Modelling of Contact Problems of Rough Surfaces

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Prolog

Macht es scheinbar keinen Sinn,
dass die Schicht schon wieder hin,
dass ein Zahnrad schon defekt
und das Schmieröl recht verdreckt,
so tut's oft dem Gewinne gut,
wenn man vorher rechnen tut.

If you always wonder why
your coating seems so shy.
Shows fractures quick and fast
or delaminates at last.
Then might it help to win
if you throw a model in.

Abstract

In this paper it is shown that a completely analytical theory based on the extended Hertzian approach together with additional considerations taking into account the geometrical conditions of a curved surface provide an appropriate model for the theoretical "simulation" of a variety of asperity contact problems. This model yields relatively fast and easy to use tools for the analysing of contact problems arising in connection with rough surfaces.
In this study the results are shown on the example of a 3µm-DLC-coating on a steel substrate with asperities of about 100µm in diameter and 15µm height. It is found, that – under a general average pressure of 1GPa – the ideal asperity tip contact situation would lead to severe damage due to plastic flow within the steel substrate. On the other hand a rather conforming contact situation appears to be completely non critical.

A Contact Model for Rough Surfaces

There is of course no problem to geometrically construct all sorts of rough surfaces by applying appropriate mathematical functions.

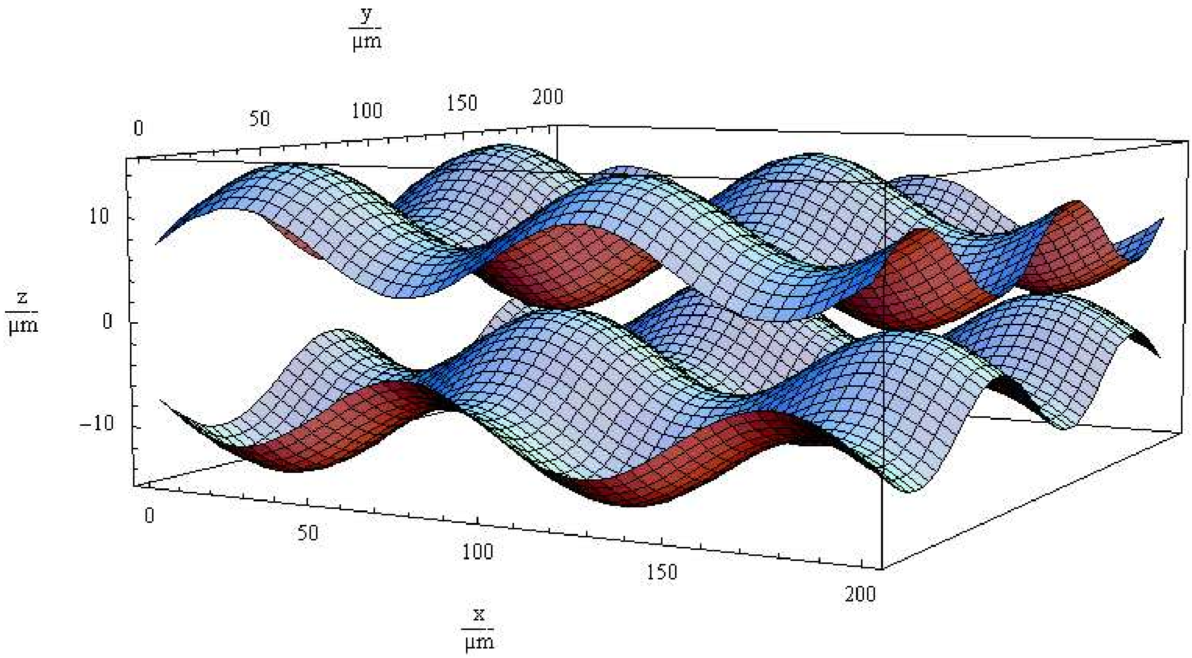

Fig. 1: Simple example for two rough surfaces in contact (worst case situation).

Fig. 1 shows an example of two surfaces of equal roughness in a mere “mathematical” contact situation (point contact). The physical modelling of this contact however, requires rather comprehensive evaluations. These evaluation methods will not be topic of this short overview. But it might be necessary to elaborate some of the basics in order to provide a proper description about the mathematical procedures necessary for a sufficiently accurate explanation of how to theoretically simulate the real contact situation.

So, simple sin-functions have been used in fig. 1 to construct a conform and homogenous roughness model. As it can be deduced from fig. 1, in the chosen contact situation, were only the tips of the asperities are in contact, there is no need to consider more than one of those “single contacts” (fig. 2).

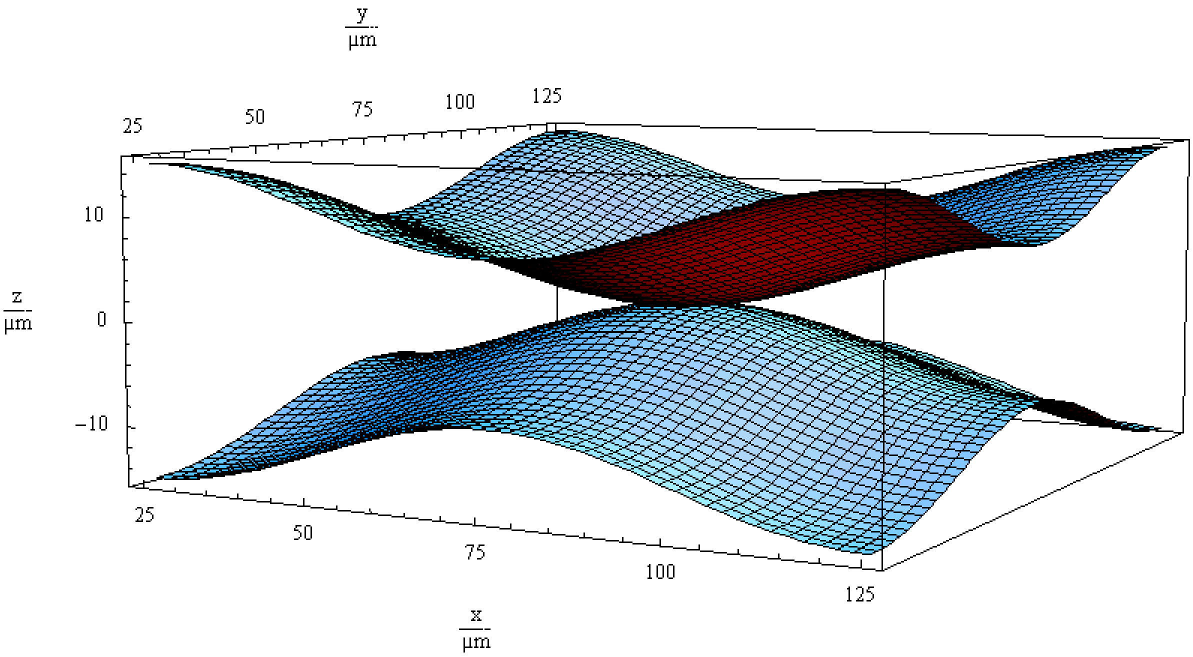

Fig. 2: Single contact.

As we are only interested in contacts within the elastic regime, there will never be contact radii filling the whole asperity area. Thus, defining $w_S(r)$ and $w_I(r)$ as the normal displacement of the sample-body (1st counterpart of the contact pairing) and the indenter (2nd counterpart of the contact pairing), respectively and denoting the overall approach with the letter h the following governing contact equation can be given:

$$w_S(r)+w_I(r)=h-\frac{r^2}{d_0}-\frac{r^4}{d_2}-\frac{r^6}{d_4}-\frac{r^8}{d_6}-\ldots \tag{1}$$

which could be considered as an extended Hertzian approach [1] and [2]. For the single contact shown in fig. 2 a good approximation of the contact-shape-function would be given due to (L giving the asperity width, H… height of asperity):

$$d_0=\frac{2L^2}{H\pi^2},\quad d_2=-\frac{24L^4}{H\pi^4},\quad d_4=\frac{720L^6}{H\pi^6},\quad d_6=-\frac{40320L^8}{H\pi^8}. \tag{2}$$

This also holds in the case of the rather conforming contact situation as shown in figure 3 as long as the contact area does not exceed about 80% of the asperity region. The d_i must then read (X gives the factor between the deepening and the asperity region, X>1 → for X=1 the asperity would fit perfectly into the deepening):

$$d_0 = \frac{4L^2}{H\pi^2}\frac{X^2}{X^2-1}, \quad d_2 = -\frac{48L^4}{H\pi^4}\frac{X^4}{X^4-1}, \quad d_4 = \frac{1440L^6}{H\pi^6}\frac{X^6}{X^6-1}, \quad d_6 = -\frac{80640L^8}{H\pi^8}\frac{X^8}{X^8-1}. \tag{3}$$

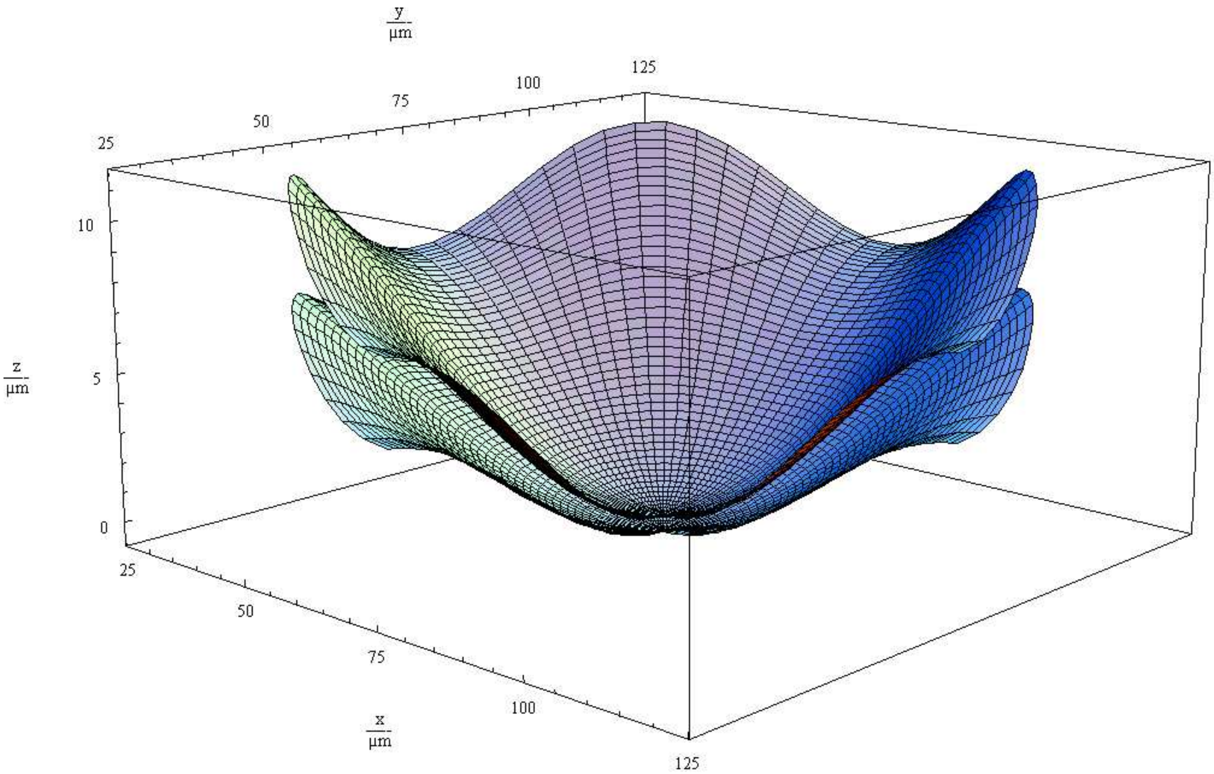

Fig. 3: Single contact with conforming contact conditions.

However even higher degrees of conformity could be modelled by simply adding more terms to the extended Hertzian approach (1). The solution of this governing contact equation is given in [1] for powers of r up to 8. The solution is complete for the case of a half space, which means, the potential function is given and the complete elastic field (stress, displacements, strains…) can be easily evaluated without any approximations. Here we present only the resulting pressure distribution beneath the indenter:

$$\sigma_{zz0}(r,\varphi) = \sum_{n=0}^{N} c_{\sigma n} r^n \sqrt{a^2 - r^2} \tag{4}$$

For higher powers of r the mathematical procedure of evaluating the potential functions is presented in the appendix of [1]. As we here intend also to discuss problems with surfaces of higher degree of curvature it is necessary to mention, that in these cases the half space approach might not be valid anymore. Assuming symmetry of revolution and applying the method of potential theory it can be demonstrated however, that by denoting the surface shape function with Z(r) the resulting pressure distribution beneath the indenter can be approximated very well by the following expressions:

$$\sigma_{zz0}(r,\varphi)=\sum_{n=0}^{N} c_{\sigma n} r^n \sqrt{1+\left[Z'(r)\right]^2} * \sqrt{a^2-r^2} \ , \tag{5}$$

$$\tau_{rz0}(r,\varphi)=-\sum_{n=0}^{N} c_{\tau n} r^n \frac{\sqrt{1+\left[Z'(r)\right]^2}}{Z'(r)} * \sqrt{a^2-r^2} \ . \tag{6}$$

It is evident, that the shearing load $\tau_{rz0}(r,\varphi)$ has nothing to do with a frictional forces acting in lateral direction on the surface but only occurs as a result of the geometrical surface curvature. This becomes clear if we bear in mind, that the stresses above are still given in cylinder coordinates, but the normal direction of our curved surface now is a function of r and can not be given by the unit vector in z-direction $\vec{e}_z$ as we have done in the half space case defined due to $z \geq 0$.

Application to some Hypothetic Contact Problems

At first we consider the conforming contact situation applying the geometrical conditions given in fig. 3. Assuming that over a sufficiently large area (covering quite a few asperities) the average contact pressure should be 1GPa we can evaluate the load for one asperity under the given geometrical (roughness) conditions (see fig. 1). We find for the contact situation geometrically described in figures 1 to 3 the load to be p=10N. Though, in principle the author has the means to perform the evaluation for layered materials correctly [3, 4] here we use effective monolithic material parameters in order to simplify and shorten the calculation process. It can be deduced from similar investigations [5], that the error is small, when the ratio of contact radius to coating thickness is relatively big, as it is here. The resulting surface stress distribution (with X=1.2) is given in figure 4.

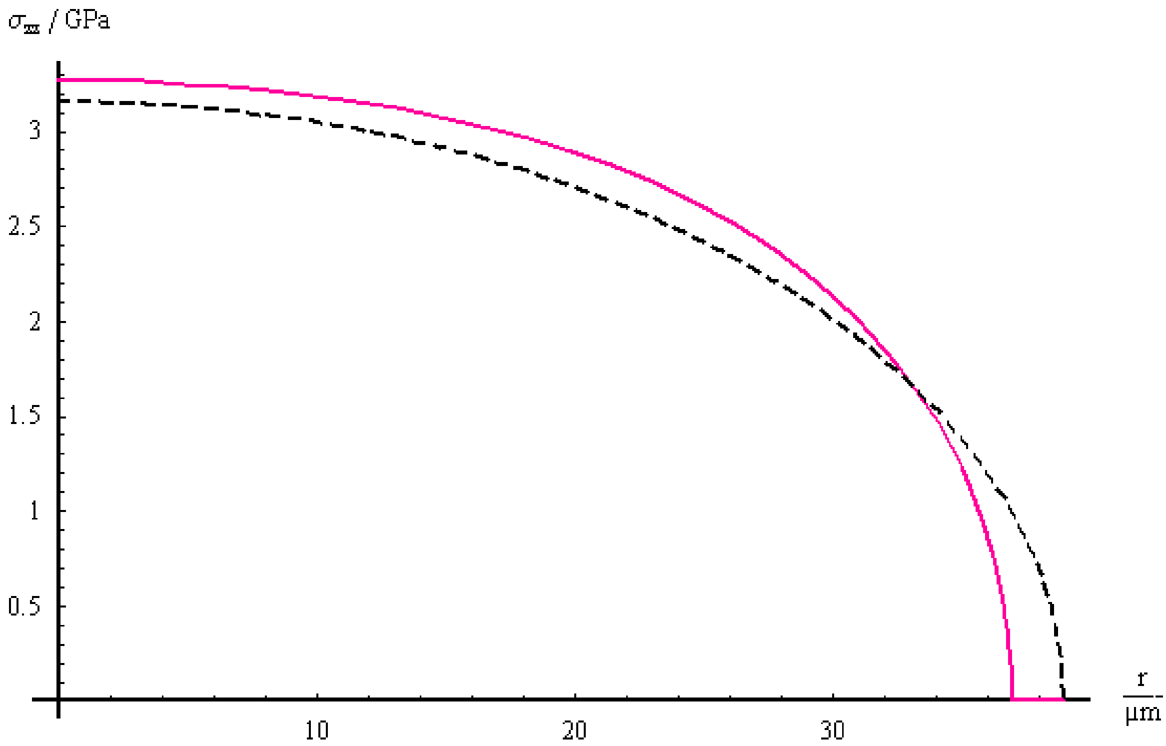

Fig. 4: Resulting stress distribution for a single contact with conforming contact conditions according to fig. 3 (red line). For comparison the equivalent Hertzian pressure distribution (same load) is given, too (black dashed line).

In order to find out whether this pressure distribution might be dangerous for a system 3µm TiN with 400GPa Young'S modulus on steel (220GPa) we evaluate the von Mises stress along the x-z-plane (fig. 5). Clearly we detect the sharp leap along the interface. The stresses are well below typical critical yield stress values of 3.5GPa for rim hardened steel and >15GPa for most DLC hard coatings. Thus, the conforming contact situation appears to be no problem for the given material combination under these geometrical conditions[1].

[1] It should be pointed out, that the conclusion about the non critical state of the conforming contact situations can of course not be drawn from the results for the von Mises stress only. All field components should be considered, which the author in fact has done, but he prefers not to present more results in order to keep this study short and because they do not add much to the problem.

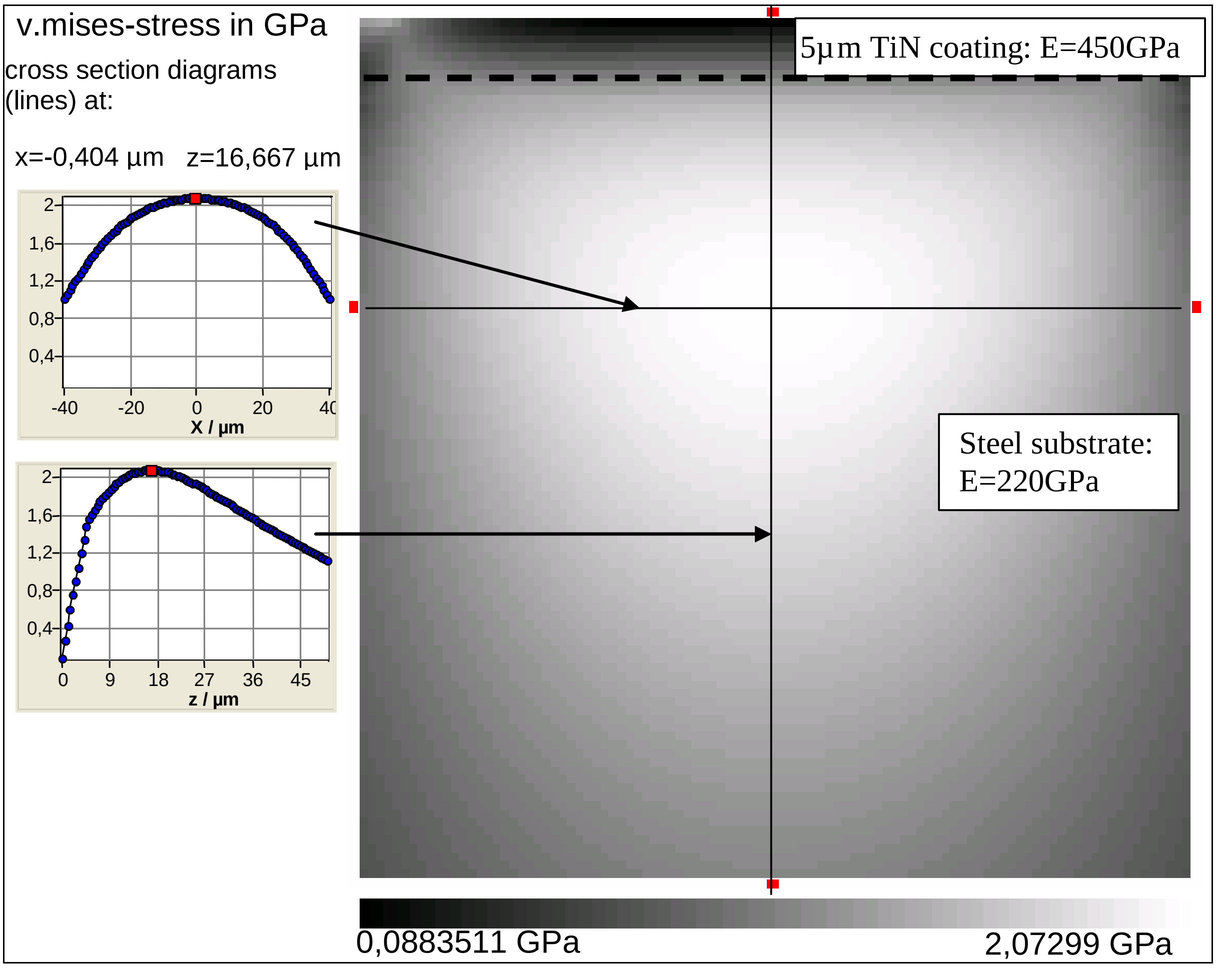

Fig. 5: Resulting von Mises stress distribution for a single contact with conforming contact conditions according to fig. 3. The evaluation has been performed using a prototype of the Software "FilmDoctor" [6].

The picture changes completely if we are going to the contact situation described due to fig. 2. Certainly, now we obtain a very Hertzian surface stress distribution (fig. 6), but with much higher absolute values. The subsequent evaluation of the von Mises stress, shows that the assumed DLC-coating-steel-substrate compound can never survive this type of contact conditions (fig. 7).

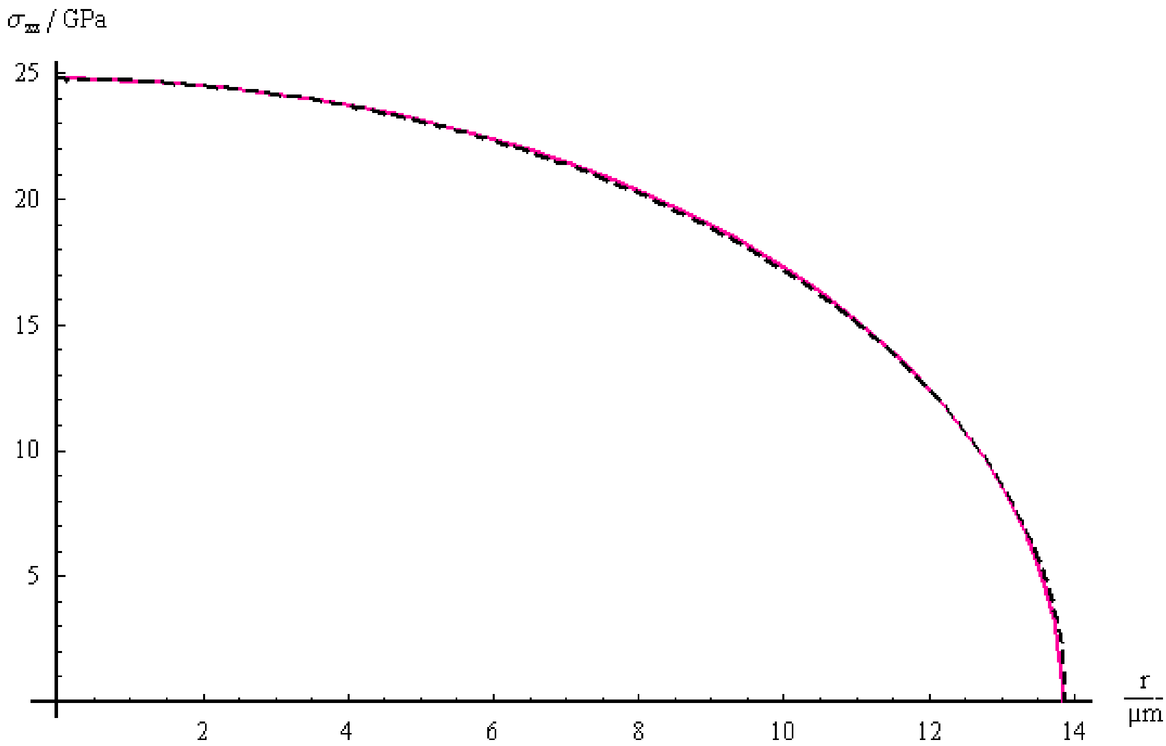

Fig. 6: Resulting stress distribution for a single contact with ideal asperity tip contact conditions according to fig. 2 (red line). For comparison the equivalent Hertzian pressure distribution (same load) is given, too (black dashed line).

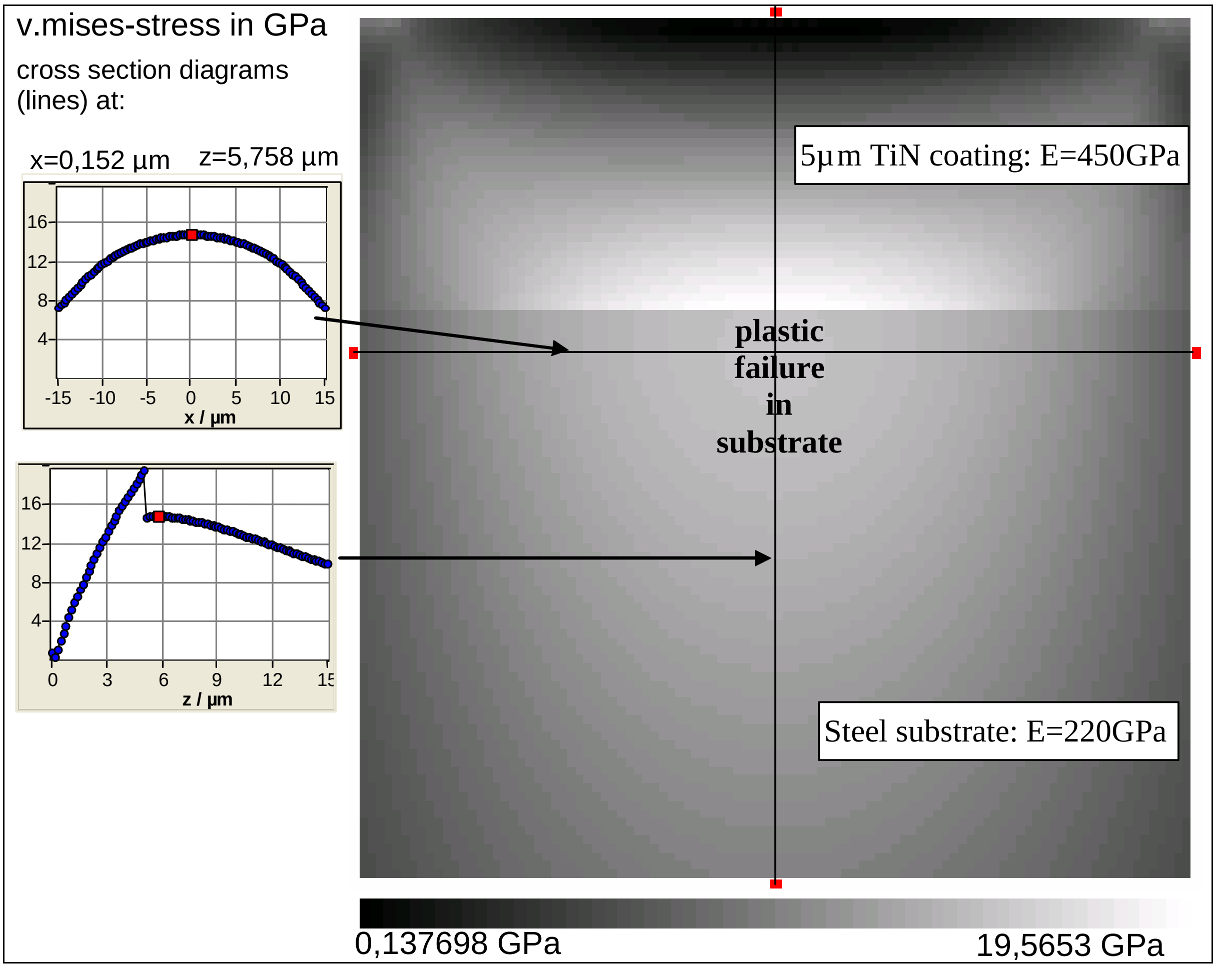

Fig. 7: Resulting von Mises stress distribution for a single contact with ideal asperity tip contact conditions according to fig. 2. The evaluation has been performed using a prototype of the Software "FilmDoctor" [6].

Conclusions and Outlook

It has been shown that a completely analytical theory based on the extended Hertzian approach together with additional considerations taking into account the geometrical conditions of a curved surface provide an appropriate model for the theoretical "simulation" of a variety of asperity contact problems. This model yields relatively fast and easy to use tools for the analysing of contact problems arising in connection with rough surfaces.

In this study the results were shown on the example of a 3µm-DLC-coating on a steel substrate with asperities of about 100µm in diameter and 15µm height. It was found, that – under a general average pressure of 1GPa – the ideal asperity tip contact situation (fig. 2) would lead to severe damage due to plastic flow within the steel substrate. A rather conforming contact situation on the other hand (fig. 3) appeared to be completely non critical. A more systematic investigation also including intermediate contact situations (contact along the inclined faces of the asperities), and friction would be necessary in order to find an optimum roughness and/or coating thickness accordingly fitted to the industrial demands of the later application. The “optimum search” should also include economic parameters like cost for surface roughness reduction and cost per µm of coating thickness.

References

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[6] FilmDoctor: software for the evaluation of the elastic field of arbitrary combinations of normal and tangential loads of the type “load~$\sum_{n=1}^{6} c_n * r^n \sqrt{a^2 - r^2}$ ” (with n=0,2,4,6), available at: http://www.siomec.de (contact: contact@siomec.de)