The Origin of Matter AND Something about the Missing f(R)-Theories in This Universe

By

Dr. Norbert Schwarzer

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1 Abstract

In this brief paper we address the question of the purely linear Ricci structure of the Einstein-Hilbert action and try to explain the fact why there are apparently no f(R)-theories realized in this universe.

As a by-product we obtain matter in a fundamental and completely non-postulated manner. According to our findings, matter is but the volume or scale jitter of space-time and / or the perturbation of the latter's curvature.

It should be pointed out that, in fact, in Einstein's original equations for the General Theory of Relativity [A1] the matter is postulated and artificially added via the so-called energy momentum tensor.

In the Hilbert derivation [A2], the tensor occurs via a Lagrange-matter density term under the variational integral, which is also only postulated.

So, both Einstein and Hilbert addressed the matter (as energy momentum tensor and matter density), but as they could not show where it mathematically comes from, they had to postulate it.

1.1 Abstract References

[A1] A. Einstein, "Grundlage der allgemeinen Relativitätstheorie", Annalen der Physik (ser. 4), 49, pp. 769–822

[A2] D. Hilbert, "Die Grundlagen der Physik, Teil 1", Göttinger Nachrichten, 1915, pp. 395–407

2 Introduction

While the classical vacuum Hilbert action [1] variational integral reads:

$$\delta_g W = 0 = \delta_g \int_V d^n x \left(\sqrt{-g} \cdot R \right) \tag{1}$$

and results in the vacuum Einstein field equations [2]:

$$\begin{gathered} \delta_g W = 0 = \delta_g \int_V d^n x \left(\sqrt{-g} \cdot R \right) \\ \delta W = 0 = \int_V d^n x \left(\sqrt{-g} \cdot \left(R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} \right) \right) \delta g^{\mu\nu}, \\ \Rightarrow 0 = R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} \end{gathered} \tag{2}$$

we know from [3] that an extension of the kernel of the integral with a kernel function of the kind $\Phi_R[R]$, results in:

$$\delta_g W = 0 = \delta_g \int_V d^n x \left(\sqrt{-g} \cdot \Phi_R[R] \right)$$

$$\delta W = 0 = \left[\begin{array}{c} \int_V d^n x \left(\sqrt{-g} \cdot \left(\begin{array}{c} \Phi'_R[R] \cdot R_{\mu\nu} - \frac{1}{2} \Phi_R[R] \cdot g_{\mu\nu} \\ + \Lambda g_{\mu\nu} - \left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \Delta_g \right) \Phi'_R[R] \end{array} \right) \right) \delta g^{\mu\nu} \\ - \int_{\text{Surface}} d^{n-1} y \left(\sqrt{|h|} \cdot \varepsilon \cdot \Phi'_R[R] \cdot N^\lambda h^{\mu\nu} \partial_\lambda \left(\delta g_{\mu\nu} \right) \right) \end{array} \right]^{1}. \tag{3}$$

So far, there is no empirical evidence or hint that such a generalization is of need.

Consequently, one unsolved question is:

Why is that?

… and the other unsolved question is:

Where does the matter in the classical theory (where it is postulated; see [1, 2]) come from?[2]

3 Why Don't We See f(R)-Lagrangians in This Universe?

In theoretical physics there was—and still is—a lot of talk about generalizations of the kernel of the Einstein-Hilbert action, leading to forms (3) and subsequent discussion about the fact why we don't see evidence for such a thing in our universe. Interestingly, namely, one only finds hints for the realization of the simple Ricci scalar linear kernel function f(R)=R. In this paper we are trying to give an answer to the question why—apparently, which is to say on first and rather shallow sight—there are no other kernels of need to describe the universe.

We start with the first line in (3) and leave the variation open to a yet unknown metric tensor $G_{\alpha\beta}$ as a scaled version of the ordinary metric $g_{\alpha\beta}$ in the form:

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f], \tag{4}$$

with the corresponding Ricci scalar R^* and the determinant G of this scaled metric tensor, fulfilling the following conditions:

$$\sqrt{-g} \cdot \Phi_R[R] = \sqrt{-\gamma} \cdot F^{\frac{n}{2}} \cdot R^* = \sqrt{-G} \cdot R^*. \tag{5}$$

The term for the scale adapted Ricci scalar was already given in our previous publications (e.g., see [4, 5, 6, 7, 8, 9, 10] and the appendix of this paper) and reads:

$$R^* = \frac{R}{F} - \frac{1}{2F^2} \left(\begin{array}{c} (n-1) \left(\overbrace{2 g^{ab} F_{,ab} + F_{,d} g^{cd} g^{ab} g_{ab,c}}^{=2\Delta F - 2 F_{,d} g^{cd}{}_{,c}} \right) \\ - n F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array} \right) - (n-1) \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^3} (n-6) \cdot \tag{6}$$

Now, when incorporating (5) into (3), we obtain:

---

[1] W=action; g=determinant of the metric tensor; $g_{\mu\nu}$=metric tensor; R=Ricci scalar; $R_{\mu\nu}$=Ricci tensor; $\Phi_R[R]$=scalar function of R; Δ_g=Laplace operator with respect to metric $g_{\mu\nu}$; ∇_μ=covariant derivative; other symbols are not of need here (see [3])

[2] Note that both Hilbert [1] and Einstein [2] had to postulate the matter in order to have some in their field equations.

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot \Phi_R [R] \right)$$

$$\xrightarrow{\delta \to \delta G_{\alpha\beta};\, \sqrt{-g} \cdot \Phi_R [R] = \sqrt{-\gamma} \cdot F^{\frac{n}{2}} \cdot R^* = \sqrt{-G} \cdot R^*} \quad . \tag{7}$$

$$\delta_{G_{\alpha\beta}} W = 0 = \delta_{G_{\alpha\beta}} \int_V d^n x \left(\sqrt{-\gamma} \cdot F^{\frac{n}{2}} \cdot R^* \right) = \delta_{G_{\alpha\beta}} \int_V d^n x \left(\sqrt{-G} \cdot R^* \right)$$

This variation, however, just gives us the classically known result from [1], namely:

$$\delta_{G_{\alpha\beta}} W = \delta_{G_{\alpha\beta}} \int_V d^n x \left(\sqrt{-G} \cdot R^* \right) = \int_V d^n x \left(\sqrt{-G} \cdot \left(R^*{}_{\alpha\beta} - \frac{R^*}{2} G_{\alpha\beta} \right) \right), \tag{8}$$

only that the scale or volume part of the metric here is considered separately. Otherwise it is exactly the Hilbert result for the vacuum case.

The fact that we have a scaled metric does not make any difference, as neither the classical Hilbert approach [1] nor the Einstein theory [2] distinguished between the pure metric $g_{\alpha\beta}$ and a scalar factor F. The classical theory always worked with the whole thing, which is to say $G_{\alpha\beta}$, without ever realizing that the consideration of the volume factor might have shown the path to a quantized General Theory of Relativity or “Theory of Everything” (see appendix in here and [4–10]). Hence, we have found a very simple explanation for the conspicuous absence of higher order kernels in R in the Einstein-Hilbert action for the General Theory of Relativity. The reason is that we can always substitute any such general kernel by a simple linearly R^*-dependent one and an adjusted metric $G_{\alpha\beta}$. As the variation does not care about the metric tensor (scaled or unscaled) until the variation is performed, the result can always be brought into the classical form (8). On the other hand, we could also bring the classical form into a multitude of metric-kernel-Ricci-configurations in accordance with (3). For this, we would just need to work ourselves backwards through the variational transformation in (7)

… but why should we?

The resulting math would only be more complicated than necessary, but—in essence—and when done correctly (despite the complicated math, which obviously currently hinders the f(R) theories to find suitable descriptions of the experimental observations) would describe the same world. So, sticking to the simpler math obviously is the smarter strategy.

However, by having worked out the connection here, we might just interpret the correspondence between the complicated option (3) and the simple classical one (8) as an extension of the “Theory of Relativity” towards a more general “**Theory of Relativity and Perspectivity**”, because it obviously depends on the observer’s model (perspective) what theory describes his reality.

In other words: We couldn’t see such other kernels, because we always had at least one eye closed.

4 The Meaning of the Volume Factor

When expanding the kernel in (8) (see appendix), we obtain:

$$
\begin{aligned}
0 &= \left(R^{*}{}_{\alpha\beta} - \frac{1}{2} R^{*} \cdot G_{\alpha\beta}\right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta\left(\frac{1}{F}\right)\right)}^{\delta G^{\alpha\beta}} \\
&= \left(\begin{array}{c}
\boxed{R_{\alpha\beta} - R\frac{g_{\alpha\beta}}{2}} \\
-\frac{1}{2F}\left(\begin{array}{c}
F_{,\alpha\beta}(n-2) + F_{,ab} g_{\alpha\beta} g^{ab} + F_{,a} g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b}\right) - \\
F_{,\alpha} g^{ab} g_{\beta b,a} - F_{,\beta} g^{ab} g_{\alpha b,a} + F_{,d} g^{cd}\left(\begin{array}{c} g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} \\ + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta} g_{ab,c} g^{ab} \end{array}\right)
\end{array}\right) \\
+\frac{1}{4F^{2}}\left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta} F_{,c} F_{,d} g^{cd}(4-n)\right) \\
+\left(\frac{(n-1)}{2F}\left(\begin{array}{c} 2\Delta F - 2F_{,d} g^{cd}{}_{,c} \\ -\frac{n}{(n-1)} F_{,d} g^{cd} g^{ab} g_{ac,b} \end{array}\right) + \frac{g^{ab} F_{,a} \cdot F_{,b}}{4F^{2}}(n-6)\right) \cdot \frac{g_{\alpha\beta}}{2}
\end{array}\right) \delta G^{\alpha\beta} \\
&= \boxed{R_{\alpha\beta} - R\frac{g_{\alpha\beta}}{2}} + \text{matter}
\end{aligned} \tag{9}
$$

and realize that, in contrast to Hilbert and Einstein [1, 2], we just found a very simple and straight forward way to fundamentally obtain matter in a completely non-postulated manner.

5 Still, "for Academic Reasons": What Happens When We Would like to Have a Certain Kind of f(R)?

Taking equation (3) and reordering as follows:

$$
\begin{aligned}
\delta W^{*} &= \int_{\text{Surface}} d^{n-1}y\left(\sqrt{|h|} \cdot \varepsilon \cdot \Phi_{R}'[R] \cdot N^{\lambda} h^{\mu\nu} \partial_{\lambda}\left(\delta g_{\mu\nu}\right)\right) \\
&= \int_{V} d^{n}x\left(\sqrt{-g} \cdot \left(\begin{array}{c} \Phi_{R}'[R] \cdot R_{\mu\nu} - \frac{1}{2} \Phi_{R}[R] \cdot g_{\mu\nu} \\ +\Lambda g_{\mu\nu} - \left(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\Delta_{g}\right)\Phi_{R}'[R] \end{array}\right)\right)\delta g^{\mu\nu},
\end{aligned} \tag{10}
$$

we might just consider this as an adjusted, non-extremal Hamilton principle.

As the kernels in both integrals are only scalars, we assume that we could always find a metric of type (4), but with a different metric core:

$$
G_{\alpha\beta} = \gamma_{\alpha\beta} \cdot F[f], \tag{11}
$$

where we could substitute the surface integral by something like:

$$
\begin{aligned}
\delta W^{*} &= \int_{\text{Surface}} d^{n-1}y\left(\sqrt{|h|}\cdot\varepsilon\cdot\Phi'_{R}\left[R\right]\cdot N^{\lambda}h^{\mu\nu}\partial_{\lambda}\left(\delta g_{\mu\nu}\right)\right) \\
&\Rightarrow \int_{V} d^{n}x\sqrt{-G}\times\Phi'_{R^{*}}\left[R^{*}\right]\cdot\not{X}^{\alpha\beta}\cdot G_{\alpha\beta} \qquad . \\
&= \int_{V} d^{n}x\left(\sqrt{-G}\cdot\left(\begin{array}{c}\Phi'_{R^{*}}\left[R^{*}\right]\cdot R^{*}_{\mu\nu}-\frac{1}{2}\Phi_{R^{*}}\left[R^{*}\right]\cdot G_{\mu\nu} \\ +\Lambda G_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-G_{\mu\nu}\Delta_{G}\right)\Phi'_{R^{*}}\left[R^{*}\right]\end{array}\right)\right)\delta G^{\mu\nu}
\end{aligned}
\tag{12}
$$

The corresponding evaluation for a simple, which is to say unscaled metric and unperturbed R-liner kernel is shown in the appendix of this paper. For a scaled metric as used here, this gives:

$$
0=\int_{V} d^{n}x\left(\sqrt{-G}\cdot\left(\begin{array}{c}\Phi'_{R^{*}}\left[R^{*}\right]\cdot R^{*}_{\mu\nu}-\frac{1}{2}\Phi_{R^{*}}\left[R^{*}\right]\cdot G_{\mu\nu}-H\cdot G_{\mu\nu}\Phi'_{R^{*}}\left[R^{*}\right]\cdot \\ +\Lambda G_{\mu\nu}\Phi'_{R^{*}}\left[R^{*}\right]-\left(\nabla_{\mu}\nabla_{\nu}-G_{\mu\nu}\Delta_{G}\right)\Phi'_{R^{*}}\left[R^{*}\right]\end{array}\right)\right)\delta G^{\mu\nu}. \tag{13}
$$

Further suitable substitution of the kind:

$$
G_{\mu\nu}\rightarrow\Gamma_{\mu\nu};R^{*}\rightarrow R^{**} \tag{14}
$$

may even allow us to construct a variational kernel with the function:

$$
\Phi_{R^{*}}\left[R^{*}\right]=\Phi\cdot R^{**} \tag{15}
$$

and to obtain the subsequent field equations:

$$
\begin{aligned}
0 &= \int_{V} d^{n}x\left(\sqrt{-\Gamma}\cdot\left(\begin{array}{c}\Phi\cdot R^{**}_{\mu\nu}-\frac{R^{**}}{2}\Phi\cdot\Gamma_{\mu\nu}-H\cdot\Gamma_{\mu\nu}\cdot\Phi \\ +\Lambda\Gamma_{\mu\nu}\Phi-\left(\nabla_{\mu}\nabla_{\nu}-\Gamma_{\mu\nu}\Delta_{\Gamma}\right)\Phi\end{array}\right)\right)\delta\Gamma^{\mu\nu} \qquad . \\
&= \int_{V} d^{n}x\left(\sqrt{-\Gamma}\cdot\left(R^{**}_{\mu\nu}-\frac{R^{**}}{2}\Gamma_{\mu\nu}-H\cdot\Gamma_{\mu\nu}+\Lambda\Gamma_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-\Gamma_{\mu\nu}\Delta_{\Gamma}\right)\right)\Phi\right)\delta\Gamma^{\mu\nu}
\end{aligned}
\tag{16}
$$

By investigating the variated kernel, we find

$$
\begin{aligned}
0 &= \left(R^{**}_{\mu\nu}-\frac{R^{**}}{2}\Gamma_{\mu\nu}-H\cdot\Gamma_{\mu\nu}+\Lambda\Gamma_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-\Gamma_{\mu\nu}\Delta_{\Gamma}\right)\right)\Phi \\
&\Rightarrow \\
0 &= \overbrace{R^{**}_{\mu\nu}-\frac{R^{**}}{2}\Gamma_{\mu\nu}}^{\text{vacuum part}}+\overbrace{\frac{\Lambda\Gamma_{\mu\nu}-H\cdot\Gamma_{\mu\nu}-\left(\nabla_{\mu}\nabla_{\nu}-\Gamma_{\mu\nu}\Delta_{\Gamma}\right)\Phi}{\Phi}}^{\text{matter}}
\end{aligned}
\tag{17}
$$

and see that we have just obtained matter again.

6 Conclusions

Not only were we able to show that any f(R) setting in a generalized Einstein-Hilbert action could be substituted by a linear kernel and a scaled metric tensor, but we also saw that this brings us a rigorous—non-postulated—derivation of matter and energy.

7 References

[1] D. Hilbert, “Die Grundlagen der Physik, Teil 1”, Göttinger Nachrichten, 1915, pp. 395–407

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8 Appendix

From Wikipedia, the free encyclopedia (https://en.wikipedia.org/wiki/Hamilton's_principle):

> *In physics, Hamilton's principle is William Rowan Hamilton's formulation of the principle of stationary action. It states that the dynamics of a physical system are determined by a variational problem for a functional based on a single function, the Lagrangian, which may contain all physical information concerning the system and the forces acting on it. The variational problem is equivalent to and allows for the derivation of the differential equations of motion of the physical system. Although formulated originally for classical mechanics, Hamilton's principle also applies to classical fields such as the electromagnetic and gravitational fields, and plays an important role in quantum mechanics, quantum field theory and criticality theories.*

So, the definition of the Hamilton principle is based on its “formulation of the principle of stationary action”. In simpler words, the variation of such an action should be zero or, mathematically formulated, should be put as follows:

$$\delta W = 0 = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L . \tag{18}$$

Here L stands for the Lagrangian, W the action, and g gives the determinant of the metric tensor, which describes the system in question within an arbitrary Riemann space-time with the coordinates x. Thereby, we used the Hilbert formulation of the Hamilton principle [1] in a slightly more general form. We were able to show in [2] that the original Hilbert variation does not only produce the Einstein field equations [3] but also contains the Quantum Theory [2, 4, 5]. It should be noted that, while the original Hilbert paper [1] started with the Ricci scalar R as the integral kernel, which is to say L=R, we here used a general Langrangian, because—as we will show later in this appendix—this generality—in principle—is already contained inside the original Hilbert formulation. Yes, even, as strange as it may sound at this point, general kernels with functions of the Ricci scalar f(R) [6] are already included (see subsection "Why Don't We See f(R)-Lagrangians in This Universe?") in the Hilbert approach.

But what if we lived in a universe where the only thing that was certain was uncertainty?

One of the authors of [7], Dr. David Martin, always used the analogy of a moving fulcrum to demonstrate his uneasiness with the formulation (18).

In [7] we were able to show that the Hamilton principle itself hinders us to localize any system or object at a certain position. We also see that this contradicts the concept of particles. Everything seems to be permanently on the move or—rather—ever-jittering [13].

But if this ever-jittering fulcrum was one of the fundamental properties of our universe, should we then not take this into account when formulating the laws of this very universe? Shouldn't we better write (18) as follows:

$$\delta W \to 0 \cong \delta \int_V d^n x \cdot \sqrt{-g} \cdot L\ ? \tag{19}$$

And while we are at it, should we not start to investigate an even more general principle like:

$$\delta W \to f\left(W, x, g_{\alpha\beta}\right) = \delta \int_V d^n x \cdot \sqrt{-g} \cdot L\ ? \tag{20}$$

The interesting aspect about this is that this investigation was already—partially—done by (surprise, surprise) e.g., Hilbert and Einstein. But instead of explaining it in this way, they have "hidden" their generalization inside other concepts like the introduction of a cosmological constant or—oh yes—the postulation of matter and its introduction via an ominous and purely postulated parameter L_M, which is to say, a Lagrange matter term.

8.1 The Classical Hamilton Extremal Principle and How to Obtain Einstein's General Theory of Relativity with Matter (!) and Quantum Theory… also with Matter (!)

The famous German mathematician David Hilbert [1], even though applying his technique only to derive the Einstein field equations for the General Theory of Relativity [3] in four dimensions,—in principle—extended the classical Hamilton principle to an arbitrary Riemann space-time with a very general variation by not only – as Hamilton and others had done – concentrating on the evolution of the given problem or system in time, but with respect to all its dimensions. His formulation of the Hamilton extremal principle looked as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-g} \cdot \left(R - 2\Lambda + L_M\right)\right). \tag{21}$$

There we have the Ricci scalar of curvature R, the cosmological constant Λ, the Lagrange density of matter L_M, and the determinant g of the metric tensor of the Riemann space-time $g_{\alpha\beta}$. For historical reasons, it should be mentioned that Hilbert's original work [1] did not contain the cosmological constant, because it was added later by Einstein in order to obtain a static universe, but this is not of any importance here. The evaluation of the so-called Einstein-Hilbert action (21) brought indeed the Einstein General Theory of Relativity [3], but it did not produce the other great theory physicists have found, which is the Quantum Theory. It was not before this author, about one hundred years after the publication of Hilbert's paper [1], extended Hilbert's approach by considering scaling factors to the metric tensor and showed that Quantum Theory already resides inside the sufficiently general General Theory of Relativity [2, 4, 7, 8, 9, 10]. We will not discuss the reason why this simple idea has not been tried out by other scientists before, but we may still express our amazement about the fact that a trivial extension of the type

$$G_{\alpha\beta} = g_{\alpha\beta} \cdot F[f] \tag{22}$$

solves one of the greatest problems in science[3], namely the unification of physics, and that it took science more than 100 years to come up with the idea. Using the symbol G for the determinant of the scaled metric tensor $G_{\alpha\beta}$ from (22) of the Riemann space-time, we can rewrite the Einstein-Hilbert action from (21) as follows:

$$\delta W = 0 = \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right). \tag{23}$$

Please note that this converges to the classical form for F→1. Here, which is to say in this paper, we will only consider examples with q=0, but for completeness and later investigation, we shall mention that a comprehensive consideration of variational integrals for the cases of general q are to be found in [4].

Performing the variation in (23) with respect to the metric $G_{\alpha\beta}$ and remembering that the Ricci curvature of such a metric (e.g., [7] appendix D) changes the whole variation to:

$$\begin{aligned} \delta W = 0 &= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(R^* - 2\Lambda + L_M \right) \right) \\ &= \delta \int_V d^n x \left(\sqrt{-G} \cdot F^q \cdot \left(\left(\begin{array}{c} \frac{R}{F} - \frac{1}{2F^2} \left(\begin{array}{l} (n-1)\left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}} \right) \\ -nF_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) \\ -(n-1)\frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^3}(n-6) \end{array} \right) - 2\Lambda + L_M \right) \right), \end{aligned} \tag{24}$$

results in:

$$\begin{aligned} 0 &= \left(R^*_{\alpha\beta} - \frac{1}{2} R^* \cdot G_{\alpha\beta} \right) \overbrace{\left(\frac{1}{F} \cdot \delta g^{\alpha\beta} + g^{\alpha\beta} \cdot \delta\left(\frac{1}{F} \right) \right)}^{\delta G^{\alpha\beta}} \\ &= \left(\begin{array}{c} \left(\begin{array}{c} R_{\alpha\beta} - \frac{1}{2F} \left(\begin{array}{c} F_{,\alpha\beta}(n-2) + F_{,ab}g_{\alpha\beta}g^{ab} + F_{,a}g^{ab}\left(g_{\beta b,\alpha} - g_{\beta\alpha,b} \right) - \\ F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} + F_{,d}g^{cd} \left(\begin{array}{c} g_{\alpha c,\beta} - \frac{1}{2}ng_{\alpha c,\beta} - \frac{1}{2}ng_{\beta c,\alpha} \\ + \frac{1}{2}ng_{\alpha\beta,c} + \frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab} \end{array} \right) \end{array} \right) \\ + \frac{1}{4F^2}\left(F_{,\alpha} \cdot F_{,\beta}(3n-6) + g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n) \right) \end{array} \right) \delta G^{\alpha\beta} \\ + \left(\frac{(n-1)}{2F} \left(\begin{array}{c} \left(\overbrace{2g^{ab}F_{,ab} + F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}} \right) \\ - \frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b} \end{array} \right) + \frac{g^{ab}F_{,a} \cdot F_{,b}}{4F^2}(n-6) - \frac{R}{(n-1)} \right) \cdot \frac{g_{\alpha\beta}}{2} \end{array} \right), \end{aligned} \tag{25}$$

when setting q=0 and assuming a vanishing cosmological constant. With a cosmological constant we have to write:

[3] This does not mean, of course, that we should not also look out for generalizations of the scaled matric and investigate those as we did in [10].

$$0=\left(R^{*}{}_{\alpha\beta}-\frac{1}{2}R^{*}\cdot G_{\alpha\beta}\right)\overbrace{\left(\frac{1}{F}\cdot\delta g^{\alpha\beta}+g^{\alpha\beta}\cdot\delta\left(\frac{1}{F}\right)\right)}^{\delta G^{\alpha\beta}}$$

$$=\left(\begin{array}{c}\boxed{\boxed{R_{\alpha\beta}-R\frac{g_{\alpha\beta}}{2}}}+\boxed{\Lambda\cdot g_{\alpha\beta}}\\
-\frac{1}{2F}\left(\begin{array}{c}F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}+F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right)-\\
F_{,\alpha}g^{ab}g_{\beta b,a}-F_{,\beta}g^{ab}g_{\alpha b,a}+F_{,d}g^{cd}\left(\begin{array}{c}g_{\alpha c,\beta}-\frac{1}{2}ng_{\alpha c,\beta}-\frac{1}{2}ng_{\beta c,\alpha}\\
+\frac{1}{2}ng_{\alpha\beta,c}+\frac{1}{2}g_{\alpha\beta}g_{ab,c}g^{ab}\end{array}\right)\end{array}\right)\\
+\frac{1}{4F^{2}}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\\
+\left(\frac{(n-1)}{2F}\left(\begin{array}{c}2\Delta F-2F_{,d}g^{cd}{}_{,c}\\
-\frac{n}{(n-1)}F_{,d}g^{cd}g^{ab}g_{ac,b}\end{array}\right)+\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^{2}}(n-6)\right)\cdot\frac{g_{\alpha\beta}}{2}\end{array}\right)\delta G^{\alpha\beta}\quad. \qquad (26)$$

For better recognition of the classical terms, we have reordered the equations a bit and boxed the classical vacuum part of the Einstein field equations (double lines) and the cosmological constant term (single line). Everything else can be—no, represents (!)—matter or quantum effects or both.

Thus, we also—quite boldly—have set the matter density L_M equal to zero, because we see that already our simple metric scaling brings in quite some options for the construction of matter. It will be shown elsewhere [10] that there is much more which is based on the same technique.

8.2 The Principle of the Ever Jittering Fulcrum and the Alternate Hamilton Principle

We might bring forward two reasons why we could doubt the fundamentality of the Hamilton principle even in its most general form of the generalized Einstein-Hilbert action:

a) The principle was postulated and never fundamentally derived.
b) Even the formulation of this principle in its classical form (21) results in a variety of options where factors, constants, kernel adaptations, etc. could be added, so that the rigid setting of the integral to zero offers some doubt in itself. A calculation process that offers a variety of add-ons and options should not contain such a dogma. The result should be kept open and general. Dr. David Martin proposed this as the "tragedy of the jittering fulcrum", and we therefore named this principle "David's principle of the ever-jittering fulcrum" [13]. It demands:

$$\begin{aligned}\delta_{g_{\alpha\beta}}W&\simeq ?\simeq\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\\
\delta_{G_{\alpha\beta}}W&\simeq ?\simeq\delta_{G_{\alpha\beta}}\int_V d^n x\sqrt{-G}\times R^{*}\end{aligned}\quad. \qquad (27)$$

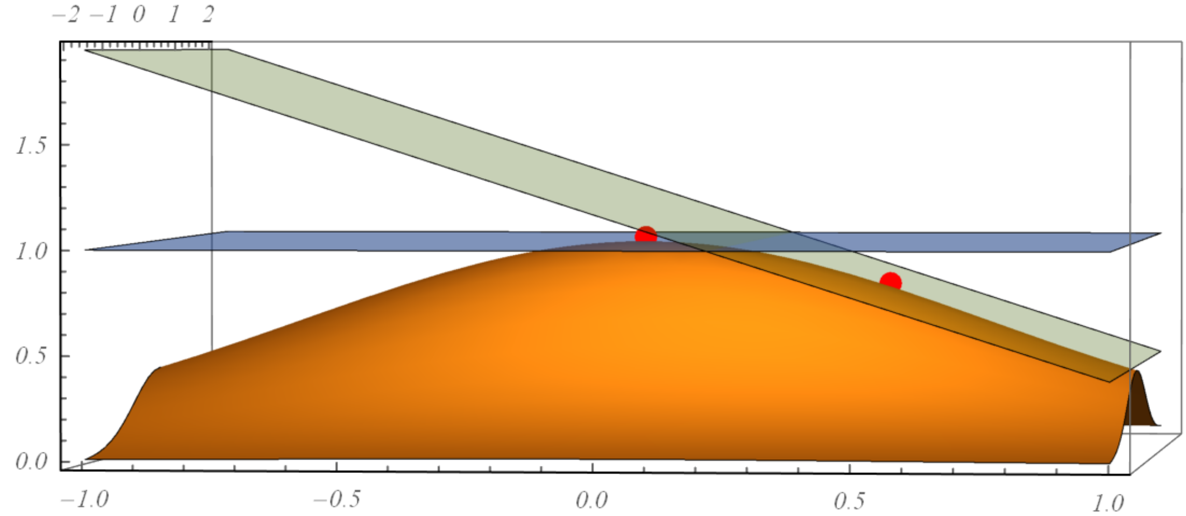

Fig. A1: David's principle of the ever-jittering fulcrum cannot accept a dogmatic insistence on a zero outcome of the Einstein-Hilbert action (21) or (generalized and also bringing about the Quantum Theory) (23). Instead, it should allow for all states and not just the extremal position (see the two red dots and the corresponding tangent planes in the picture).

One of the simplest generalizations of the classical principle could be the linear one, which is illustrated in figure A1. It could be constructed as follows:

$$\int_V d^n x \sqrt{-g} \times \not{\hbar}^{\alpha\beta} \cdot g_{\alpha\beta} = \delta_{g_{\alpha\beta}} W = \delta_{g_{\alpha\beta}} \int_V d^n x \sqrt{-g} \times R \,. \tag{28}$$

Thereby we have used the classical form with the unscaled metric tensor, respectively without setting the factor apart from the rest of the metric. Performing the variation on the right-hand side and setting

$$\not{\hbar}^{\alpha\beta} = H \cdot \delta g^{\alpha\beta} \tag{29}$$

or—for the reason of maximum generality—even

$$\not{\hbar}^{\alpha\beta} = H_{ab}^{\alpha\beta} \cdot \delta\gamma^{ab} = H \cdot \delta g^{\alpha\beta} \tag{30}$$

just gives us the same result as we would obtain when assuming a non-zero cosmological constant, because the evaluation yields:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H \cdot \delta g^{\alpha\beta} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned} \,, \tag{31}$$

respectively:

$$\begin{aligned} \int_V d^n x \sqrt{-g} \times H_{ab}^{\alpha\beta} \cdot \delta\gamma^{ab} \cdot g_{\alpha\beta} &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} \right) \delta g^{\alpha\beta} \\ \Rightarrow 0 &= \int_V d^n x \sqrt{-g} \times \left(R_{\alpha\beta} - \frac{R}{2} g_{\alpha\beta} - H g_{\alpha\beta} \right) \delta g^{\alpha\beta} \end{aligned} \,. \tag{32}$$

Simply setting $H=-\Lambda$ (c.f. single-line boxed term in equation (26)) demonstrates this.

Nothing else is the usage of a general functional term T, being considered a function of the coordinates of the system (perhaps even the metric tensor) in a general manner, as follows:

$$\int_V d^n x\sqrt{-g}\times T=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,. \tag{33}$$

As before, performing the variation on the right-hand side and setting

$$T=T_{\alpha\beta}\cdot\delta g^{\alpha\beta} \tag{34}$$

gives us something that was classically postulated under the variational integral, namely the classical energy-matter tensor. This time, however, it simply pops up as a result of David's principle of the jittering fulcrum and is equivalent to the introduction of the term L_M under the variational integral. Evaluation yields:

$$\begin{aligned}\int_V d^n x\sqrt{-g}\cdot T_{\alpha\beta}\cdot\delta g^{\alpha\beta}&=\int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}\right)\delta g^{\alpha\beta}\\ \Rightarrow 0&=\int_V d^n x\sqrt{-g}\times\left(R_{\alpha\beta}-\frac{R}{2}g_{\alpha\beta}-T_{\alpha\beta}\right)\delta g^{\alpha\beta}\end{aligned}\,. \tag{35}$$

So, we see that in introducing a cosmological constant and in postulating a matter term, even Einstein and Hilbert already—in principle—"experimented" with a non-extremal setting for the Hamilton extremal principle.

Apart from linear dependencies and other functions or functional terms, we could just assume a general outcome like:

$$f(W)=f\left(\int_V d^n x\sqrt{-g}\times R\right)=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,\cdot \tag{36}$$

This, however, would not give us any substantial hint where to move on, respectively, which of the many possible paths to follow. We therefore here start our investigation with the assumption of an eigen result for the variation as follows:

$$\not\lambda\cdot W=\not\lambda\cdot\int_V d^n x\sqrt{-g}\times R=\delta_{g_{\alpha\beta}}W=\delta_{g_{\alpha\beta}}\int_V d^n x\sqrt{-g}\times R\,. \tag{37}$$

This leads to:

$$\int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda}-R\cdot\left(\frac{1}{2}\cdot g_{\kappa\lambda}\delta g^{\kappa\lambda}+\not\lambda\right)\right)=0\,. \tag{38}$$

As the term $\not\lambda$ could always be expanded into an expression like:

$$\not\lambda=H\cdot g_{\kappa\lambda}\delta g^{\kappa\lambda}, \tag{39}$$

we obtain from (38):

$$\begin{aligned}0&=\int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}\delta g^{\kappa\lambda}-R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\delta g^{\kappa\lambda}\right)\\ &=\int_V d^n x\sqrt{-g}\left(R_{\kappa\lambda}-R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}\right)\delta g^{\kappa\lambda}\\ &\Rightarrow R_{\kappa\lambda}-R\cdot\left(\frac{1}{2}+H\right)g_{\kappa\lambda}=0\end{aligned}\,. \tag{40}$$

We realize that the term H can be a general scalar even if we would demand the term $\not\lambda$ to be a constant.

The complete equation when assuming a scaled metric tensor of the form (22) would read:

$$\left(\begin{array}{l} \left(R_{\alpha\beta} - \frac{1}{2F}\left(\begin{array}{l} F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab} \\ +F_{,a}g^{ab}\left(g_{\beta b,\alpha}-g_{\beta\alpha,b}\right) - F_{,\alpha}g^{ab}g_{\beta b,a} - F_{,\beta}g^{ab}g_{\alpha b,a} \\ +F_{,d}g^{cd}\left(g_{\alpha c,\beta} - \frac{1}{2} n g_{\alpha c,\beta} - \frac{1}{2} n g_{\beta c,\alpha} + \frac{1}{2} n g_{\alpha\beta,c} + \frac{1}{2} g_{\alpha\beta}g_{ab,c}g^{ab}\right)\end{array}\right) \\ \qquad + \frac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\right) \\ -\left(\begin{array}{l} R - \frac{1}{2F}\left((n-1)\left(\overbrace{2g^{ab}F_{,ab}+F_{,d}g^{cd}g^{ab}g_{ab,c}}^{=2\Delta F - 2F_{,d}g^{cd}{}_{,c}}\right) - nF_{,d}g^{cd}g^{ab}g_{ac,b}\right) \\ \qquad -(n-1)\frac{g^{ab}F_{,a}\cdot F_{,b}}{4F^2}(n-6)\end{array}\right)\cdot\left(\frac{1}{2} + H\right)g_{\alpha\beta}\end{array}\right) = 0, \tag{41}$$

and in the case of metrics with constant components this equation simplifies to:

$$\left(\begin{array}{l} \left(\begin{array}{l} R_{\alpha\beta} - \frac{1}{2F}\left(F_{,\alpha\beta}(n-2)+F_{,ab}g_{\alpha\beta}g^{ab}\right) \\ +\frac{1}{4F^2}\left(F_{,\alpha}\cdot F_{,\beta}(3n-6)+g_{\alpha\beta}F_{,c}F_{,d}g^{cd}(4-n)\right)\end{array}\right) \\ -\left(R - \frac{(n-1)}{2F}\left(2g^{ab}F_{,ab} + \frac{g^{ab}F_{,a}\cdot F_{,b}}{2F}(n-6)\right)\right)\cdot\left(\frac{1}{2}+H\right)g_{\alpha\beta}\end{array}\right) = 0 \cdot \tag{42}$$

8.2.1 The Question of Stability

From purely mechanical considerations, one might assume that extremal solutions of the variational equation (27) correspond to more stable states than non-extremal solutions, and in fact we have found this in connection with the 3-generations problem, which we have discussed in the main part of [12].

8.3 Appendix References

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