
(n-1)\left(2m^2\cdot f+m\cdot f\cdot\left(g^{00}g^{ab}g_{ab,0}\right)_{,0}\right) \\
-n\cdot m\cdot f\cdot\left(g^{c0}g^{a0}g_{ac,0}\right)_{,0}
\end{array}\right)\cdot\left(\frac{1}{2}+H\right) \\
+\frac{g^{00}{}_{,0}}{4F^2}\left(\left(3(F')^2-2FF''\right)(n-2)\{m\}_\alpha m\cdot f^2+\delta^0_\alpha m^2f^2E^2\right) \\
+m\cdot f\left(\begin{array}{c}
-\frac{F'}{2F}\left(\begin{array}{c}
g^{00}\{m\}_\alpha m\cdot f(n-2)+g^{0\nu}m\cdot f\cdot\left(g^{0b}g_{\nu b,\alpha}-g^{00}g_{\nu\alpha,0}\right) \\
-g^{0\nu}\{m\}_\alpha f\cdot g^{0b}g_{\nu b,0} \\
-g^{00}m\cdot f\cdot g^{0b}g_{\alpha b,0}+m\cdot f\cdot\left(\begin{array}{c}
g^{c0}g^{00}g_{\alpha c,0}-g^{c0}\frac{g^{00}}{2}ng_{\alpha c,0} \\
-g^{c0}\frac{g^{0\nu}}{2}ng_{\nu c,\alpha}+\frac{g^{0\nu}}{2}ng^{00}g_{\alpha\nu,0}
\end{array}\right)
\end{array}\right) \\
-\delta^0_\alpha\frac{F'}{2F}\left(m^2\cdot f+m\cdot f\cdot g^{00}\frac{1}{2}g_{ab,0}g^{ab}\right) \\
+\delta^0_\alpha\frac{F'}{2F}\left(\begin{array}{c}
(n-1)\left(2m^2\cdot f+m\cdot f\cdot g^{00}g^{ab}g_{ab,0}\right) \\
-n\cdot m\cdot f\cdot g^{c0}g^{a0}g_{ac,0}
\end{array}\right)\cdot\left(\frac{1}{2}+H\right) \\
+\frac{1}{4F^2}\left(\left(3(F')^2-2FF''\right)(n-2)g^{00}\{m\}_\alpha m\cdot f^2+\delta^0_\alpha m^2f^2g^{00}E^2\right)
\end{array}\right)_{,f}
\end{array}\right) \quad (104)
$$

that we have now obtained a set of n polynomials with at least one polynomial of third order in m, and are "just" left with the task of finding:

a) solutions to all such n equations (indexed α=0,1,…,n-1) and
b) suitable metrics for the coding of elementary particles.

As a hint, it shall just be said that metrics with time dependencies, completely solving the quantum gravity field equations, are given in the appendix N of our book [9].

We realize that without the generalized Hamilton principle the problem of the three generations of elementary particles cannot be solved as we require the parameter *H* to be non-zero.

6.4 Appendix References

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