



## How to measure intrinsic stresses via nanoindentation – an example

### Abstract

In the following study the measurement and analysis of intrinsic stresses and yield strength via nanoindentation and their separation from each other will be elaborated using a specific example. The results for the intrinsic stresses are compared with those determined by other means. The agreement is excellent.

### Introduction

Yield strength values obtained via nanoindentation are often flawed by the intrinsic stresses residing in the surface area of the samples in question. Within this study the procedure necessary to separate the intrinsic stresses and determine correct yield strength values is demonstrated on thin films with known biaxial intrinsic stresses.

In [1] the author has proposed the following measuring procedure:

1. The yield strength is determined using the method of the effectively shaped indenter as presented in [1]. However, during unloading, the indenter is only drawn back to a distinct fraction of the maximum load  $p_0$ . This load shall be called  $p_1$ . The reader should note that  $p_1$  must be chosen such that on the one hand there is “enough unloading curve” for the determination of the shape of the effective indenter, and on the other hand the load is still big enough in order to avoid strong and dominant inelastic unloading effects like e.g. unloading fractures. In addition, a  $p_1$  close to  $p_0$  also assures that the shape of the effective indenter only changes in an insignificant manner during unloading. We call the determined yield strength  $\sigma_M^{crit}$ . Here we now have to add the two elastic fields resulting from the intrinsic stresses  $\sigma_{ij}^I$  and the nanoindenter loading  $\sigma_{ij}^L$ . The von Mises stress can be written in the following form:

$$\sigma_M = \sqrt{\frac{1}{2} \left( (\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{zz} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6 * (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{zy}^2) \right)}$$

with  $\sigma_{ij} = \sigma_{ij}^I + \sigma_{ij}^L$ .

2. Now a slowly oscillating tangential load component  $t_x$  with increasing amplitude is added and the resulting lateral shift is measured. So we now have mixed load conditions and assume our - in step one determined - effectively shaped indenter acting with the combined load components  $p_1$  and  $t_x$  onto the coating substrate compound.

3. The slowly oscillating tangential loading with increasing amplitude is monitored with very high resolution (as well as the static normal load and displacement, of course) until nonlinear behavior can be detected. Thus, the value of maximum tangential load  $t_x = t_{crit}$  or maximum lateral displacement  $u_\sigma$  is determined. Now we introduce the following assumption: The

combined stresses add up to a mechanical stress field producing a maximum von Mises stress  $\sigma_M = \sigma_M^{crit}$  somewhere within the investigated coating material.

4. With these two measurements and the resulting measured values  $p_0$ ,  $p_1$ , the corresponding penetration depth,  $t_{crit}$  and  $u_\sigma$  we can construct two linear independent equations

$$\sigma_M^{crit} = \sqrt{\frac{1}{2} \left( \left( \sigma_{xx}^L - \sigma_{yy}^L + \sigma_{rr}^f (f_{xx}^I - f_{yy}^I) \right)^2 + \left( \sigma_{zz}^L - \sigma_{yy}^L + \sigma_{rr}^f (f_{zz}^I - f_{yy}^I) \right)^2 \right) + \left( \sigma_{xx}^L - \sigma_{zz}^L + \sigma_{rr}^f (f_{xx}^I - f_{zz}^I) \right)^2 + 6 * \left( (\tau_{xy}^L + \sigma_{rr}^f f_{xy}^I)^2 + (\tau_{xz}^L + \sigma_{rr}^f f_{xz}^I)^2 + (\tau_{zy}^L + \sigma_{rr}^f f_{zy}^I)^2 \right)}$$

for the critical von Mises stress respectively yield strength with two different nanoindenter stress distributions  $\sigma_{ij}^L$  resulting from the pure normal loading  $\sigma_{ij}^L = \sigma_{ij}^{normal\ load}$  and the mixed loading experimental setup  $\sigma_{ij}^L = \sigma_{ij}^{mixed\ load}$ . We have used the fact, that according to our approach (eq. (8) in [1]) for the intrinsic stresses we can write the intrinsic stress field as  $\sigma_{ij}^I = \sigma_{rr}^f * f_{ij}^I(x, y, z) \equiv \sigma_{rr}^f * f_{ij}^I$  with a suitable function  $f(x, y, z)$ . Due to the linear independence of the two loading conditions we can now extract the intrinsic stress value  $\sigma_{rr}^f$  residing in the coating and the critical von Mises stress of a corresponding unstressed material ( $\sigma_{rr}^f = 0$ ), meaning the von Mises stress this unstressed material would require in order to reach its yield strength limit  $\sigma_M = \sigma_M^{crit}$ .

### ***The measurement and analyzing procedure in praxis***

The above described analyzing procedures shall now be demonstrated on some very first experimental examples. The measurements have been performed by T. Chudoba and V. Linss from the company ASMEC using a so called UNAT measurement system [2] equipped with a lateral force unit (LFU) which can generate und measure lateral forces und displacements with the same resolution like common nanoindenters in normal direction.

Four samples with 3 $\mu$ m CrN-coatings on silicon have been investigated with different but known biaxial stresses. A detailed description of the intrinsic stress determination and the nanoindentation procedures will be published elsewhere [3]. Here we want to concentrate on the analysis of the experimental data. At first, Young's modulus, hardness H and yield strength Y have been determined using classical normal nanoindentation. The results are presented in [1, table 1]. Now we want to follow the concrete procedure for one of those samples. For this we chose sample number 3 of [1].

## Analyzing procedure step I: pure normal indentation with max. load $p_0$

**FilmDoctor v 0.997z56**  
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material load calculate Calculation

Line graph 2D graph 3D graph Animation Comparison Value browser

**step 1: select your material**

Poisson's ratio  $\nu$  Young's modulus E select from database layer thickness h intrinsic stresses  $\sigma_x$   $\sigma_y$

gradient  gradient  gradient

<input checked="" type="checkbox"/> layer 1: $\nu$ : 0.25 E: 295 GPa	select from database	h: 3 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 2: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 3: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 4: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 5: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 6: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 7: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 8: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 9: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
<input type="checkbox"/> layer 10: $\nu$ : 0.208 E: 82 GPa	select from database	h: 5 $\mu\text{m}$	in x: 0 GPa	in y: 0 GPa
substrate: $\nu$ : 0.223 E: 165 GPa	select from database		in x: 0 GPa	in y: 0 GPa

edit database

OK

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**step 2: define the load parameters**

choose load definition method: fit load-depth curve

use effective half space accuracy: 10 fit: 10 points 95 % of curve

indenter:  $\nu$ : 0.1 E: 1100 GPa R:  $\mu\text{m}$

effective:  $\nu$ : 0.25 E: 205 GPa F: 249,98734 mN a: 1.75  $\mu\text{m}$   fixed

indenter shape:  $Z(r) = \frac{r^2}{d_0} + \frac{r^4}{d_2} + \frac{r^6}{d_4} + \frac{r^8}{d_6}$

0,805 100 1000 10000 1000000

$h_0$   $d_0$   $d_2$   $d_4$   $d_6$

0 0 0 0 0

$d_0$  28.299  $d_2$  -4.61E8  $d_4$  229.101  $d_6$  -2941.02

$h_0$  autofit  autofit  $h_0$

show indenter-shape calculate  $c_n + a$

load graph clear graph  draw points  $c_0$ : 0.1102455  $c_2$ : 0.023109157  $c_4$ : 0.01581775  $c_6$ : -0.001735221

OK

Fig. 1 and 2: Material data-input and fit of effective indenter to unloading curve

Using the software FilmDoctor, we first type in the material data Young's modulus, Poissons's ratio and thickness. As substrate we have silicon with a known Young's modulus of 165GPa and Poisson's ratio of 0.223. For the film we are estimating the Poisson's ratio and use the values determined by the means of the procedure described in [4]. Now we chose "fit load-depth-curve" from the load definition page, load the indentation curve and fit a paraboloid indenter to the same. The fit can be done by hand or automatically. The next step is the evaluation of the elastic field of the effective indenter in the moment of beginning unloading (maximum load  $p_0$ ). So, after setting up the parameters for the calculation (fig. 3).

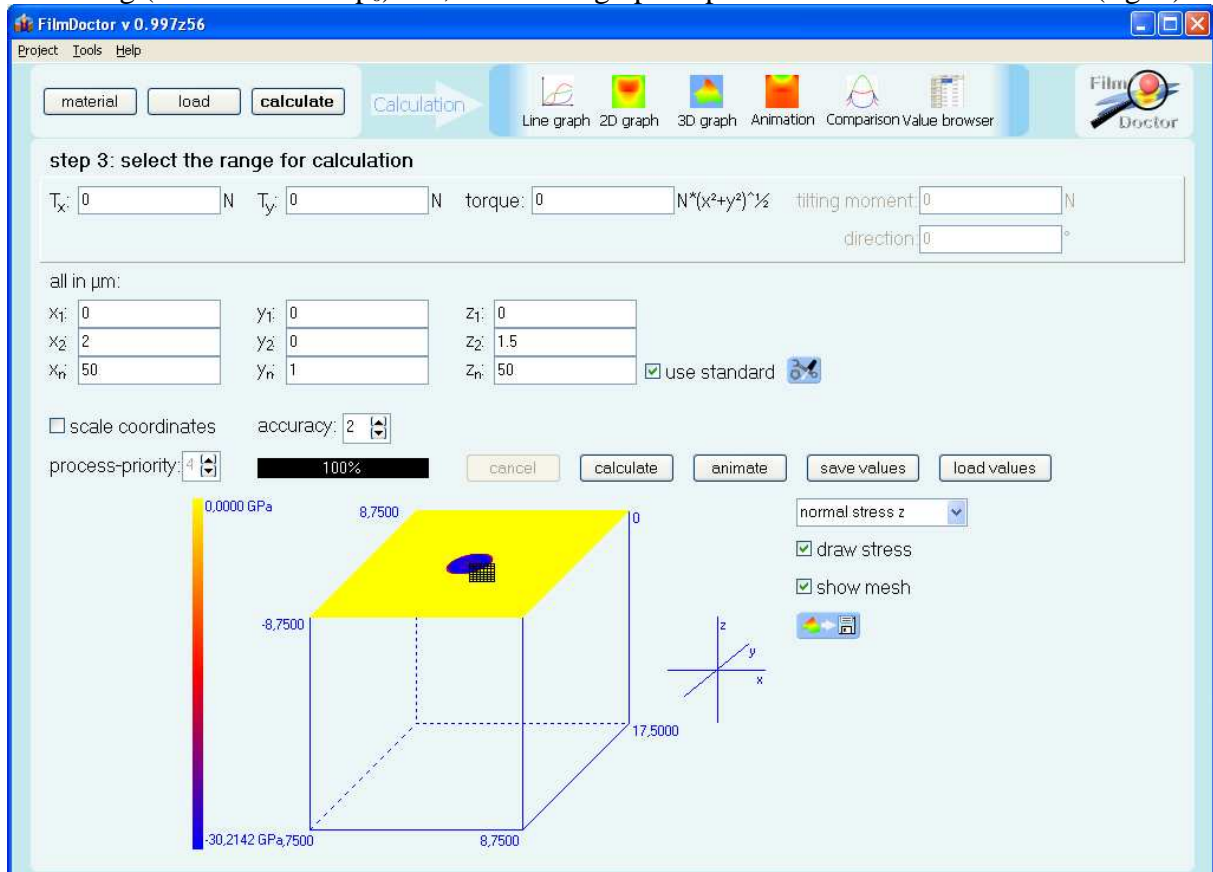


Fig. 3: Setting up the calculation-parameters and starting the evaluation

We find the von Mises Maximum (fig. 4 with 18.8357GPa), which is still mixed with the yet unknown intrinsic stress.

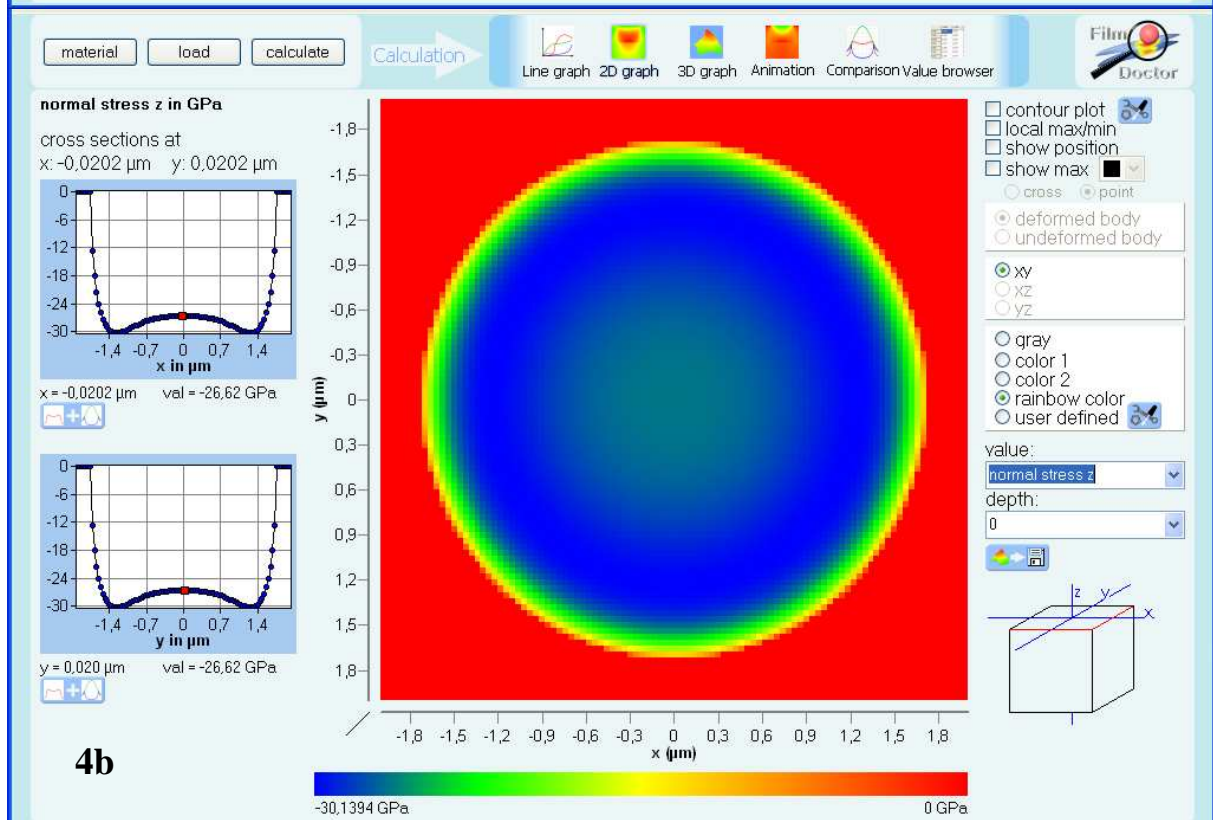
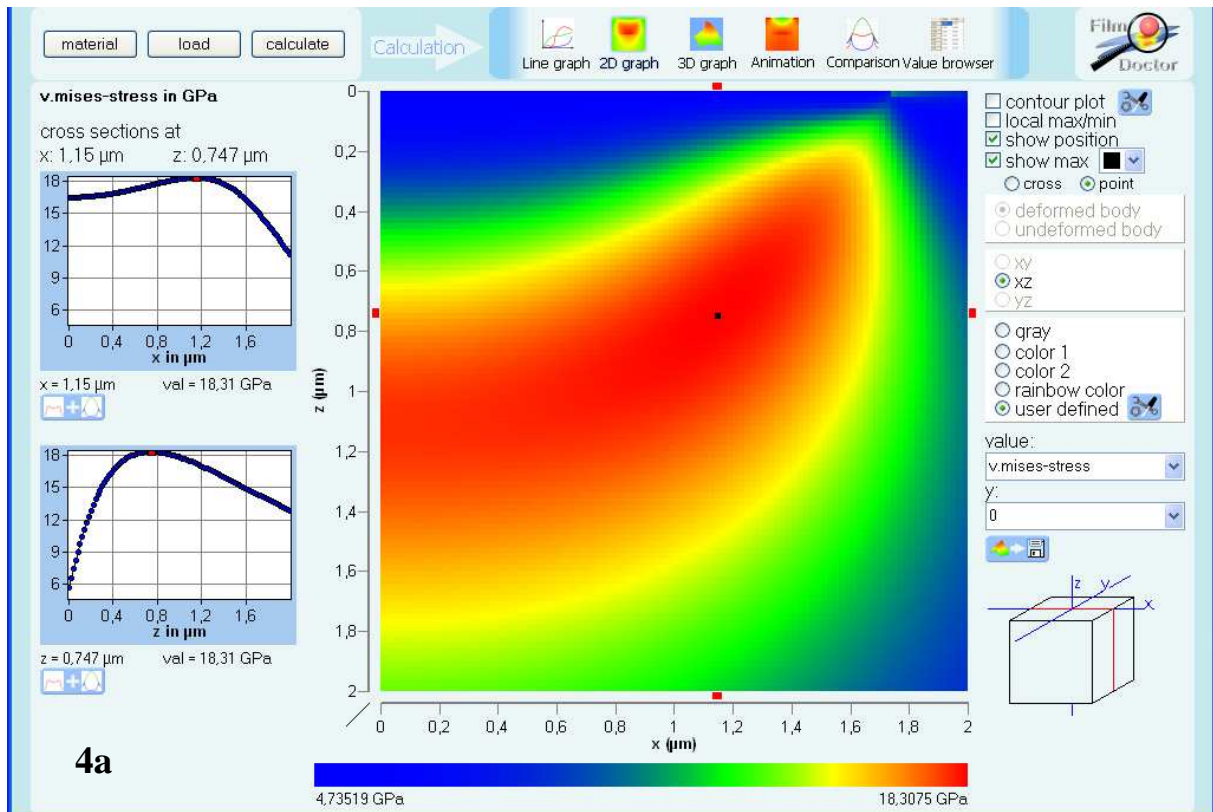


Fig. 4a: Resulting von Mises stress for the contact parameters given in fig. 2 at max. load  $p_0$ .  
 Fig. 4b: As the effective indenter produces a rather non-Hertzian normal surface pressure distribution (normal stress  $z$ ), the von Mises maximum is to be found outside the axis of indentation.

## Analyzing procedure step II: mixed normal and lateral indentation with maximum load $0.7 \cdot p_0$

Now we reduce the normal load to  $0.7 \cdot p_0$  and start the lateral loading of the sample surface until again plastic flow respectively any other inelastic behavior can be observed. As effective indenter we either determine the new one for the reduced load by again fitting a paraboloid to the unloading curve after the lateral loading procedure or simply use the effective indenter from step I at reduced normal load. However, the latter is only possible when the surface shape of the inelastically deformed surface is not significantly changed during the lateral loading procedure, which would result in an discontinuity of the unloading curve at the position of the introduced load (blue ellipse in fig. 2).

The author wants to point out here, that instead of a lateral load also inclined normal indentation should allow to obtain a second measurement sufficiently linear independent from the pure normal loading. However, this would require a well calibrated equipment clearly assigning lateral  $t_x$  and tilting load  $M_t$  to the indenter inclination and normal load.

## Analyzing procedure step III: evaluation of the von Mises stress for the mixed normal and lateral indentation with normal load $0.7 \cdot p_0$ and lateral load $t_x$

We assume that at a normal load of  $0.7 \cdot p_0$  and a lateral load  $t_x$  inelastic behavior has been detected. Now we evaluate the von Mises stress for this mixed loading situation. As described in [1] we have to take into account, that usually lateral or inclined indentation also produces indenter tilting. In the example considered here we found the following loading conditions:

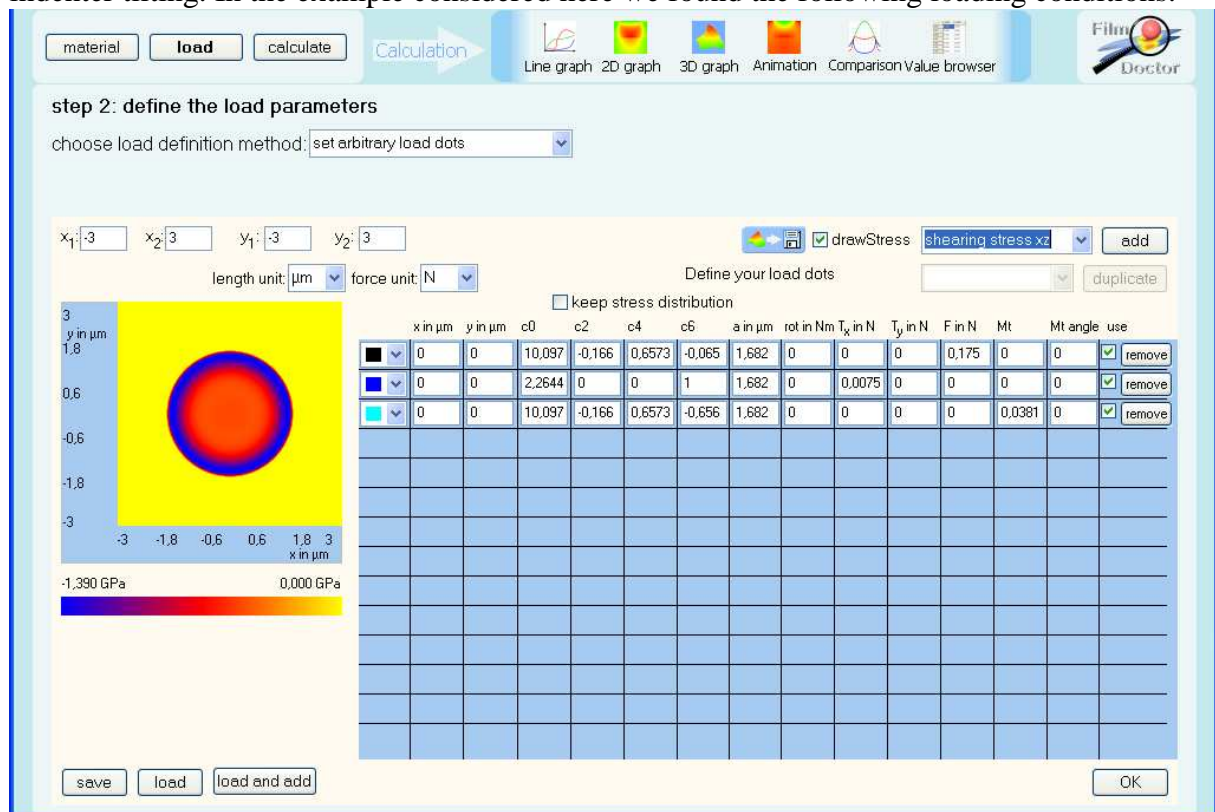


Fig. 5: Parameters for the mixed load evaluation

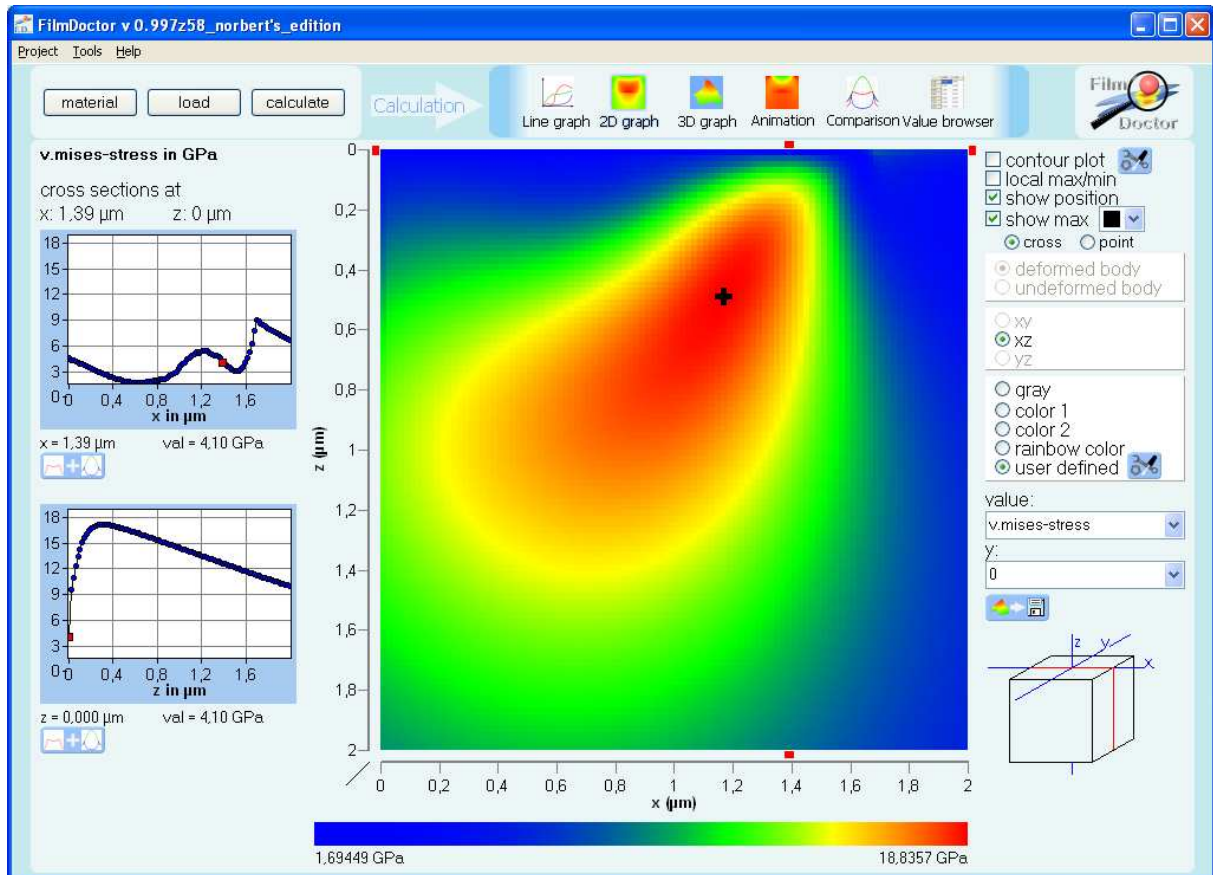


Fig. 6: Resulting von Mises stress for the mixed load situation.

## Analyzing procedure step IV: evaluation of the “pure” yield strength and the intrinsic stress

From the equations given above we can easily obtain a formula for the intrinsic film stress

$$\sigma_{rr}^f :$$

$$\begin{aligned}
& -6(f_{xy}^I \tau_{xy}^L + f_{xz}^I \tau_{xz}^L + f_{yz}^I \tau_{yz}^L) + f_{xx}^I (-2\sigma_{xx}^L + \sigma_{yy}^L + \sigma_{zz}^L) \\
& + f_{yy}^I (-2\sigma_{yy}^L + \sigma_{xx}^L + \sigma_{zz}^L) + f_{zz}^I (-2\sigma_{zz}^L + \sigma_{yy}^L + \sigma_{xx}^L) \\
& \pm \sqrt{4 \begin{pmatrix} f_{xx}^{I^2} + f_{yy}^{I^2} + 3(f_{xy}^{I^2} + f_{xz}^{I^2} + f_{yz}^{I^2}) \\ -f_{yy}^I f_{zz}^I + f_{zz}^{I^2} - f_{xx}^I (f_{yy}^I + f_{zz}^I) \end{pmatrix} + \begin{pmatrix} \sigma_M^{crit^2} - \sigma_{xx}^{L^2} - \sigma_{yy}^{L^2} - 3(\tau_{xy}^{L^2} + \tau_{xz}^{L^2} + \tau_{yz}^{L^2}) \\ + \sigma_{yy}^L \sigma_{zz}^L - \sigma_{zz}^{L^2} + \sigma_{xx}^L (\sigma_{yy}^L + \sigma_{zz}^L) \end{pmatrix} + \begin{pmatrix} -6(f_{xy}^I \tau_{xy}^L + f_{xz}^I \tau_{xz}^L + f_{yz}^I \tau_{yz}^L) + f_{xx}^I (-2\sigma_{xx}^L + \sigma_{yy}^L + \sigma_{zz}^L) \\ + f_{yy}^I (-2\sigma_{yy}^L + \sigma_{xx}^L + \sigma_{zz}^L) + f_{zz}^I (-2\sigma_{zz}^L + \sigma_{yy}^L + \sigma_{xx}^L) \end{pmatrix}^2} \\
\sigma_{rr}^f = & \frac{\pm \sqrt{\dots}}{2 \begin{pmatrix} f_{xx}^{I^2} + f_{yy}^{I^2} + 3(f_{xy}^{I^2} + f_{xz}^{I^2} + f_{yz}^{I^2}) \\ -f_{yy}^I f_{zz}^I + f_{zz}^{I^2} - f_{xx}^I (f_{yy}^I + f_{zz}^I) \end{pmatrix}}. \tag{1}
\end{aligned}$$

Assuming now only biaxial stress one simply has to compare the expected value for the unstressed case with the measured one  $\sigma_M^{crit}$  and could evaluate  $\sigma_{rr}^f$  using equation (1), which can be dramatically simplified due to the fact that within the coating  $f_{xx}^I = f_{yy}^I = 1$  and all other  $f_{ij}^I = 0$ .

$$\sigma_{rr}^f = \frac{-\sigma_{xx}^L - \sigma_{yy}^L + 2\sigma_{zz}^L \pm \sqrt{4\sigma_M^{crit^2} - 3((\sigma_{xx}^L - \sigma_{yy}^L)^2 + 3(\tau_{xy}^{L^2} + \tau_{xz}^{L^2} + \tau_{yz}^{L^2}))}}{2}. \tag{2}$$

So the two measurements provide us with two equations we can solve with respect to  $\sigma_M^{crit}$  and  $\sigma_{rr}^f$  [5].

For the sample considered here we obtain  $\sigma_M^{crit} = 16.6 \text{ GPa}$  and  $\sigma_{rr}^f = -2.03 \text{ GPa}$ , with the latter being in very good agreement with the value measured by other means ( $-1.9 \text{ GPa}$ , c.f. [3]). By taking the biaxial intrinsic stress fixed to its directly measured value of  $\sigma_{rr}^f = -1.9 \text{ GPa}$  one would obtain two possible values for the yield strength, namely  $16.79 \text{ GPa}$  in mixed loading and  $16.71 \text{ GPa}$  in the pure normal loading case.

From the small difference of these two values and the considerations presented in [5] one can easily deduce, that measurement of intrinsic stresses via nanoindenter requires a very high accuracy and well calibrated equipment. However, the procedure described here could also be used as a simple estimator for the maximum value the intrinsic stress can not exceed and thus, giving more precise error bars for the yield strength (respectively hardness) determined from nanoindentation data. This way big hardness or yield strength values only obtained due to huge intrinsic compressive stresses will not pass as absolute material properties. Such an information is of special importance when nanoindenter results from pure normal loading states are going to be used in applications with mixed loading conditions.

All evaluations have been performed using a special prototype of the software FilmDoctor [6].



## References:

- [1] N. Schwarzer, "Intrinsic stresses – Their influence on the yield strength and their measurement via nanoindentation", publication of the Saxonian Institute of Surface Mechanics, online at [www.siomec.de/pub/2007/001](http://www.siomec.de/pub/2007/001)
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- [3] T. Chudoba, N. Schwarzer, V. Linss, Thin Solid Films, to be submitted
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- [5] N. Schwarzer, "Modelling of the mechanics of thin films using analytical linear elastic approaches", Habilitationsschrift der TU-Chemnitz 2004, FB Physik Fester Körper, published in the online-archive of the Technical University of Chemnitz available at: <http://archiv.tu-chemnitz.de/pub/2004/0077>
- [6] FilmDoctor: software package for the evaluation of the elastic field of arbitrary combinations of normal, rotating and lateral loads of the type  $\sim r^n \sqrt{a^2 - r^2}$  (with  $n=0,2,4,6$ ), available from the internet at: <http://www.siomec.de/downloads> (contact: [service@siomec.de](mailto:service@siomec.de)).