3. School Transport and Circadian Rhythmic

Optimized getting up and working or school start times have become issues of rather heated discussions the moment it became more and more evident that there is a dramatic influence of our sleeping habits on the general health and ability to work efficiently. It has been shown that whole societies, mainly out of tradition, suboptimum internal organization or simply ignorance, are severely suffering under an effect being known as social jetlag. Some authors with sever background and after having collected numerous data and statistics are even calling this "Genetic Discrimination" a majority of a whole society (over 70%) has to endure. This negative influence is even more pronounced for children, because their brains are still under development and therefore we want to apply our new optimization method here on the problem of school transport and school starting times. Application to work starting times and duty strokes, as it might be of more interest for the thin film community, is straight forward and a simple substation procedure. It is meanwhile a well-established fact that social jetlag leads to severe health risks like obesity [24] and undesired social effects, like increased depression rates [25], smoke addiction [26], lower academic performance [27, 28], elevated accident risk [29- 31], aggressivity, learning problems [27, 28, 32, 33] etc. ect.. Here are some numbers about a few effects only one hour of social jetlag does result in:

1. We find an increased risk of 33% for obesity [24]
2. Depression increases by 31% [25]
3. Increase of risk to become a smoker by 41% [26]
4. Negative correlation with academic performance, which is to be found 22% worse [27]
5. Increase of traffic accidents [29-31]

This already rather impressive collection of negative effects of the social jetlag is not even remotely complete. Still we have to point out that some of the numbers are coming from studies with adults, while these effects on children and teens are probably even more severe simply due to the fact that their brains are still under development [32, 33, 34].
Thus, incorporation of these health effects as constraints into school transport optimization seems to be an important issue.
Assuming we have the transport problem as given in the figures 8a and 8b, we immediately see that a bifurcating "optimum" solution is possible using either one (fig. 8a) or two bus-lines (fig. 8b). Without properly taking long-term costs like depreciation into account, most modern optimization systems as i.e. given in [11 - 19] would immediately suggest version 8a with only one bus and simply different school starting times for the two schools. Of course, considered purely economically in the short term and ignoring depreciation and all health or follow up costs due to imperfect education, this is in fact the optimum solution. However, there might be soft boundary conditions coming into play completely contradicting this "optimum" the moment they are considered, too.
In the introduction section, we have already given an overview on how important a "healthy" get-up time can be and it is clear that badly chosen school starting times in connection with long school transportation times have a great negative impact on health and educational success of the children.

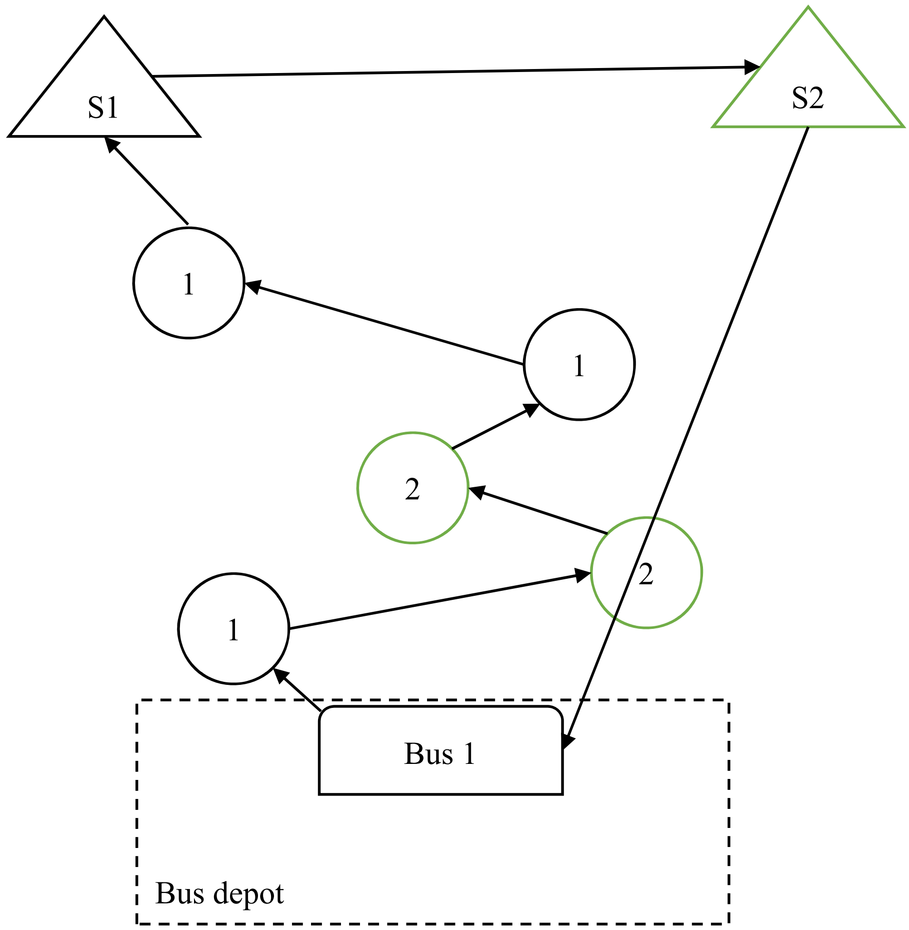

Fig. 8a: Schematic presentation of the school transport problem as discussed in the text. The spheres are denoting the stop positions while their numbers give the school to which the pupils have to be brought. The triangles are defining school one and two with the corresponding numbers given in the triangle.

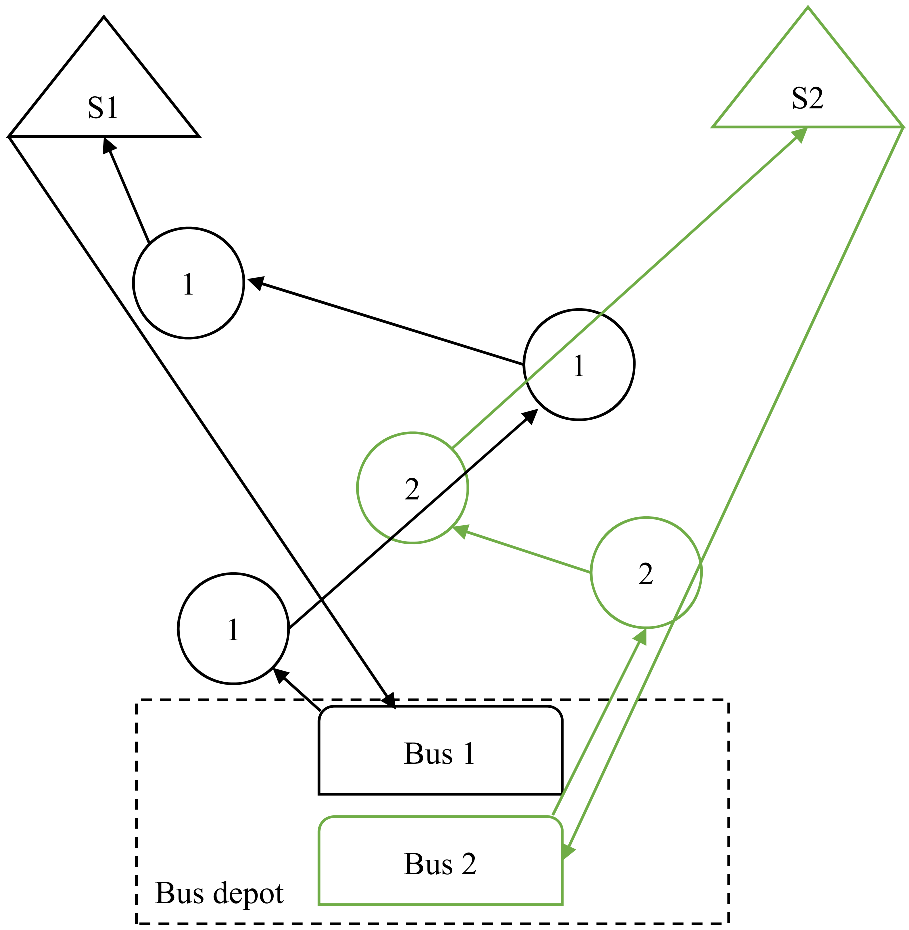

Fig. 8b: Schematic presentation of the school transport problem as discussed in the text. The spheres are denoting the stop positions while their numbers give the school to which the pupils have to be brought. The triangles are defining school one and two with the corresponding numbers given in the triangle.

In our fictive example it is evident that not only the starting time of school 1 has to lay before the one of school 2, but also that all pupils have to get up much earlier if one wants to serve the route with only "one bus" as shown in fig. 8a.
At first, we want to find out whether the "one bus" solution considered purely hard-economical (no health or education effects being taken into account) truly is the best solution when taking all costs and depreciation properly into account.
The following parameters will be applied in our fictive example:

Price per bus	350,000€ (e.g. Mercedes Benz Citaro G)
Number of pupil one bus can transport	150
Costs per km incl. driver	10€
Amortization rate per year	35,000€
Percentage of use of bus for school transport only	100%
Percentage of pupil affected by the social jetlag	0% (soft effects are not considered here)
Total number of pupil in school 1	500
Total number of pupil in school 2	500
Total distance of whole tour	2 x 250 km = 500 km

For reasons of convenience we have used an adapted version of one of Mathematica's traveling sales man demonstration packages [35] in order to find the optimum paths for all constellations being considered (figure 9).

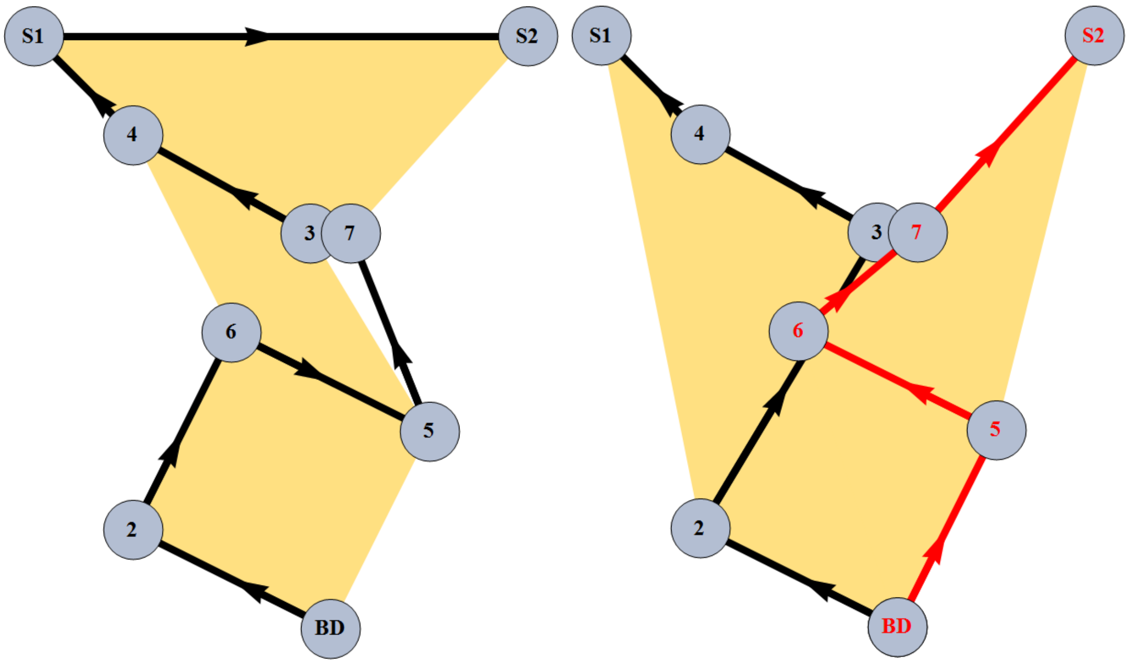

Fig. 9: Mathematica evaluation of the school transport problem illustrated in figures 8a and 8b with the additional condition that the stop numbered 3=7 serves for pupils from both schools S1 and S2. This constellation has a low disorder-grade and always the solution on the right hand side should win as the more economical one (long term considered). BD=Bus Depot.

It is found that for a great variety of constellations always the "2-bus-line" solution beats the "1-bus-line" solution. Thus, usually it is economically clever to serve the schools with separate bus lines instead of connecting various schools. In such a case all schools can have a relatively late starting time and transportation is been kept to a minimum. As an important side effect, this would also simplify the situation for the children with respect to get-up time and duration of transportation. This holds even in situations where some stopping points have to serve several schools as shown in figure 9. We might call such systems "systems of high order".

Still, in practice one often finds lesser optimal solutions being realized, meaning, even if the 2-bus-line solution was the better one we find a 1-bus-line structure. Possible reasons why politicians and officials under such conditions are not going for the economically cheaper solution could be:

a) Money is been extracted from the system.
b) The optimization is not sufficiently long term orientated (e.g. depreciation time of a bus).
c) The necessary starting investment (more busses and drivers) cannot be made.
d) The planning is not optimized solely for the school transport but also other boundary conditions and applications are considered that have nothing to do with the school transport.

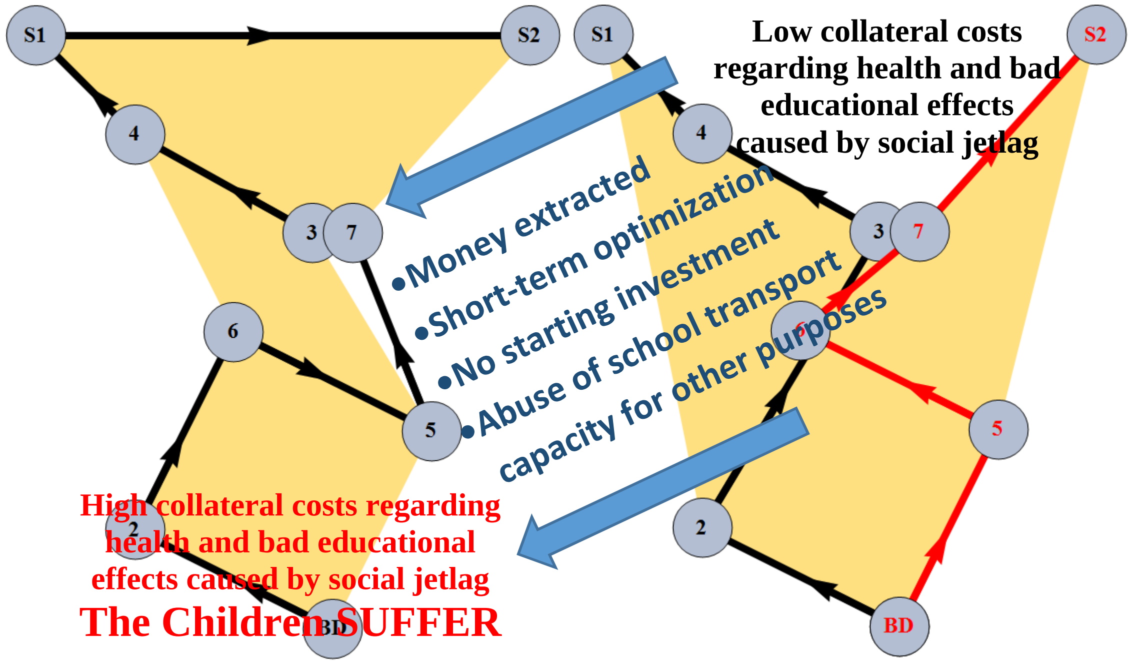

Fig. 10: Typical reasons for political choice of the bad solution of the school transport example given in figure 9.

Interestingly we have one such - rather extreme - example in Northern Germany right in the electoral district of federal chancellor Dr. A. Merkel.
In the interest of optimum conditions for the children however, politicians and officials should rethink their set of priorities in such cases, especially as we will show further below that immense collateral damage is been created with such suboptimum solutions.

The situation changes completely however, in the moment more stopping points are serving various schools. By introducing a grade of disorder, one always finds critical values of disorder where the system tips over and the 2-bus- or multiple-bus-line solution becomes less economical than the 1-bus-line solution (figure 11). In this case, of course, we will have all the disadvantages coming into play having to do with imperfect school starting times and long transport duration potentially negatively affecting the circadian boundaries of the children.

It seems that especially in some rural areas of North East Germany such a chaotic constellation is to be found. Surprisingly, even though there is a system of “local responsible schools” supported and enforced in Mecklenburg-Vorpommern, such high disorder is often to be found. The reasons given by parents insisting on giving their children to different schools, often very far away, are:

- Local responsible school is often found to be too bad or inappropriate
- Parents do not approve of the public school system and prefer independent private schools instead (free schools)
- Social composition in “local responsible school” is to be found problematic for own children
- School density is so low and transportation so bad anyway that it does not matter much where to send the children

This indicates that good (short) transportation, a critical school density and good education in the schools are important factors to end up with healthy get-up times being in congruence with the main circadian rhythm of the majority of the pupils. Here politicians and officials should make it severely more attractive for parents to choose the local responsible school

close by, rather than a more distant one and thereby contributing to an increased disorder grade.
But still, the question should be asked why individual school selection of some parents shall determine and deteriorate the school transport for all pupils. Here, the author is of the opinion that the free individual choice of a few shall not worsen the health situation for all, at least not in such a dramatic manner as it is found in many regions of Mecklenburg-Vorpommern.

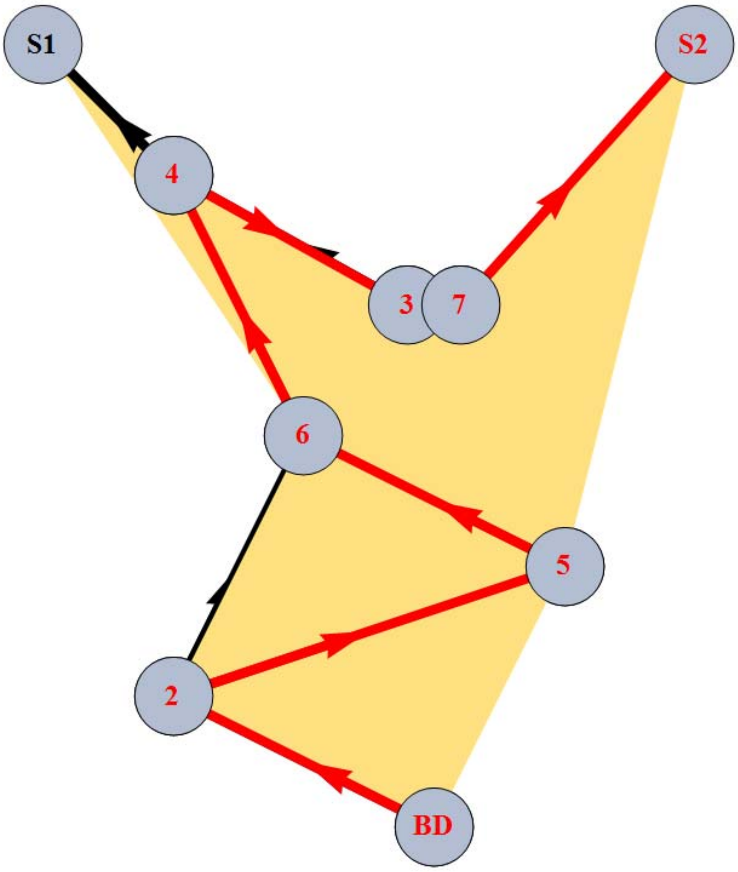

Fig. 11: Mathematica evaluation of the school transport problem illustrated in figures 8a and 8b with the additional condition that all stop-positions serving for pupils from both schools S1 and S2. This constellation has a high disorder-grade and always the "1-bus-line" solution should win as the more economical one (heath effect being ignored).

If one would take at least some of these negative social jetlag effects (especially the health and learning effects [24-34]) as given above into account and incorporate them into the optimization of our simple school transport problem, one might find a situation where the costs created by those negative effects are becoming significant. They might even become so high that they overtake those additional costs one would have if applying additional busses as schematically shown in fig. 8b. Unfortunately, within the standard optimization methods in public transport, even those with school starting time scheduling, these "soft" boundaries cannot be taken into account. That is why we are now introducing the Higgs field mechanism as described in the introduction part in order to account for these negative effects.

At first and only to have something to start with, we apply fields of the type:

$$U_k = \sum_{i=1}^{N} q_{ki}^2 e^{-\frac{(x_i - x_{i0})^2}{\sigma_{ki}}} . \qquad (21)$$

The parameters here have to be adapted to the economic impact the social jetlag has. The difficult question is how we can find economical correlations between the negative effects mentioned above and real costs. Thus, at first and for the reason of simplicity, we will leave the concrete collateral and health costs an open parameter to play with.
The following change of parameters will be applied in our fictive example:
Percentage of use of bus for school transport only 25%
Percentage of pupil affected by the jetlag of 1 h 30% (in fact, this is a very low number, because according to [24] even 70% would be fully justified)

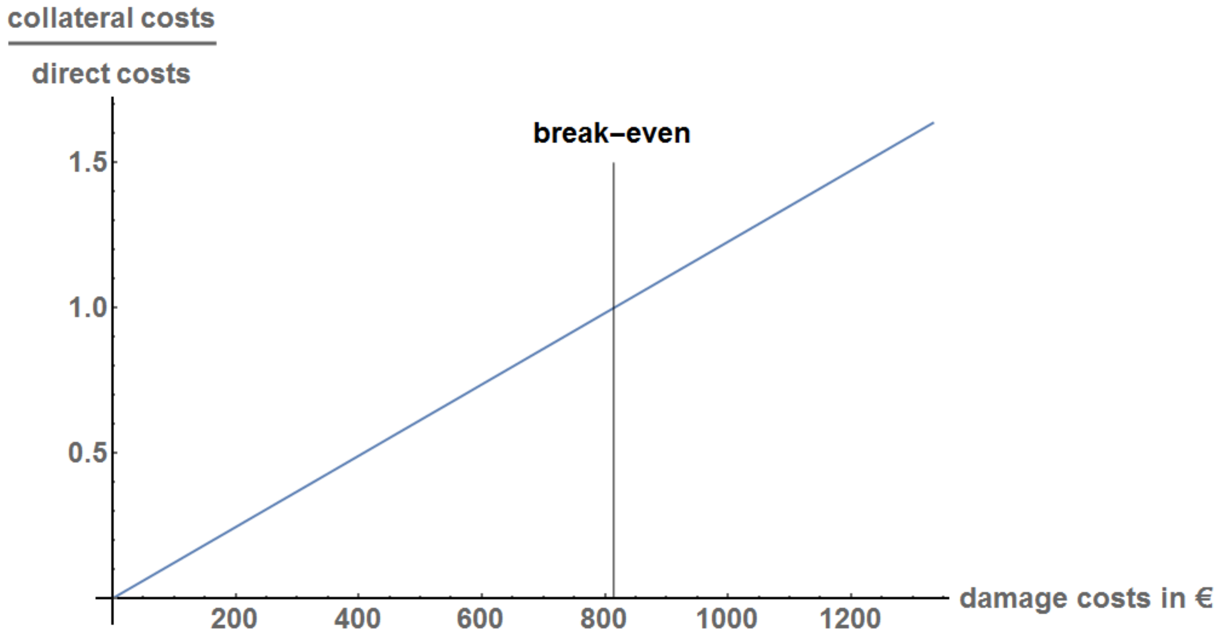

Figure 12: Development of the total follow-up and health effect costs ("collateral costs") over classical economic costs ("direct costs") caused by the social jetlag of one hour as a function of these costs per affected individual per year ("damage costs"). It was assumed that only the 1-bus-line solution is applied, which leads to the severe collateral damage, because of the early get-up times for all pupils.

Figure 12 now gives us the development of the total follow-up and health effect costs caused by the social jetlag of one hour as a function of these cost per affected individual per year. Here we considered a typical depreciation period of 10 years for relating the total collateral costs (health and other follow-up costs) to the direct costs. Interestingly we exceed the direct economic costs at an extremely low value of the individual damage costs of only 814.41 € per year. Considering the many negative effects listed above and taking the likelihood in which these effects will affect a great portion of the individuals (pupil) stressed by such a social jetlag the author considers this an almost shocking low number. Apart from the direct health costs, one only needs to consider follow-up costs like stoppage (lack of work) in order to understand that these 814 euro per year and damaged individual are rather small. If taking the results of Roenneberg et al [24] and assuming even 70%, the break even comes down to under 400€, which really gives a very bad light on those politicians and officials who are still favoring suboptimal, even bad transport solutions in combination with by far too early school starting times.
Thus, with better and more holistic parameter considerations, these total collateral costs of badly planned school transportation leading to social jetlag, we are probably to expect very severe cost effects at public and private expenses.

However, before we start to investigate these costs we have to answer the question of how to alter the usual optimization procedures like [10] to make them take such effects into account. In our fictive example we consider only four effects of the many mentioned above (risk of obesity, depression, risk to become a smoker and negative academic performance). At first we relate them to potentials as follows:

$$U_k = \sum_{i=1}^{N} U_{ki} = \sum_{i=1}^{N} q_{ki}^2 \cdot \left[e^{-\frac{\left\{\sin\left[\left(12+t-t_{i0}-\frac{d_i}{2}\right)\cdot\frac{\pi}{24}\right]\right\}^2}{\sigma_{ki}}} - e^{-\frac{1}{\sigma_{ki}}} \right]^n . \tag{22}$$

Where N gives the number of chronotypes (c.f. [24, 34]) being considered with their individual, mostly age and gender dependent parameters for jetlag-effect (obesity, depression etc.), affinity q_{ik}, midpoint of sleep t_{i0} (in hours), total sleep duration d_i (in hours) and the deviation parameter σ_{ki}. The variable t runs from 0 to 24 and is also measured in hours. For reasons of simplicity, we set t=0 as the time, where there is no average social jetlag. We also measure the individual midpoint of sleep plus half of sleep duration as difference from the average. We immediately realize, that by choosing Fourier series of periods 24 h times m with m=1,2,3… in (22) we are able to describe a great variety of different potential distribution. Following the typical chronotype distribution as given in [34], for each effect one would obtain a sum of N "individual" potentials as shown in figure 13. There it was taken into account that the distribution is not completely symmetric, meaning that there are usually more "owls" (late chronotype) than "larks" (early chronotype).

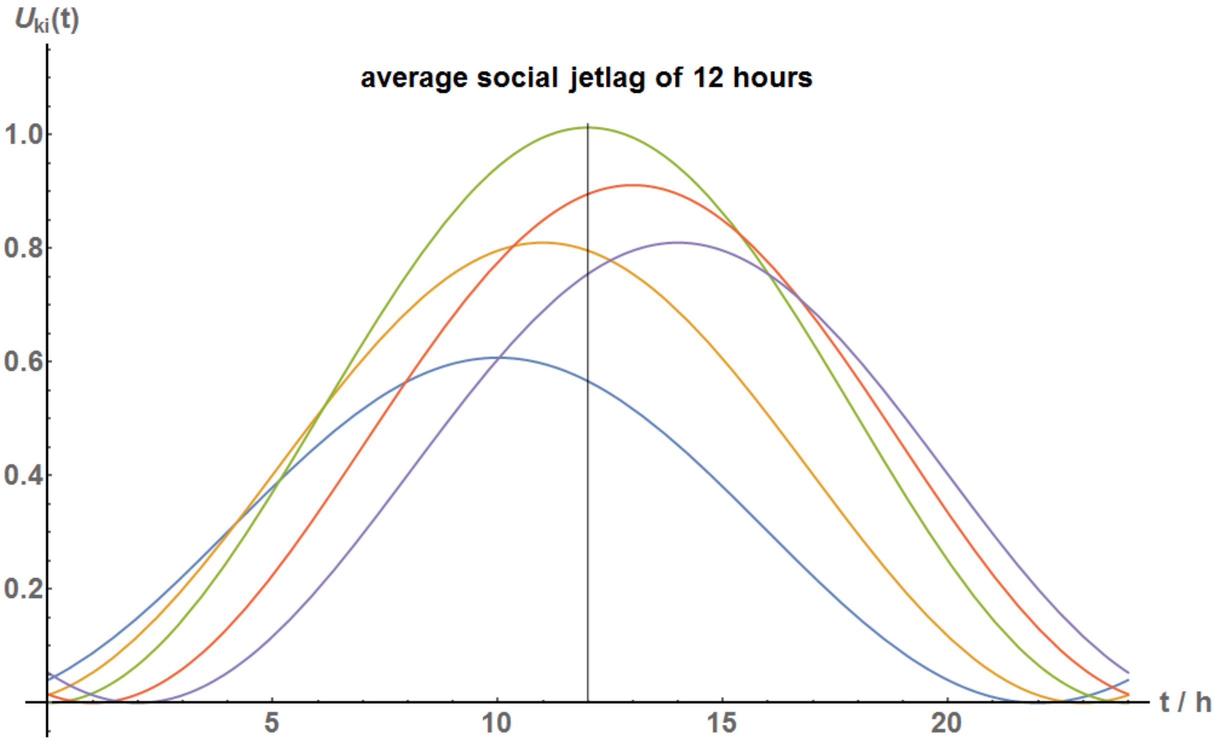

Figure 13: Set of fictive individual social jetlag potentials taking the typical chronotype distribution into account (only for illustration, set not complete; exponent n set to 1). The potential has to be adjusted to the estimates for lower social jetlag values like 1 or 2 hours. Of course, mathematically this could be continued to a jetlag of 12 hours, but it has to be pointed out explicitly here, that this value is completely hypothetic respectively artificial and only a result of the applied mathematical potential. As a result of this we will consider other potentials further below.

Now we follow the evaluation as given in the introduction and example 2 and evaluate the total "masses" for each social jetlag effect.
In order to couple the jetlag damage into the transport optimization problem without changing its internal structure completely, we simply use the cumulated masses in dependence of the various school starting times and distort the distances of the schools being effected most severely by the resulting social jetlag. Taking our example from figure 8a this makes the distance between the schools S1 and S2 variable (s. figure 14).

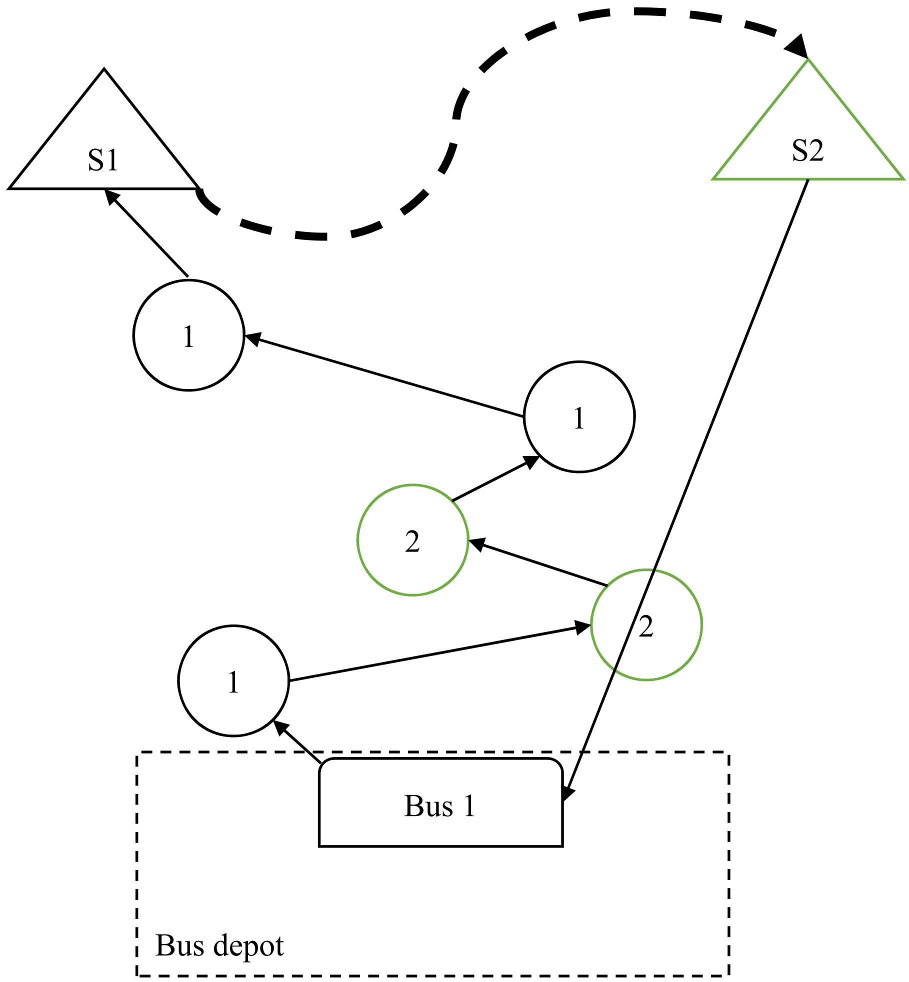

Fig. 14: Schematic presentation of the school transport problem as discussed in the text with distorted distance between the two schools S1 and S2 for more holistic optimization also considering collateral effects like social jetlag (c.f. figure 8a).

By connecting the social jetlag "mass" production with a stretching factor for the distances between the schools, the optimization procedure would be forced to switch from the 1-Bus-line solution to the 2-Bus-line-solution (figure 8b) the moment the collateral effects caused by the social jetlag are exceeding a certain point even in constellations with high grades of disorder.
In general, a public transport optimization procedure added up with this kind of "length scaling Higgs-mechanisms" would now automatically realize when limits of social jetlag effects are severely exceeded and changes from a low-number-bus-scenario to a higher number of busses with better school starting times for all schools, plus shorter transportation. In short, in this manner the optimization procedure is now reaching a much more holistic and realistic optimum.

Examples with concrete numbers for the health effects will be considered in subsequent publication.

It is illustrative to consider the Higgs mechanism within this example by just observing one or two negative effects of an unhealthy (too early) school start time like the increased risk of obesity or diabetes. Only for demonstration we will play with various types of potential functions and discuss the effects on the resulting Higgs fields respectively mass production processes.

3.1. Symmetric bump function

At first, we can write the Lagrangian as follows:

$$L(x) = Lc\left(\sum_{\forall j}\Phi_j(x_1,x_2,x_3...),\sum_{\forall i}\left(\frac{\partial\left(\sum_{\forall j}\Phi_j(x_1,x_2,x_3...)\right)}{\partial x_i}\right)\right), \qquad (23)$$
$$-U\left(\Phi_1(x_1,x_2,x_3...),\Phi_2(x_1,x_2,x_3...)\right)$$

with Lc(x) denoting the "classical task" not containing any "soft" boundaries respectively not considering health effects. We have only taken two field parameters, $\{\Phi_1, \Phi_2\}$, to couple into the total budget, namely school starting time and total duration of transportation. These two are both field parameters in the classical transport optimization and contributors to the potential of social jetlag U.

Now we expand the potential U near the minimum of the potential part in (3) if considering the whole space:

$$V = \int\left\{Lc - U\left(\Phi_1,\Phi_2\right)\right\}dr^n. \qquad (24)$$

As before we assume a minimum of the expression (5) at the position $\{\Phi_1, \Phi_2\}^{(0)}$ being also a maximum of U (c.f. figure 13). Taylor expansion of U around this point now yields:

$$\begin{aligned}U\left(\Phi_1,\Phi_2\right) &= U\left(\Phi_1^{(0)},\Phi_2^{(0)}\right)\\
&+\frac{1}{2}\left[\frac{2\partial^2 U}{\partial\Phi_1\partial\Phi_2}\left(\Phi_1-\Phi_1^{(0)}\right)\left(\Phi_2-\Phi_2^{(0)}\right)+\frac{\partial^2 U}{\partial\Phi_1^2}\left(\Phi_1-\Phi_1^{(0)}\right)^2+\frac{\partial^2 U}{\partial\Phi_2^2}\left(\Phi_2-\Phi_2^{(0)}\right)^2\right]\\
&+\frac{1}{6}\left[\begin{aligned}&\frac{\partial^3 U}{\partial\Phi_1^3}\left(\Phi_1-\Phi_1^{(0)}\right)^3+\frac{2\partial^3 U}{\partial\Phi_1^2\partial\Phi_2}\left(\Phi_1-\Phi_1^{(0)}\right)^2\left(\Phi_2-\Phi_2^{(0)}\right)\\&+\frac{2\partial^3 U}{\partial\Phi_1\partial\Phi_2^2}\left(\Phi_1-\Phi_1^{(0)}\right)\left(\Phi_2-\Phi_2^{(0)}\right)^2+\frac{\partial^3 U}{\partial\Phi_2^3}\left(\Phi_2-\Phi_2^{(0)}\right)^3\end{aligned}\right]\\
&+O(\Phi_1^4,\Phi_2^4)\end{aligned} \qquad (25)$$

The linear terms disappear because per definition U was expanded around an extremum leading to zero for the first derivative with respect to the "coordinates" $\{\Phi_1, \Phi_2\}$.

Again concentrating only on the terms of the lowest order, we obtain a matrix of the form:

$$M_{ij}^2 = \begin{pmatrix} \frac{\partial^2 U}{\partial \Phi_1^2} & \frac{\partial^2 U}{\partial \Phi_1 \partial \Phi_2} \\ \frac{\partial^2 U}{\partial \Phi_1 \partial \Phi_2} & \frac{\partial^2 U}{\partial \Phi_2^2} \end{pmatrix}. \tag{26}$$

As before the main axis transformation leads us to the two Eigenvalues M_α^2 of this matrix:

$$M_{ij}^2 R_{\alpha i} R_{\beta j} = \begin{pmatrix} \frac{\partial^2 U}{\partial \tilde{\Phi}_1^2} & 0 \\ 0 & \frac{\partial^2 U}{\partial \tilde{\Phi}_2^2} \end{pmatrix} = M_\alpha^2 \delta_{\alpha\beta} = M_\alpha^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{27}$$

Written in the field components $\tilde{\Phi}_1, \tilde{\Phi}_2$ and ignoring all terms of order higher than 2, the Lagrangian now reads:

$$L(x) = Lc(x) - \frac{1}{2}\left[M_1^2 \tilde{\Phi}_1^2 + M_2^2 \dot{\tilde{\Phi}}_2^2 \right] + O\left(\Phi_1^3, \Phi_2^3\right) - U\left(\Phi_1^{(0)}, \Phi_2^{(0)}\right). \tag{28}$$

Regarding the optimization procedure to follow, the last term only is an unimportant constant. Variation of the resulting action and comparing it with physic publications or text books (e.g. [22], [23]) now shows us that the scalar fields have produced two massive particles with masses M_1 and M_2, while all higher order terms, being summed up in the expression $O\left(\Phi_1^3, \Phi_2^3\right)$, just contribute to the interaction between these particles. Translated into our schools transport problem these masses are or contribute to the total long term costs of the whole transportation task. Ignoring them would lead to imperfect or even wrong solutions regarding transportation routes and school starting times.

Now we correlate, within our fictive example, the parameters M_1 and M_2 to some cost factors. We assume that the immense negative effects of social jetlag will amount to only about 1,000€ per year and pupil being affected. We explicitly distinguish between school starting time 60% and transport duration 40%, because we assume the effect to the learning respectively school advertence, of course, being of no importance in the busses but only in school itself. With the numbers at hand and mentioned at the beginning we consider one hour of average social jetlag for the pupils and are now able to determine the parameters in equation (22). As we have no information about social jetlag effects at the 12-hour point, we simply assume an increase of factors two and four compared to the value at t=1 (1 hour of social jetlag) for transport duration and school starting time, respectively. In order to achieve this we could either use Fourier series expansions within our approach (22) or apply a shorter expression, containing a lower degree of freedom but more directly giving us the required functionality. Here we resort to the so-called "bump" function in the form:

$$U_k = \sum_{i=1}^{N} U_{ki} = \sum_{i=1}^{N} q_{ki}^2 \cdot \left[e^{-\frac{1}{1-\left\{\left[\left(12+t-t_{i0}-\frac{d_i}{2}\right)\cdot\frac{1}{12}\right]\right\}^2}} \right]^{n_k}. \tag{29}$$

Considering only the average chronotype (N=1) and using the boundaries discussed above one determines the following parameter sets (fig. 15):

U_1=potential for transport duration: n_1=0.131755; $(q_1)^2$=0.912663

U_2=potential for school starting time: n_2=0.26351; $(q_2)^2$=3.12358

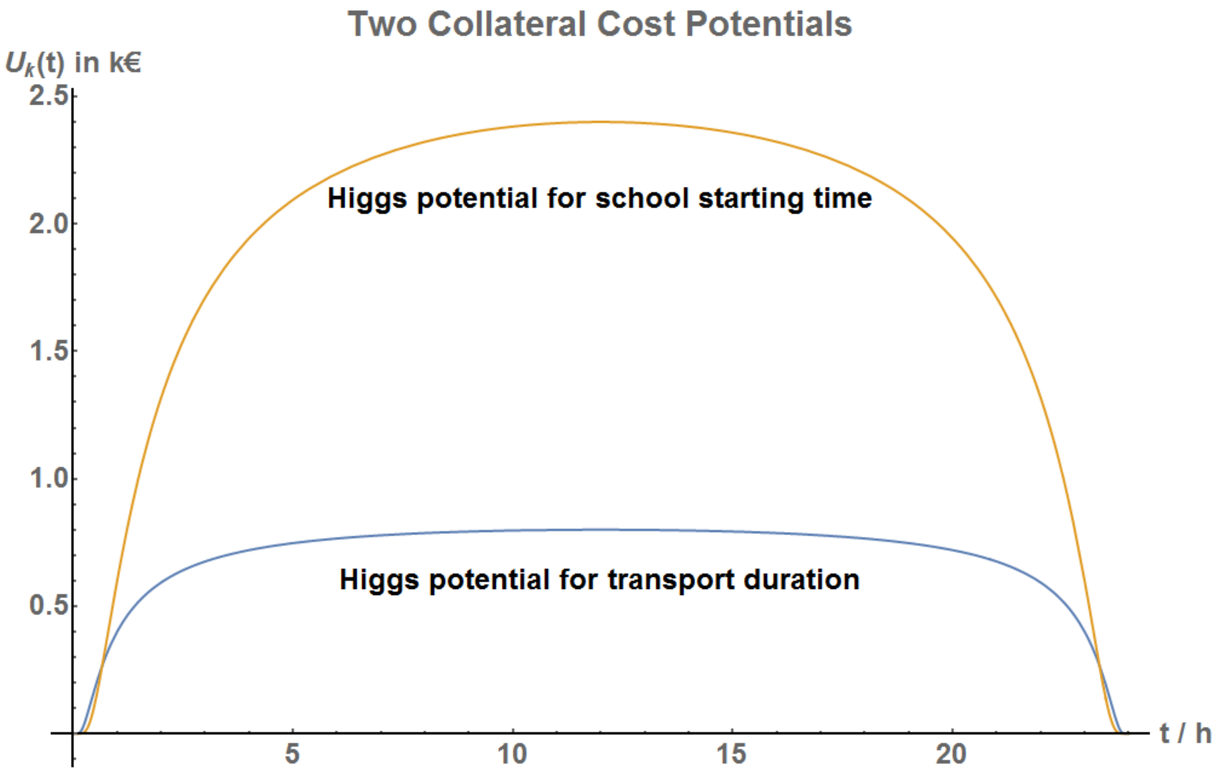

Fig. 15: Two potentials for consideration of collateral costs in a fictive school transport problem.

In should be pointed out, that in principle one has to distinguish (weigh) the various contributions of school starting time and transport duration to the total jetlag effects as a probably rather complex function. However, as we here have not enough information about this kind of effect-distribution, we simply add the two potentials up and assume a constant weighing ratio of 40/60 for transport duration to school starting time as mentioned above. This way, a badly set, rather early school starting time can be, at least partially, compensated by an effective transportation. On the other hand, somewhat longer transport times will have no or only small effects when there are suitable later school starting times. However, in such cases we are bound to also consider the rest of the day. This is, because too long transportation times combined with a late school start might become problematic regarding homework, spare time or even bedtime. This fact will be considered in section 3.3..
In order to account for the fact that school starting time, transport duration do not directly contribute to a permanent social jetlag because there are also the holidays and weekends, we divide all results by a factor of 2. This way, we treat the circadian effects rather conservatively, if not to say recessively. Still, we obtain relatively impressive results for early school starting times combined with long transportation (figures 16 and 17). It has to be noted, that it does not make much difference whether or whether not the chronotype [34] distribution with its pronounced asymmetry favoring the owl-types (figure 18). As the figures 19 to 22 show, this difference is only significant around the collateral costs minimum.
It is important to point out that due to the chronotype distribution as illustrated in figure 18 as well as the averaging over various ages of pupils there is never an optimum for all pupil (figures 17, 19 to 22 with their minima not being zero). Some are always going to suffer under individual imperfect school starting times or transportation duration. This however, should not hinder politicians and bureaucrats to seek for the optimum with minimal collateral damage

and negative health effects with respect to the circadian rhythm. In fact, as the effect of averaging over various ages (figure 22 with higher minimum) is much more pronounced than the averaging over the chronotypes alone (figure 21 with lower minimum), officials should consider school starting time scheduling in dependence of these ages.

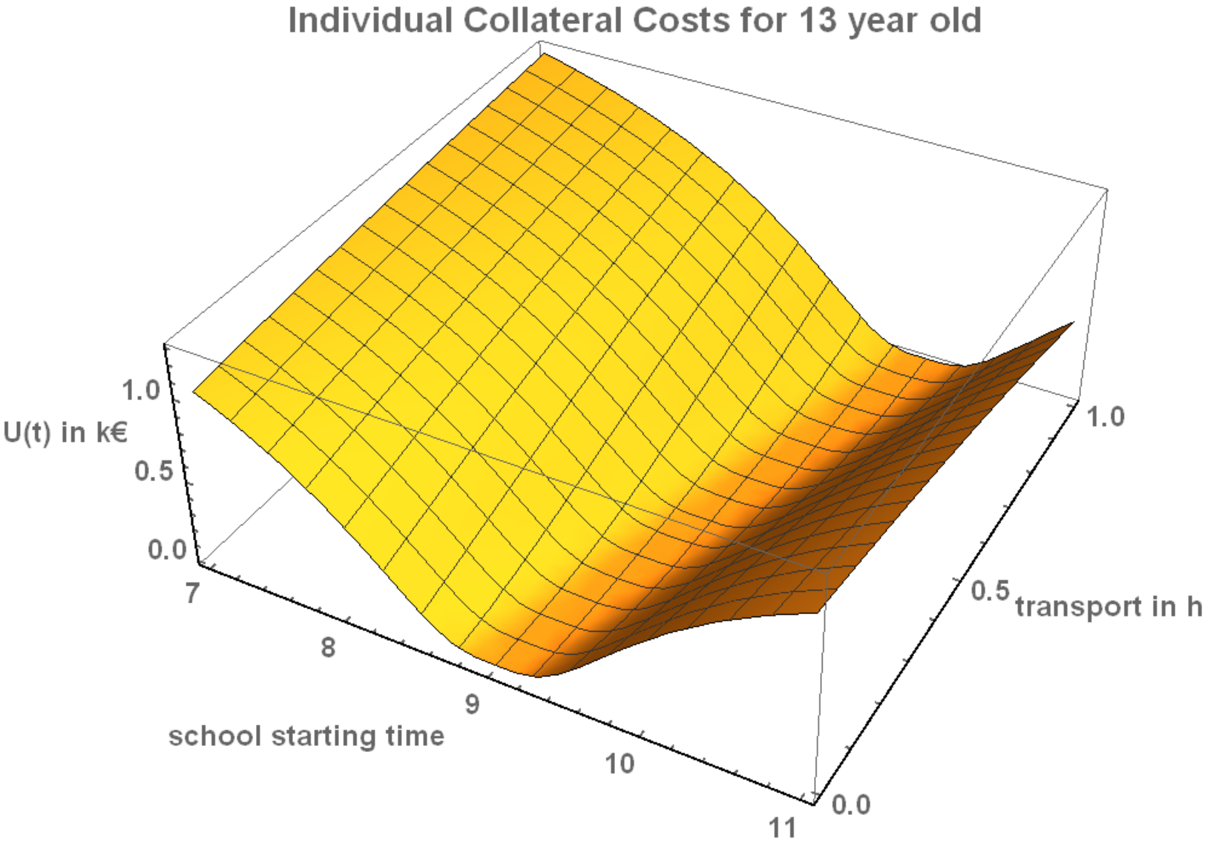

Fig. 16: Individual collateral costs for a 13 year old “average chronotype” child.

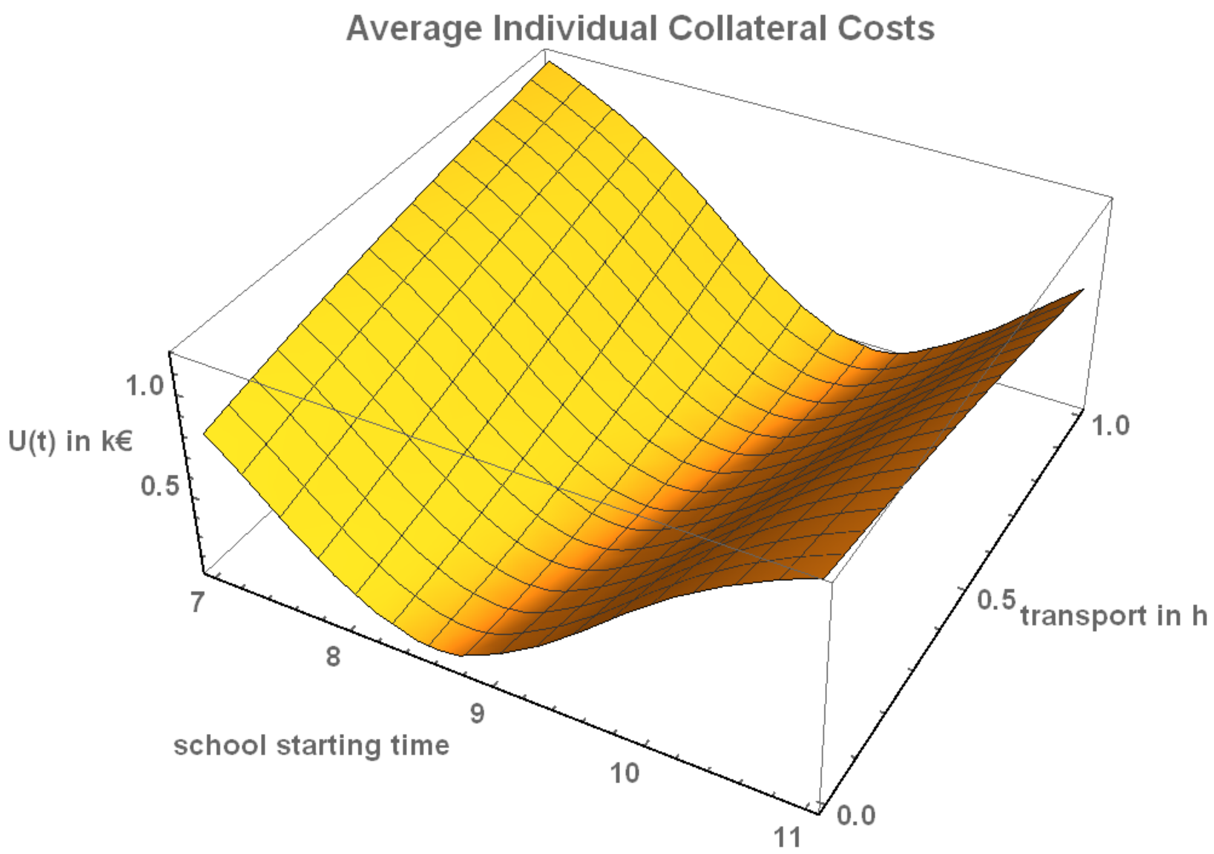

Fig. 17: Individual collateral costs for pupils averaged between 6 to 16 years.

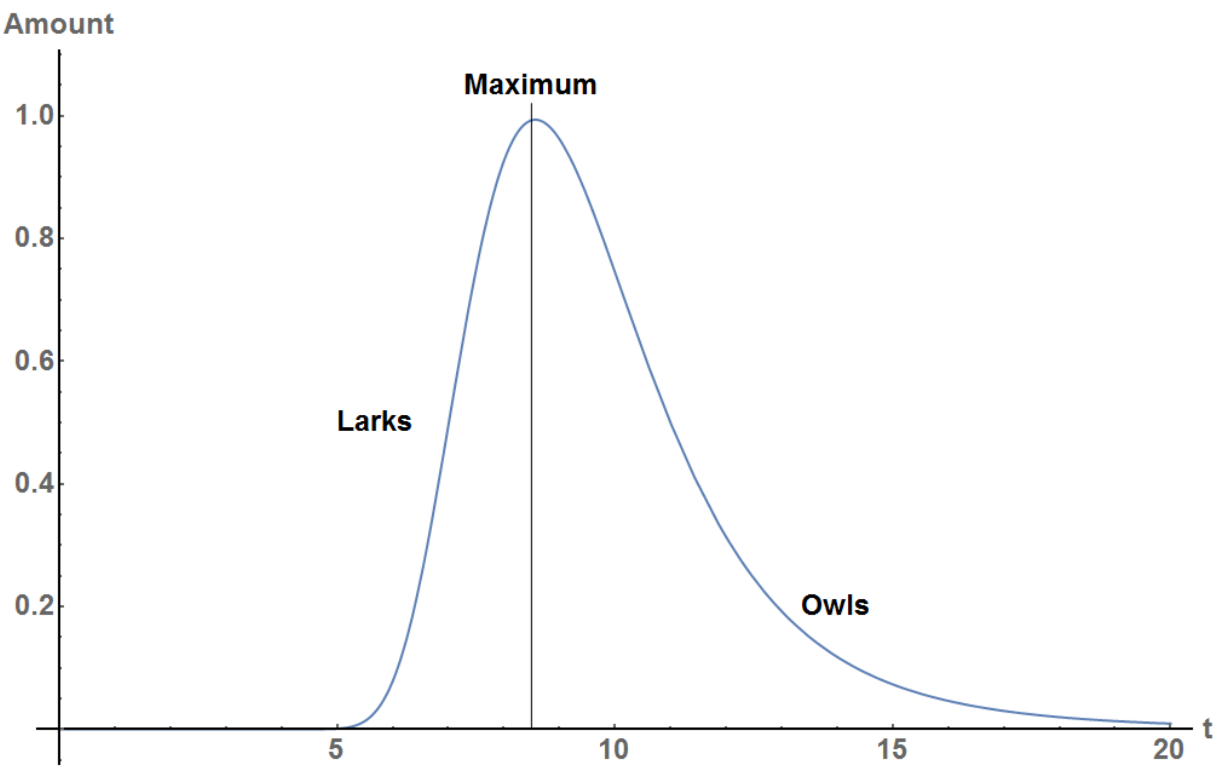

Fig. 18: Chronotype distribution schematically after [34]. Natural circadian wake-up time distribution for 15 year old children.

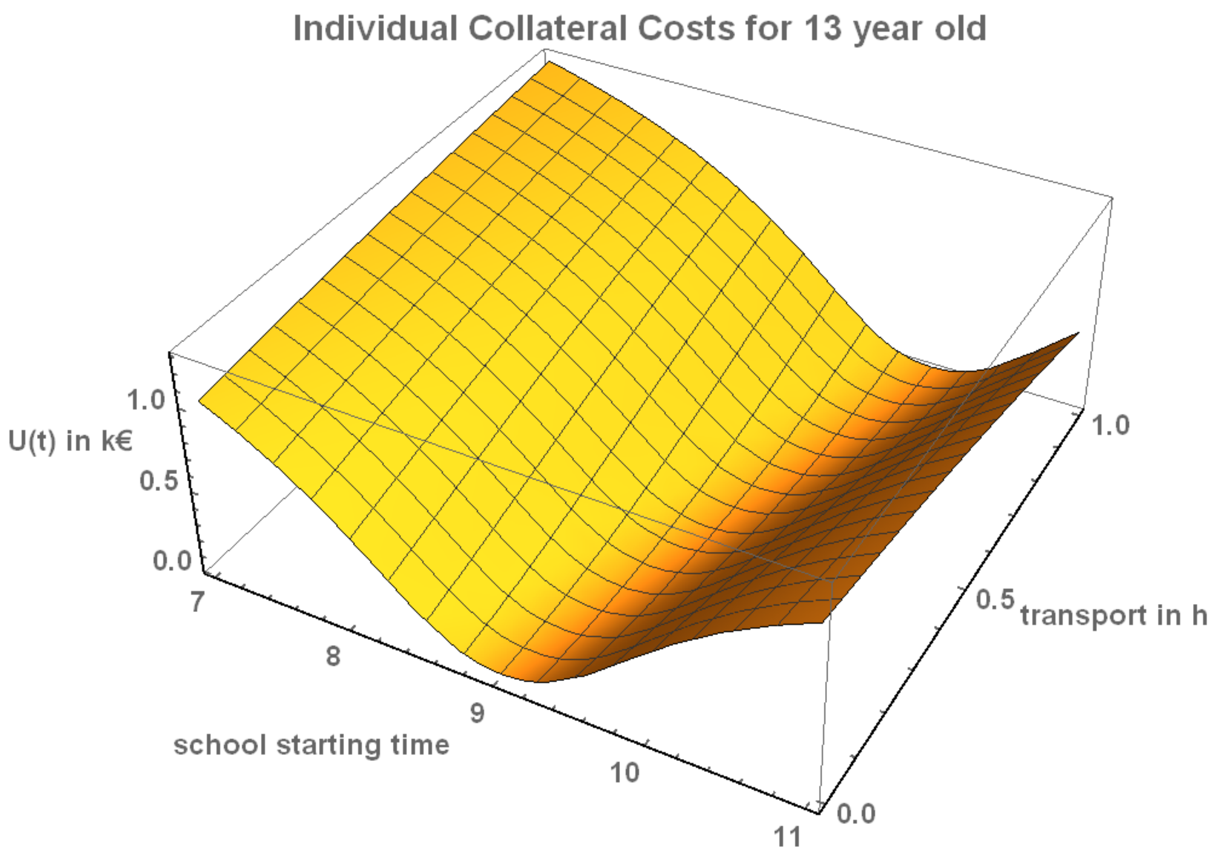

Fig. 19: Individual collateral costs for a 13 year old "average chronotype" child. Total chronotype distribution after [34] taken into account.

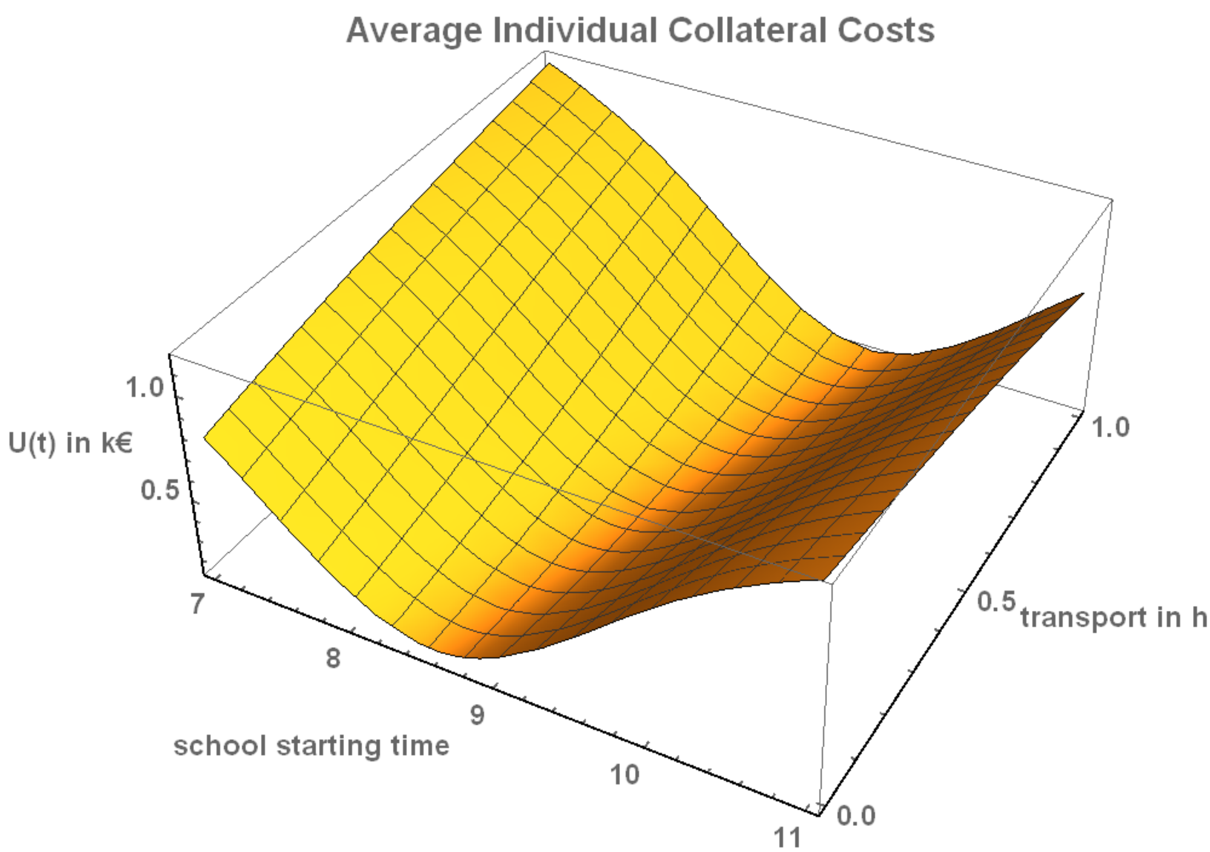

Fig. 20: Individual collateral costs for pupils averaged between 6 to 16 years. Total chronotype distribution after [34] taken into account.

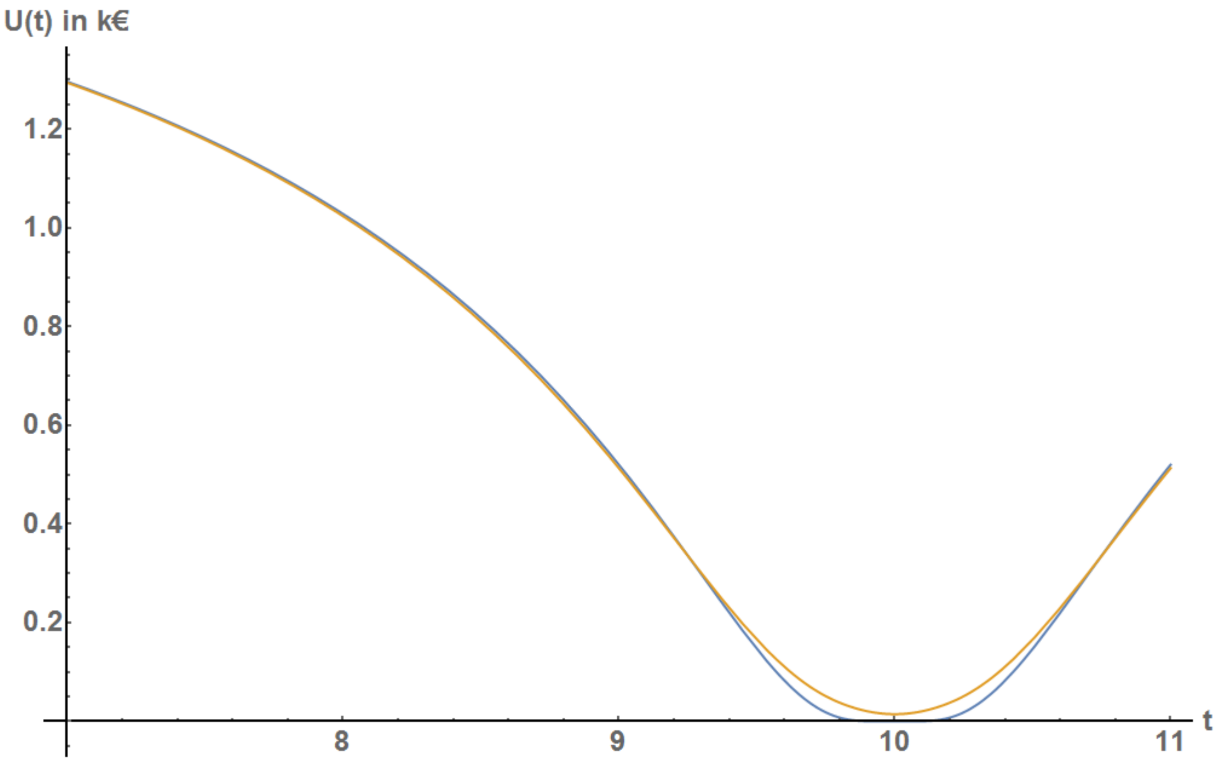

Fig. 21: Individual collateral costs for a 13 year old “average chronotype” child for transport duration of 1 hour and a preparation time of 45 minutes. Difference if total chronotype distribution after [34] taken into account (brown line) or not (blue line).

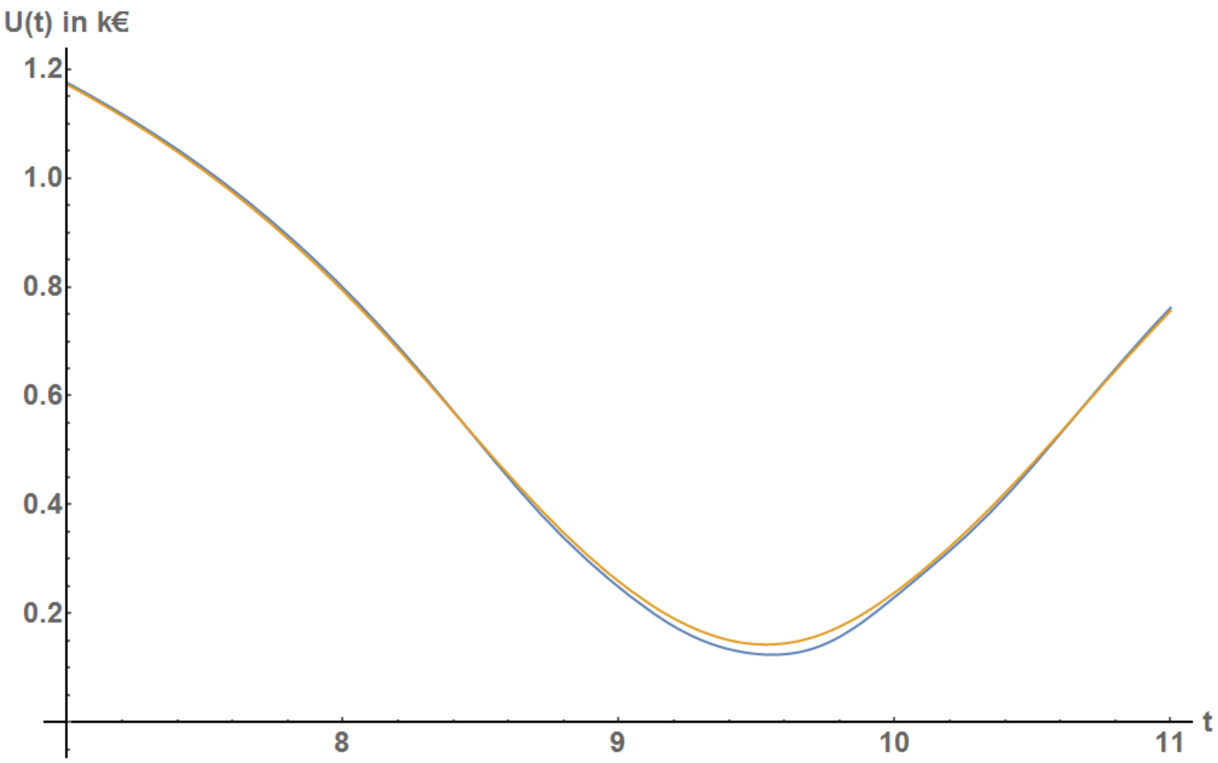

Fig. 22: Individual collateral costs for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Difference if total Chronotype distribution after [34] taken into account (brown line) or not (blue line).

With respect to the total amount of collateral damage (costs) we need to evaluate the number of pupils being affected by the social jetlag in order to obtain an estimate for the accumulated costs. For this, we assume a constant permanent social jetlag for all pupils caused by an average of 1 hour transportation due to inefficient or imperfect organization (according to figure 8a) plus improper school starting time. This is leading to a quite significant total jetlag (something which is for instance easily realized, partially even significantly exceeded in the district of Ruegen in Mecklenburg-Vorpommern, where the author started this study). We perform our practical consideration with a total number of about 10,000 pupils subjected to this treatment. In order to make our consideration holistic we have to take into account that not only the current pupils will suffer but also those who are leaving school after the regular school time is over. Thus, the output of affected pupils will have doubled after one typical pupil's generation of 10 years increasing the total (figure 23) and subsequently also the accumulated total damage (figure 24).
With the parameters and models developed above, we evaluate the total collateral damage and couple it back into the transport optimization problem via the Higgs field mechanism as described above.

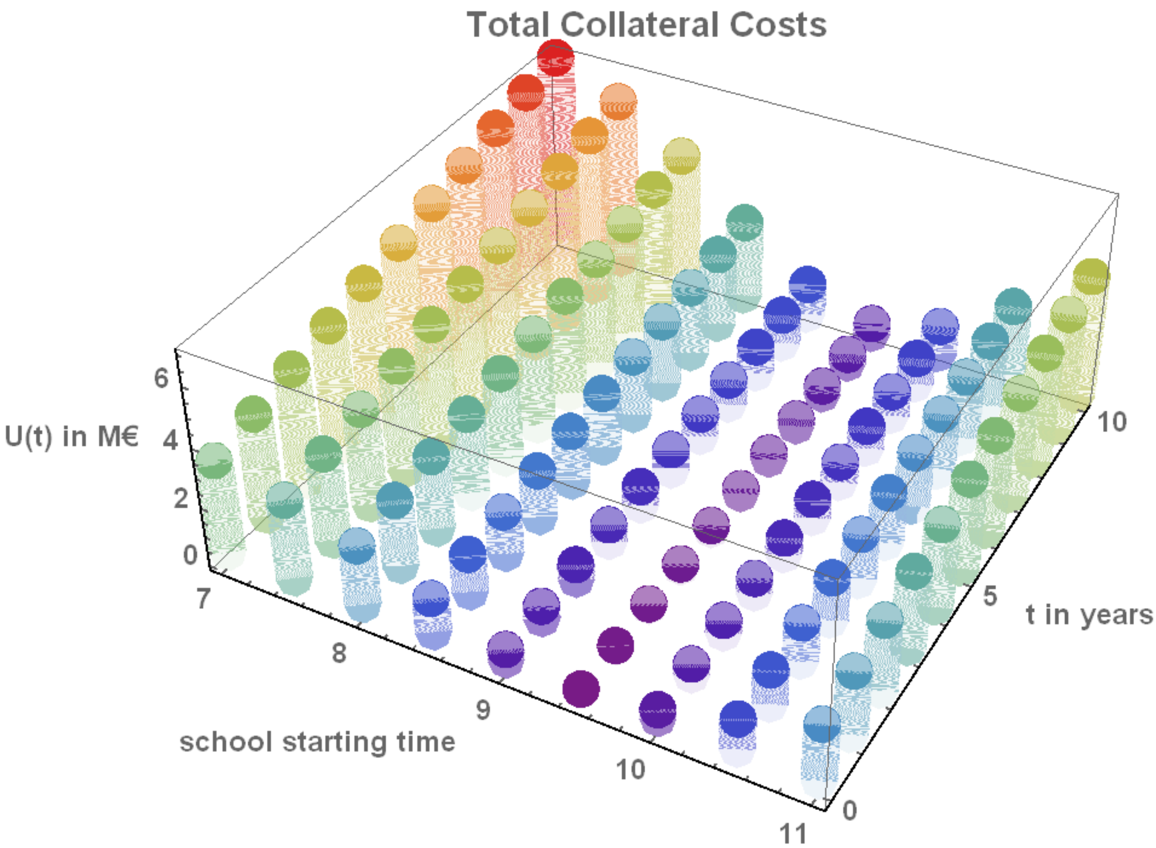

Fig. 23: Total collateral costs per year for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Total chronotype distribution after [34] was taken into account.

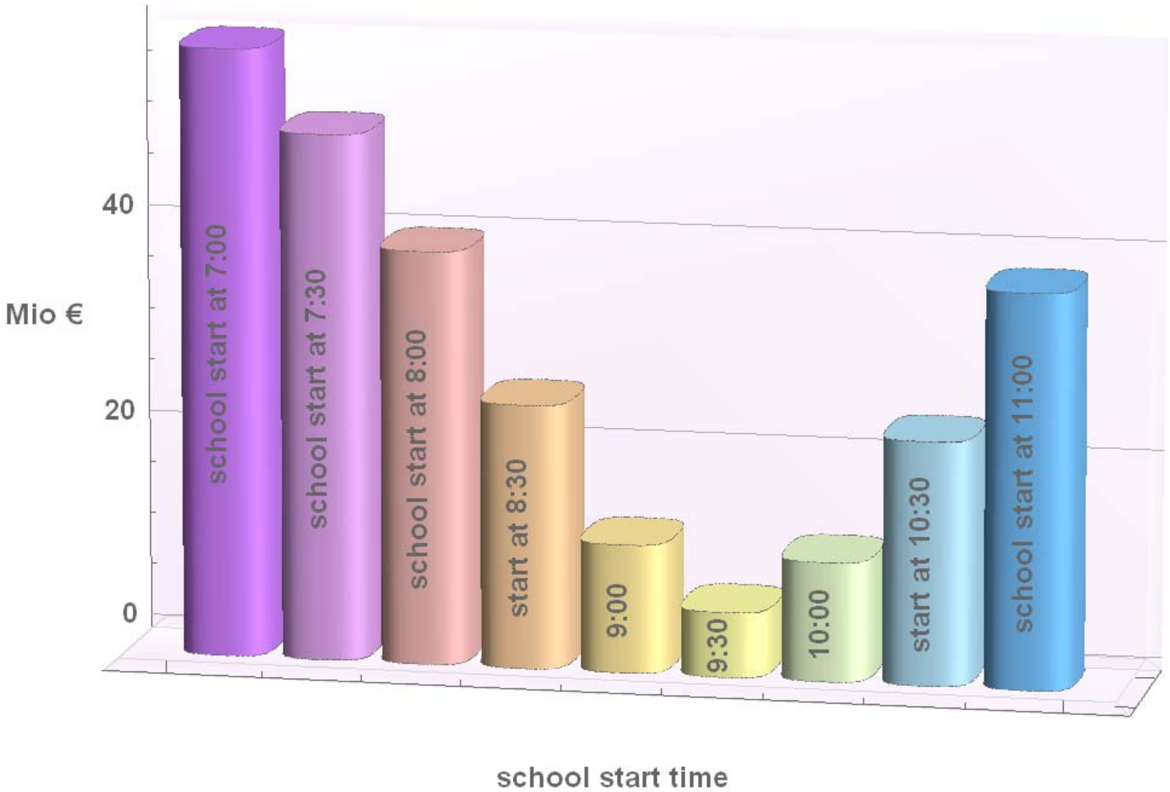

Fig. 24: Total collateral costs after one pupil-generation (10 years) for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Total chronotype distribution after [34] was taken into account.

We emphasize again that our consideration has to be considered fictive, because some of the parameters used are only estimates. Also the Higgs-potential being applied must be considered artificial because we only adjusted it to a limited number of social jetlag data points. It should also be pointed out that the positive definiteness of our current model always produces mass even when in principle no collateral damage is to be expected. A simple offset would compensate for this. However, this would require better knowledge about the true damage effects and therefore is been omitted within this section.

3.2. Applying the activity diagram as Higgs Potential

In a next step and in order to overcome the pronounced fictive character of our current model, we now introduce a new potential we will base on the activity diagram. The diagram is shown in figure 25.

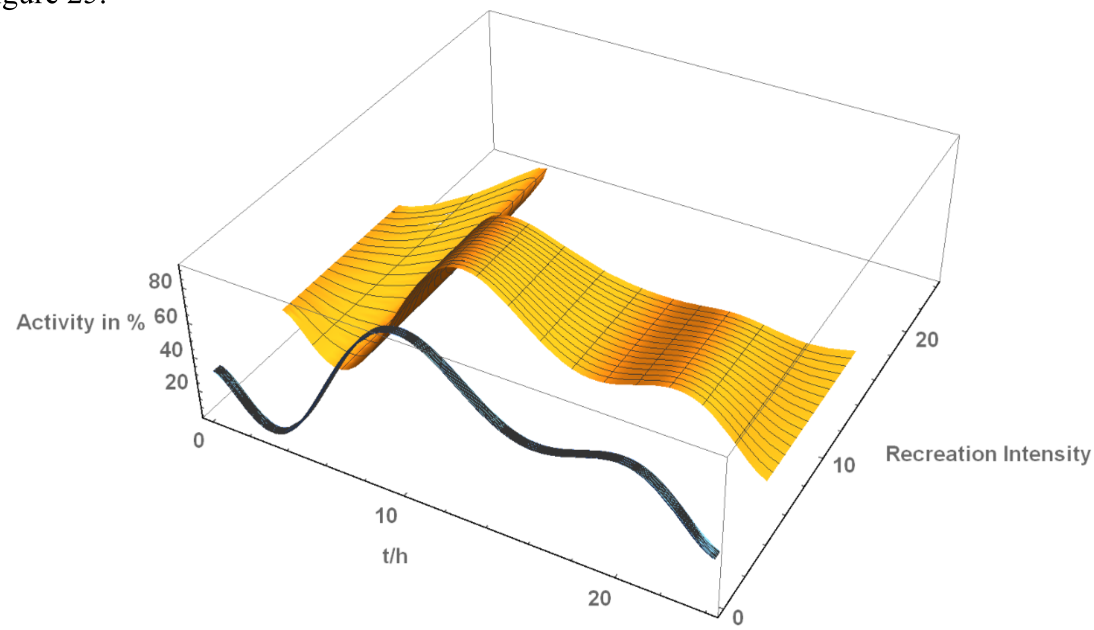

Fig. 25: Activity diagram. The width of the surface shall illustrate the importance of certain phases with respect to needs regarding recreation, long term memory built up, brain development, link-up and so on.

In order to work with this as an ordinary one-dimensional potential function, we reconstruct the classical one-dimensional activity function fc[t] (c.f. thick dark line in fig. 25) in the following form:

$$f(t) = M * \left(100 - fc\left[t + t_{min} - t_0 + p * \sin\left[t * \pi / 12\right]\right]\right)^m . \tag{30}$$

Here M, p and m are adjustable parameters, while t_{min} and t_0 are giving the minimum of fc[t] and the midpoint of sleep for the chronotype being considered. With this function, arbitrary chronotypes with the individual midpoint of sleep and sleep length can be considered. An example is given in figure 26.
The activity diagram function fc[t] was approximated via sin- and cos-functions as follows:

$$\begin{aligned}
\mathrm{fc}[t] = & -1.17257\cdot\sin\left(\frac{\pi t}{2}\right)+1.66303\cdot\sin\left(\frac{\pi t}{3}\right)-1.05586\cdot\sin\left(\frac{2\pi t}{3}\right)\\
& -0.786704\cdot\sin\left(\frac{\pi t}{4}\right)-21.4491\cdot\sin\left(\frac{\pi t}{6}\right)-6.55312\cdot\sin\left(\frac{\pi t}{12}\right)\\
& +1.98088\cdot\sin\left(\frac{5\pi t}{12}\right)+1.13119\cdot\sin\left(\frac{7\pi t}{12}\right)-0.233447\cdot\cos\left(\frac{\pi t}{2}\right)\\
& +2.34023\cdot\cos\left(\frac{\pi t}{3}\right)-0.0707073\cdot\cos\left(\frac{2\pi t}{3}\right)+0.69798\cdot\cos\left(\frac{\pi t}{4}\right)\\
& +0.772338\cdot\cos\left(\frac{\pi t}{6}\right)-26.4989\cdot\cos\left(\frac{\pi t}{12}\right)-0.883034\cdot\cos\left(\frac{5\pi t}{12}\right)\\
& +0.666839\cdot\cos\left(\frac{7\pi t}{12}\right)+53.1731
\end{aligned} \quad . \qquad (31)$$

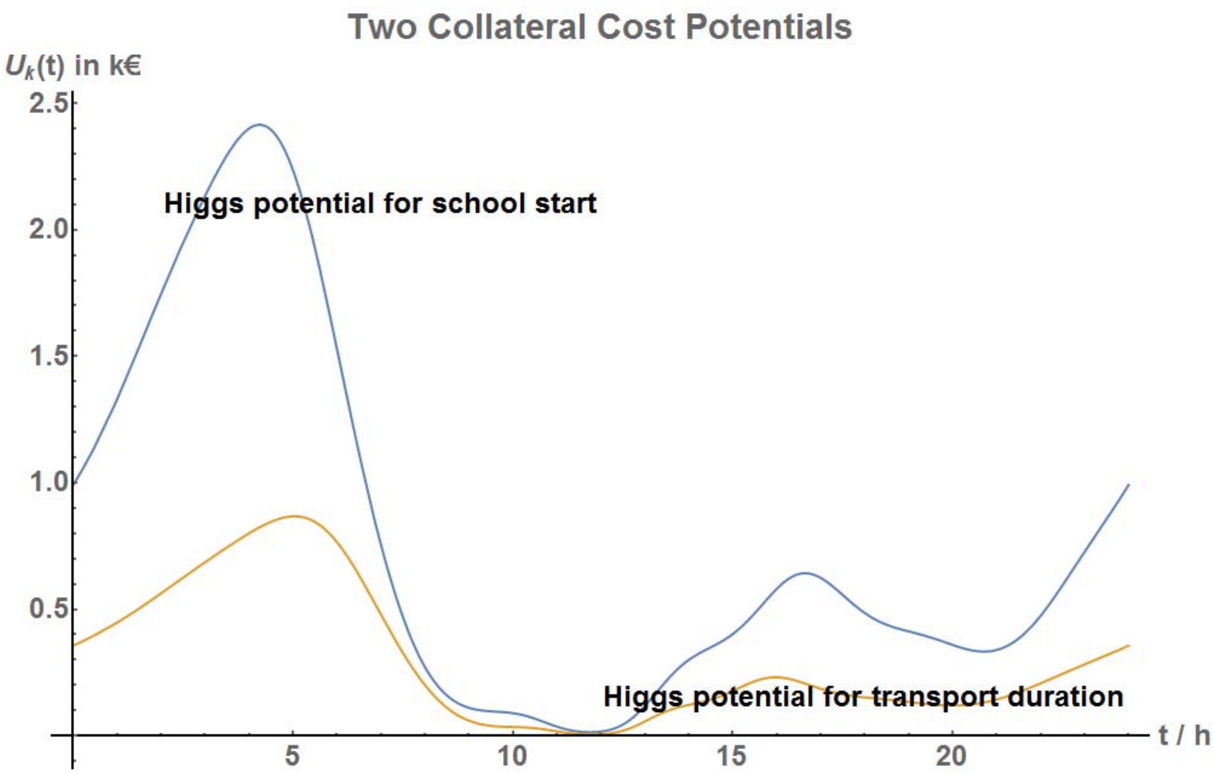

Fig. 26: Fictive collateral cost potentials derived from the activity diagram

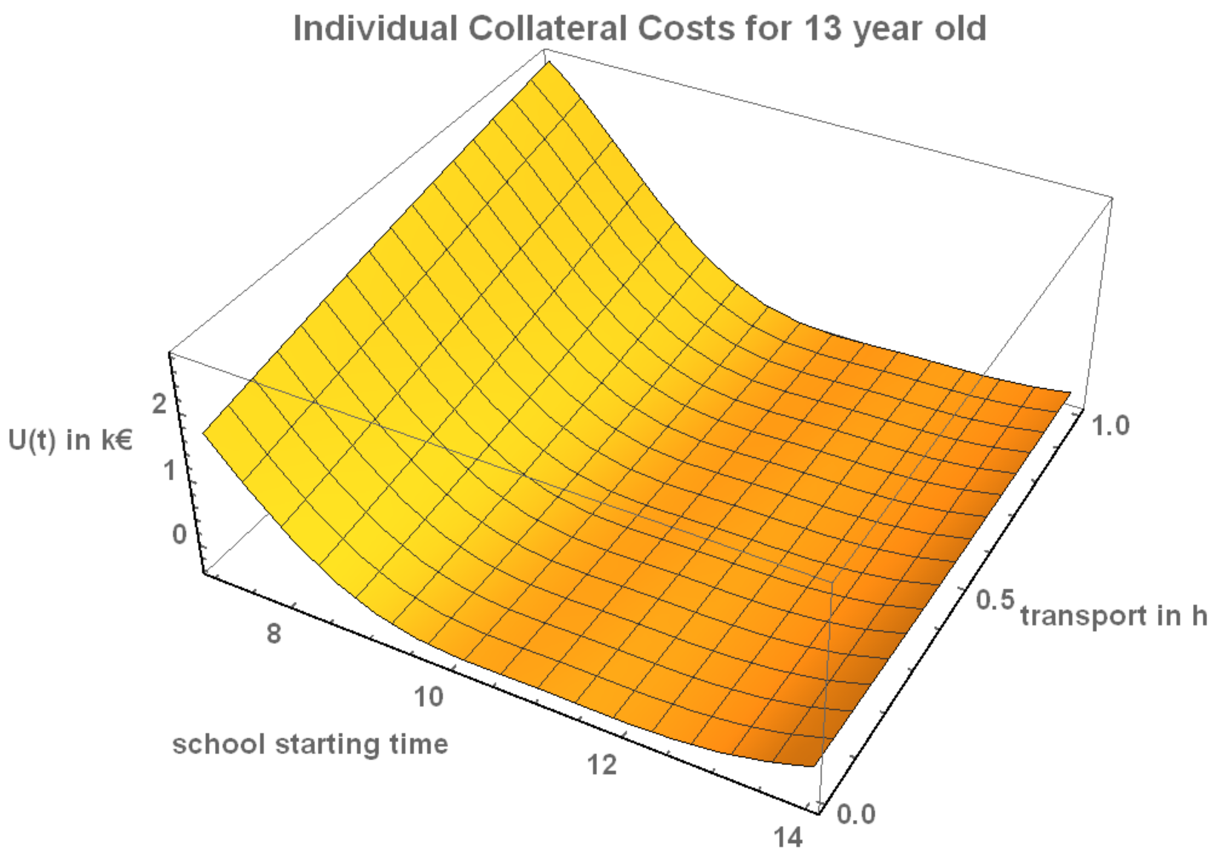

Fig. 27: Individual collateral costs for a 13 year old "average chronotype" child.

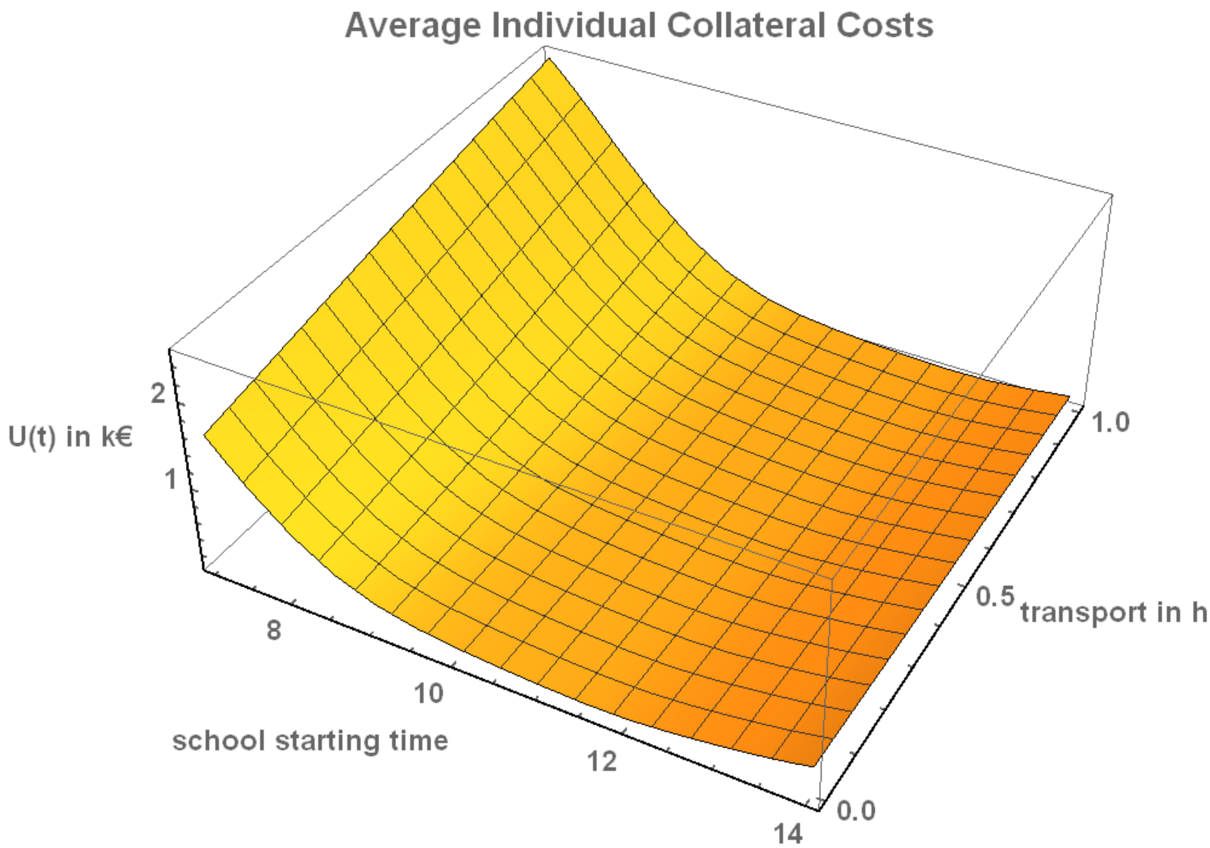

Fig. 28: Individual collateral costs for pupils averaged between 6 to 18 (not just 16 as before) years.

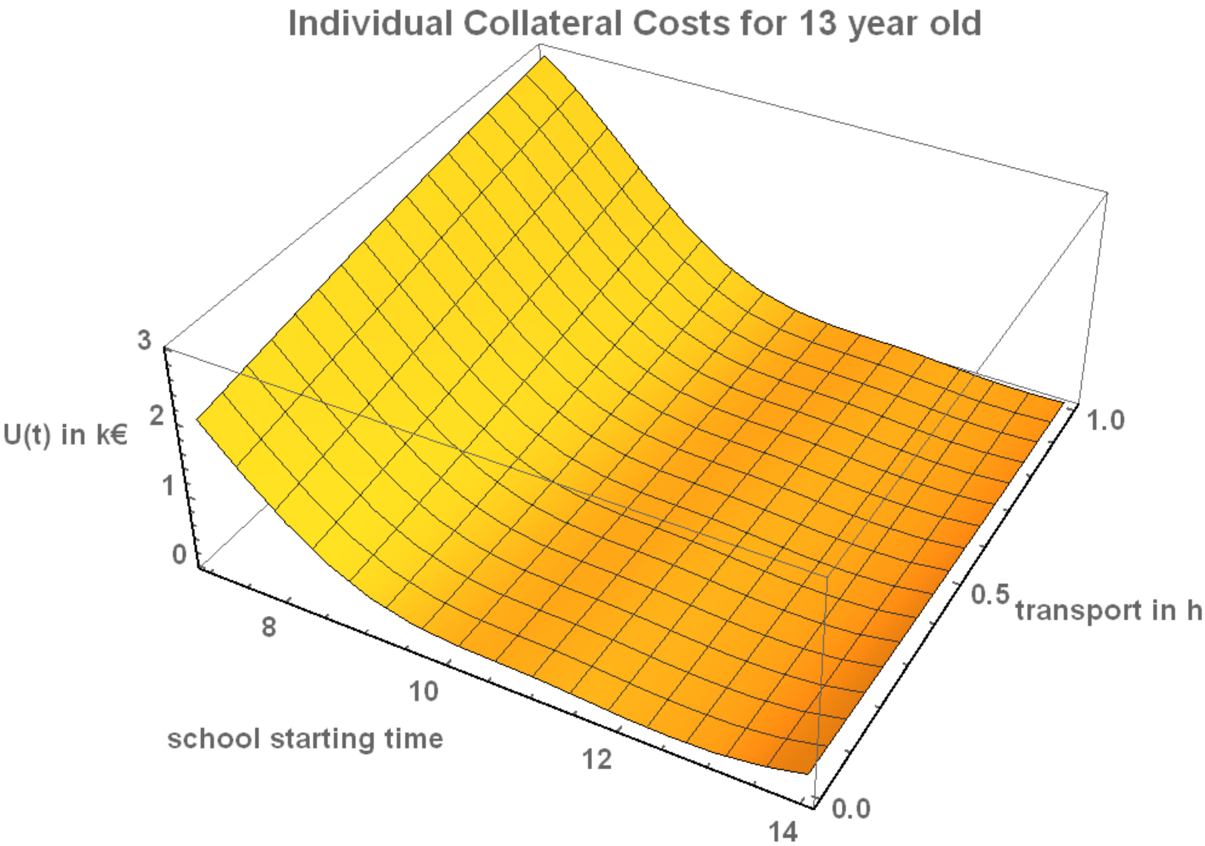

Fig. 29: Individual collateral costs for a 13 year old "average chronotype" child. Total chronotype distribution after [34] taken into account.

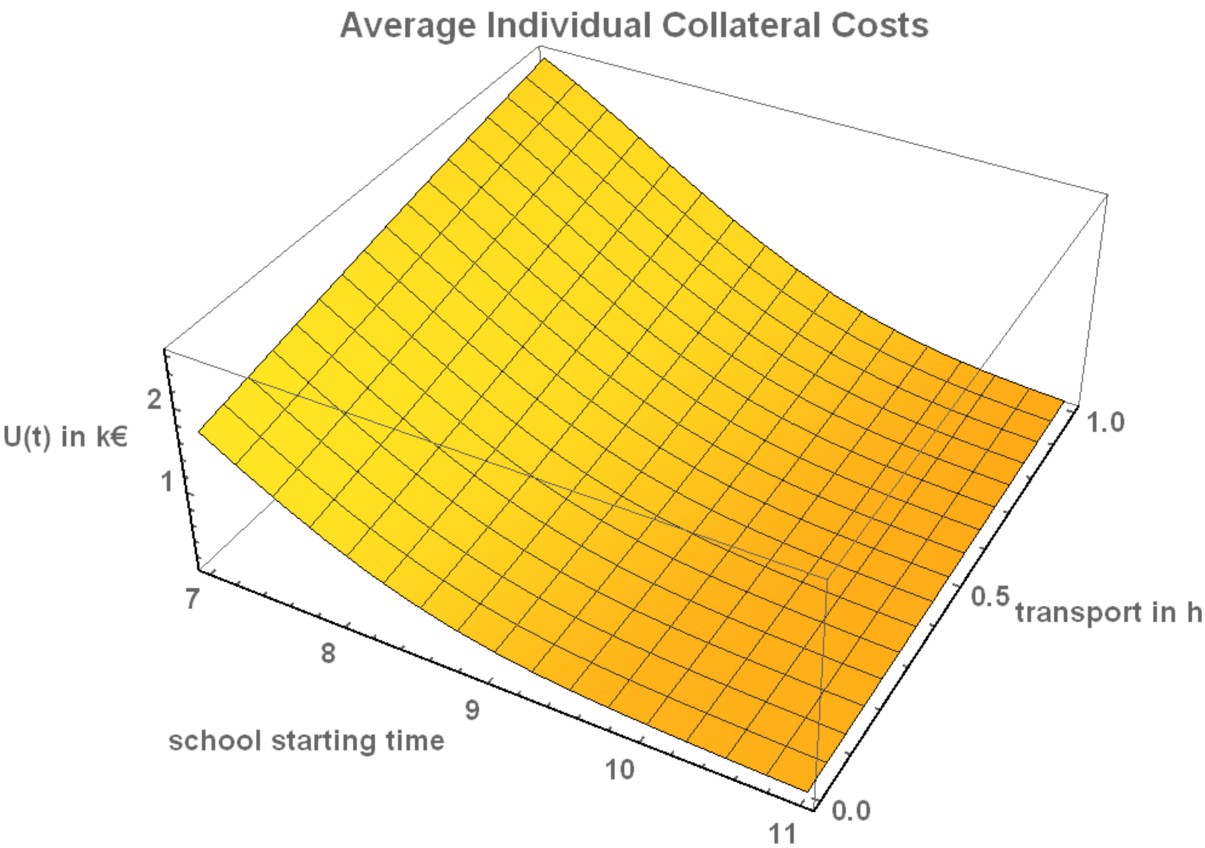

Fig. 30: Individual collateral costs for pupils averaged between 6 to 18 years. Total chronotype distribution after [34] taken into account.

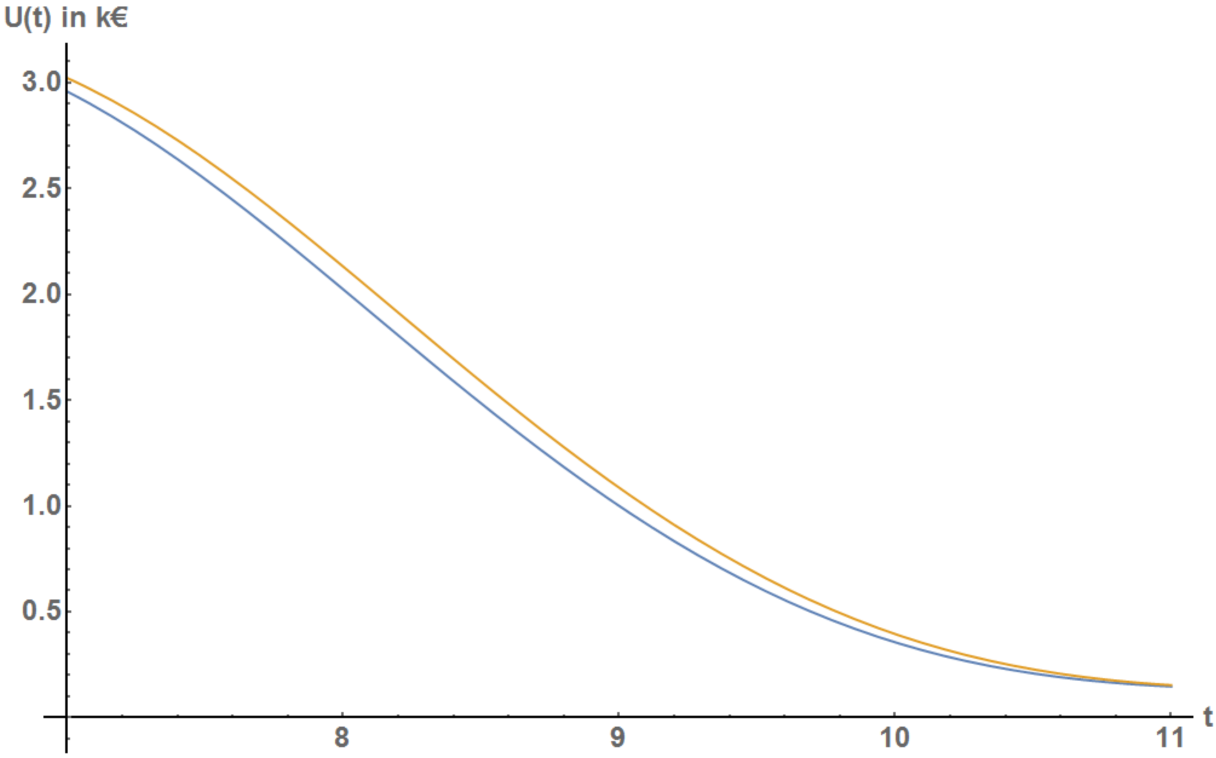

Fig. 31: Individual collateral costs for a 13 year old "average chronotype" child for transport duration of 1 hour and a preparation time of 45 minutes. Difference if total chronotype distribution after [34] taken into account (brown line) or not (blue line).

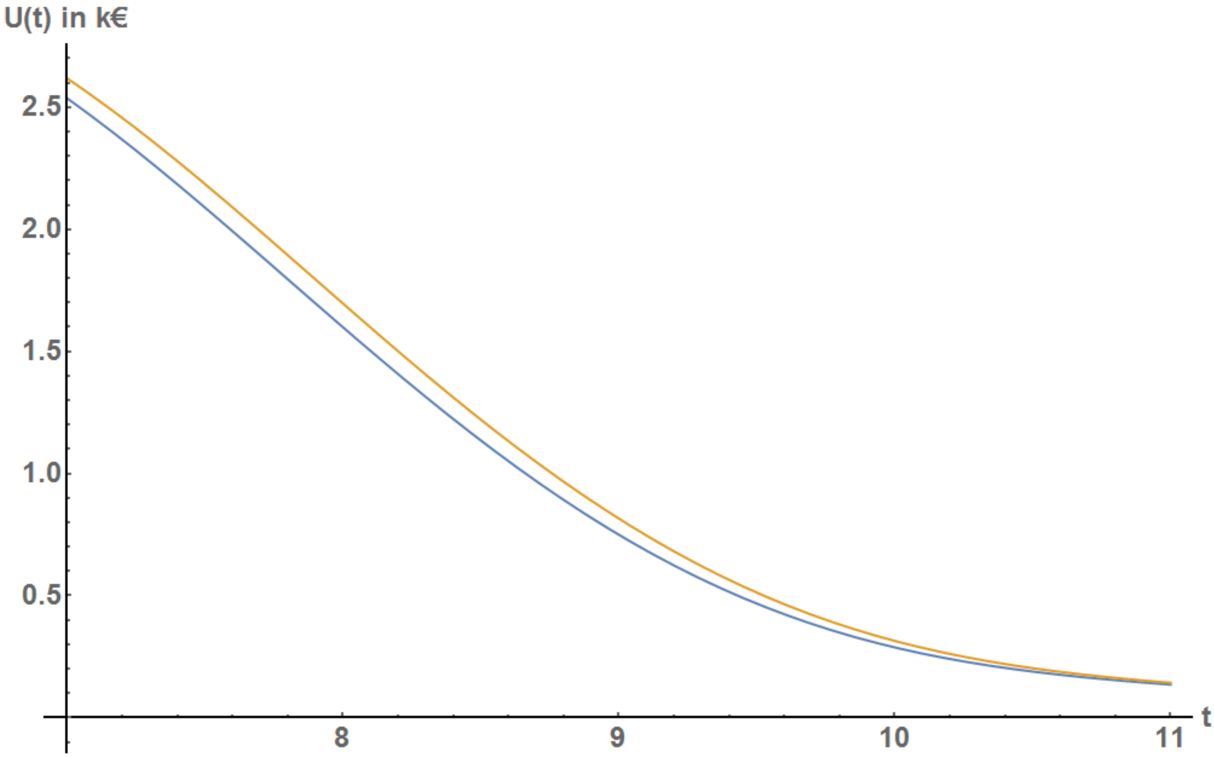

Fig. 32: Individual collateral costs for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Difference if total chronotype distribution after [34] taken into account (brown line) or not (blue line).

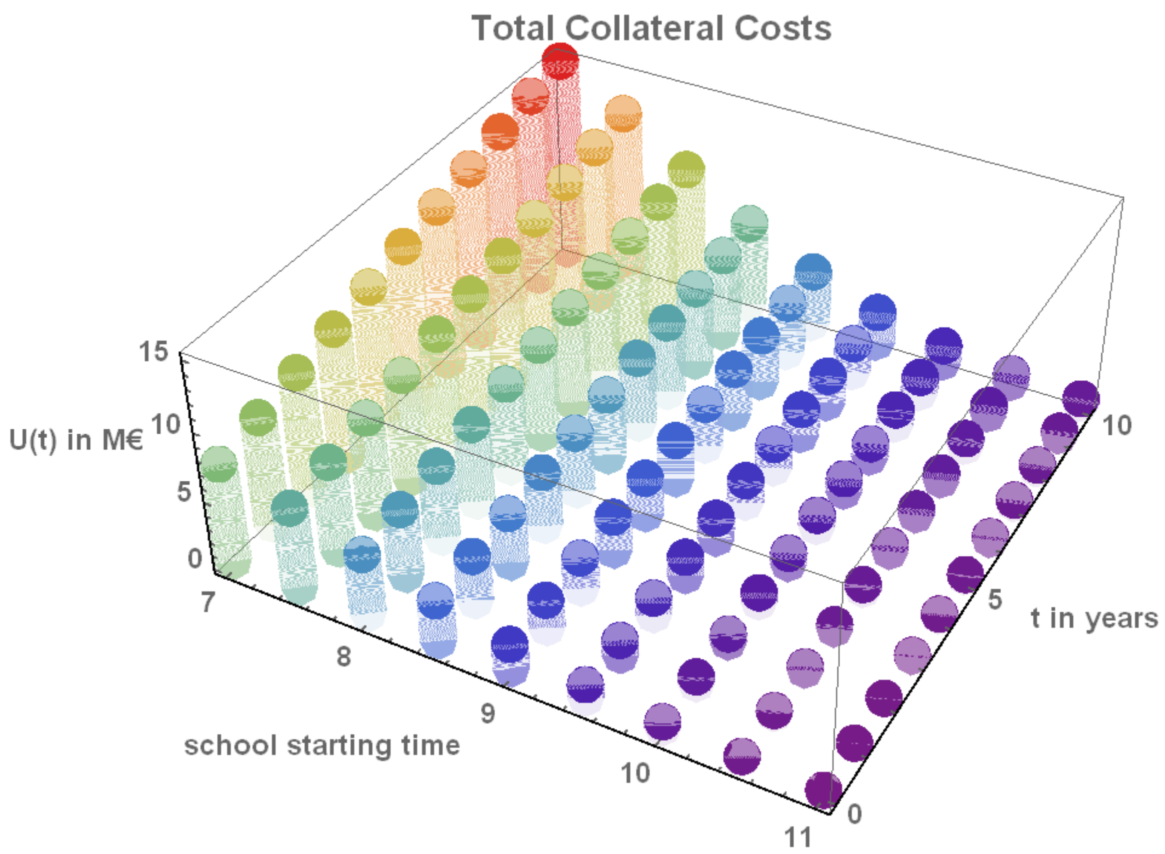

Fig. 33: Total collateral costs per year for pupils averaged between 6 to 18 years for transport duration of 1 hour and a preparation time of 45 minutes. Total Chronotype distribution after [34] was taken into account.

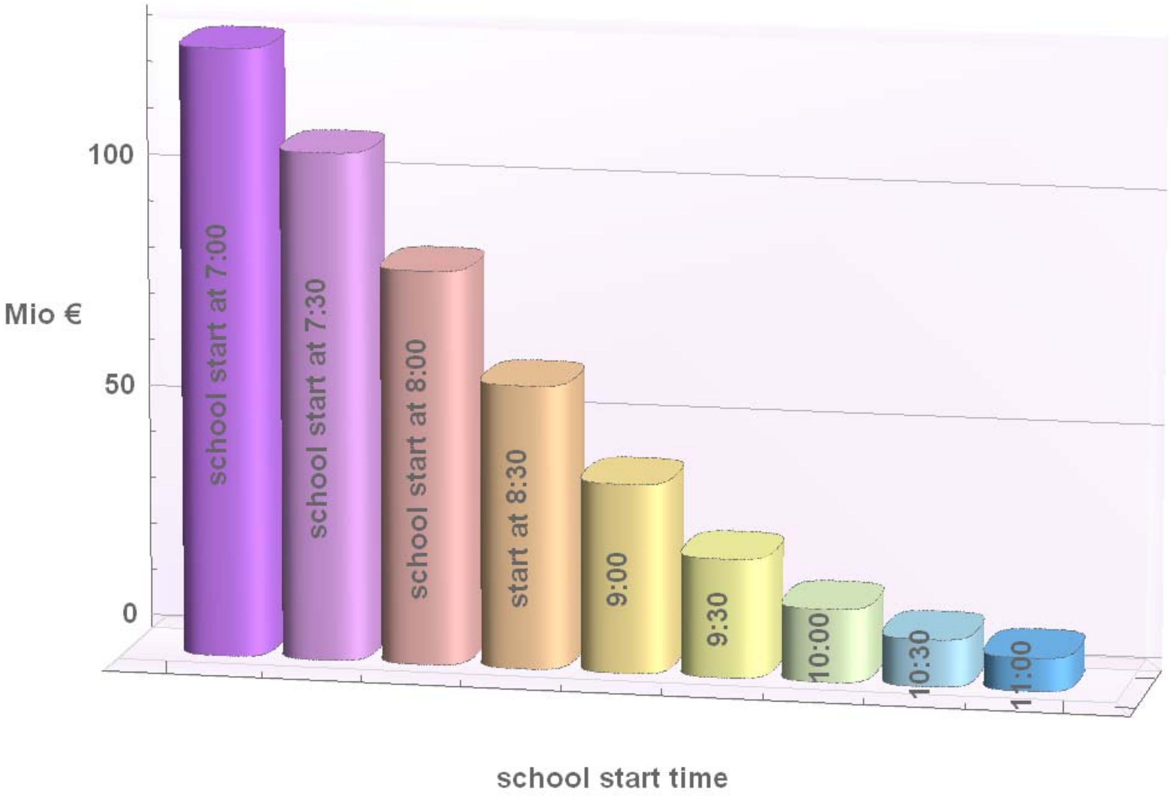

Fig. 34: Total collateral costs after one pupil-generation (10 years) for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Total chronotype distribution after [34] was taken into account. Hint: an offset should be applied in order to account the principle positive effect of education (proper education!).

As in the section before we have evaluated all relevant figures with respect to the principle collateral damage. Again, we point out that our results have to be considered fictive and need to be adjusted regarding a suitable offset.

3.3. Applying the Activity Diagram as potential but also taking "the rest of the day" into account

By considering the results in the section above one immediately realizes, that by only concentrating on the time where the children have to get up any potential clashes with the "rest of the day" cannot be taken into account. These clashes might be due to long school and transportation times, subsequence lack of spare time activity or even homework. That is why we simply add a potential where we integrate over such daytime activities. This way we not only take their different demands (e.g. with respect to attentiveness) into account but can also detect a potentially too long day for the children possibly cutting them of some early night sleeping time and leading to another social jetlag with just a different sign. Again, we do built this integrated potential on the activity diagram. Figure 35 shows 3 such potentials with the areas filled over which it has to be integrated.

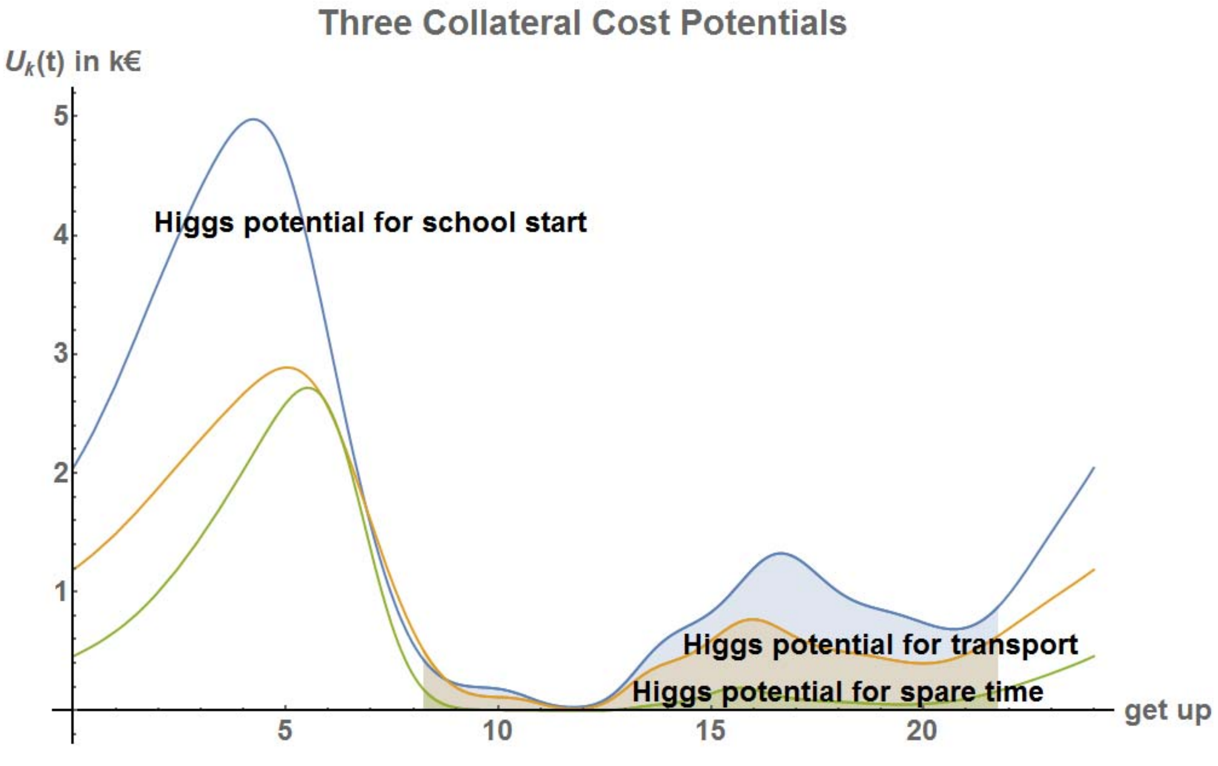

Fig. 35: Fictive collateral cost potentials derived from the activity diagram. The filled regions are showing areas over which it has to be integrated in order to take the daily work part into account and detect potentially too long or too exhausting days.

Now again, we evaluate some results for this new and more holistic potential function for some fictive yet reasonably estimated parameter sets.
This time however, we avoid the positive definiteness by introducing a certain setoff. In essence, this offset is nothing else but a parameter accounting for the fact that in principle it is to be considered positive if children are educated at all. As this parameter among all others

has here been chosen in an arbitrary manner, its absolute value is not to be considered as practically relevant but fictive.

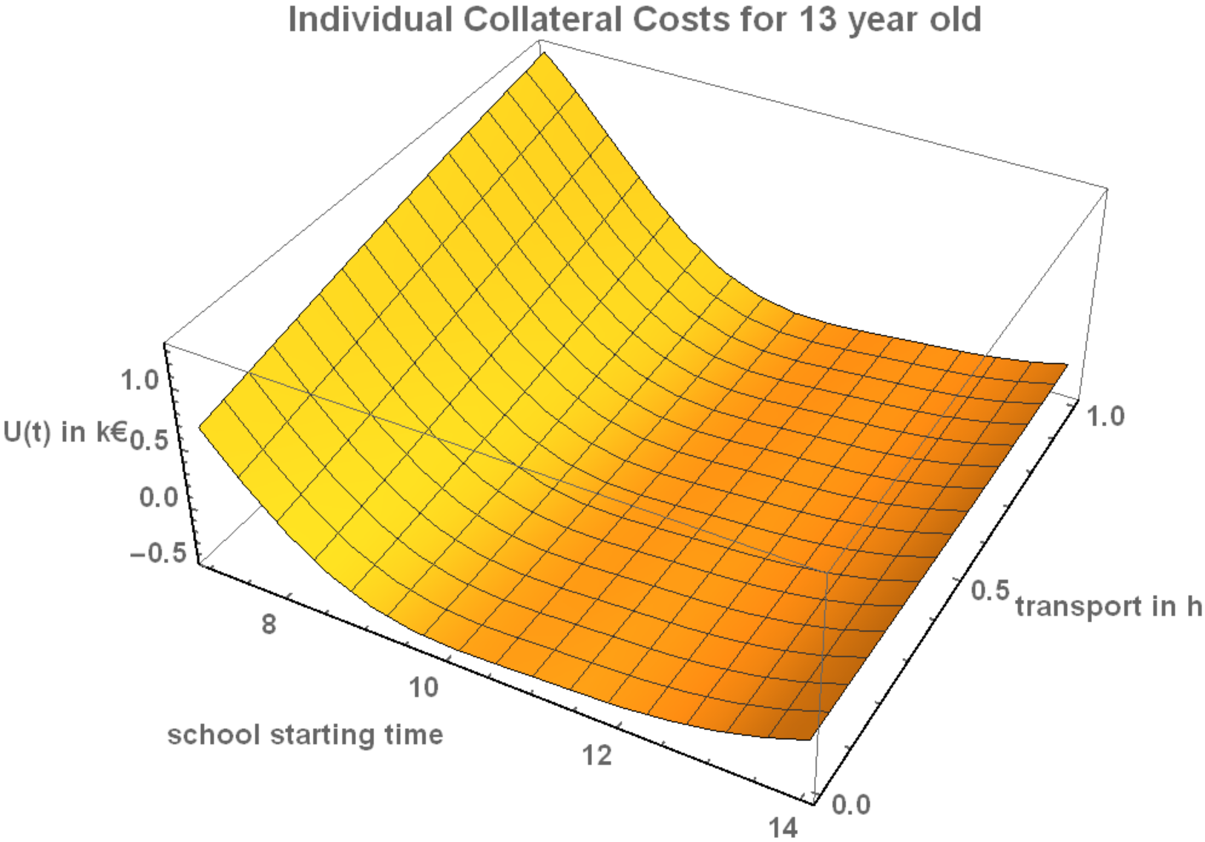

Fig. 36: Individual collateral costs for a 13 year old "average chronotype" child.

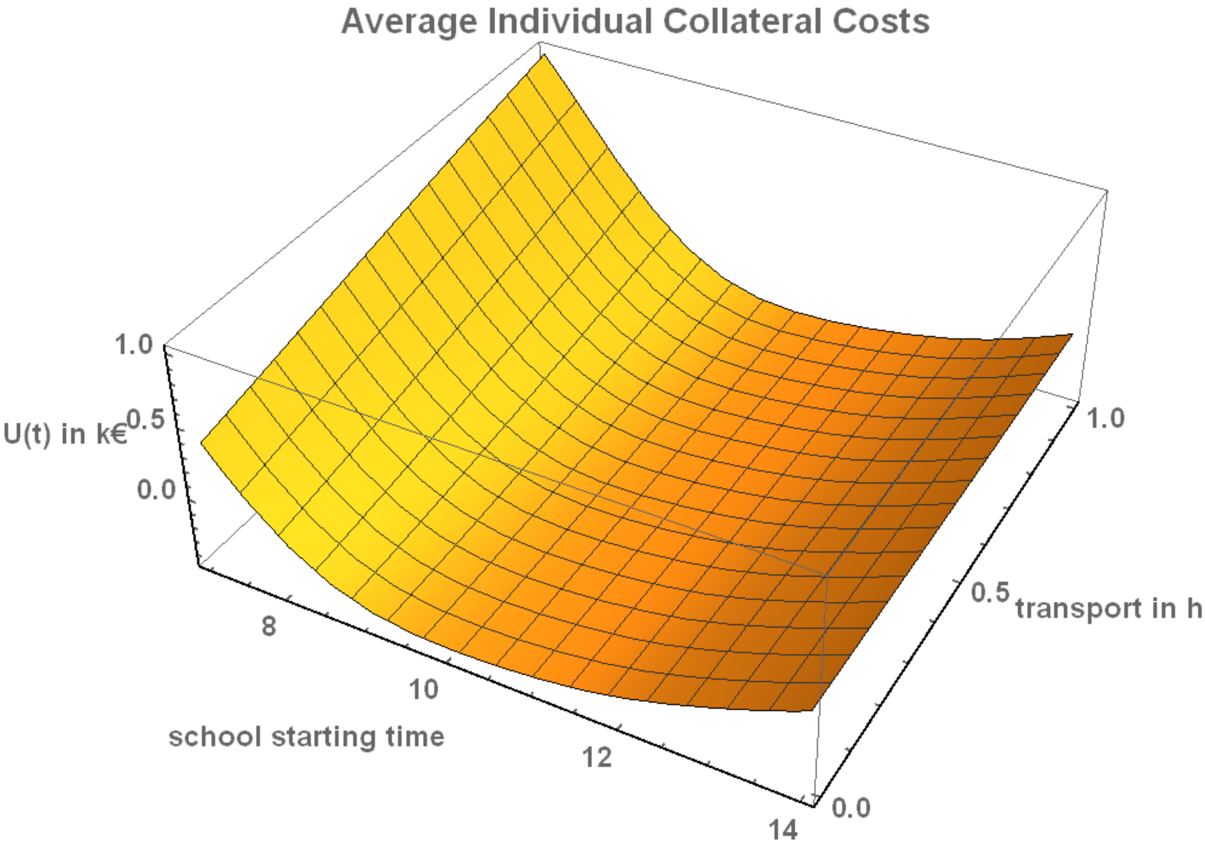

Fig. 37: Individual collateral costs for pupils averaged between 6 to 18 years.

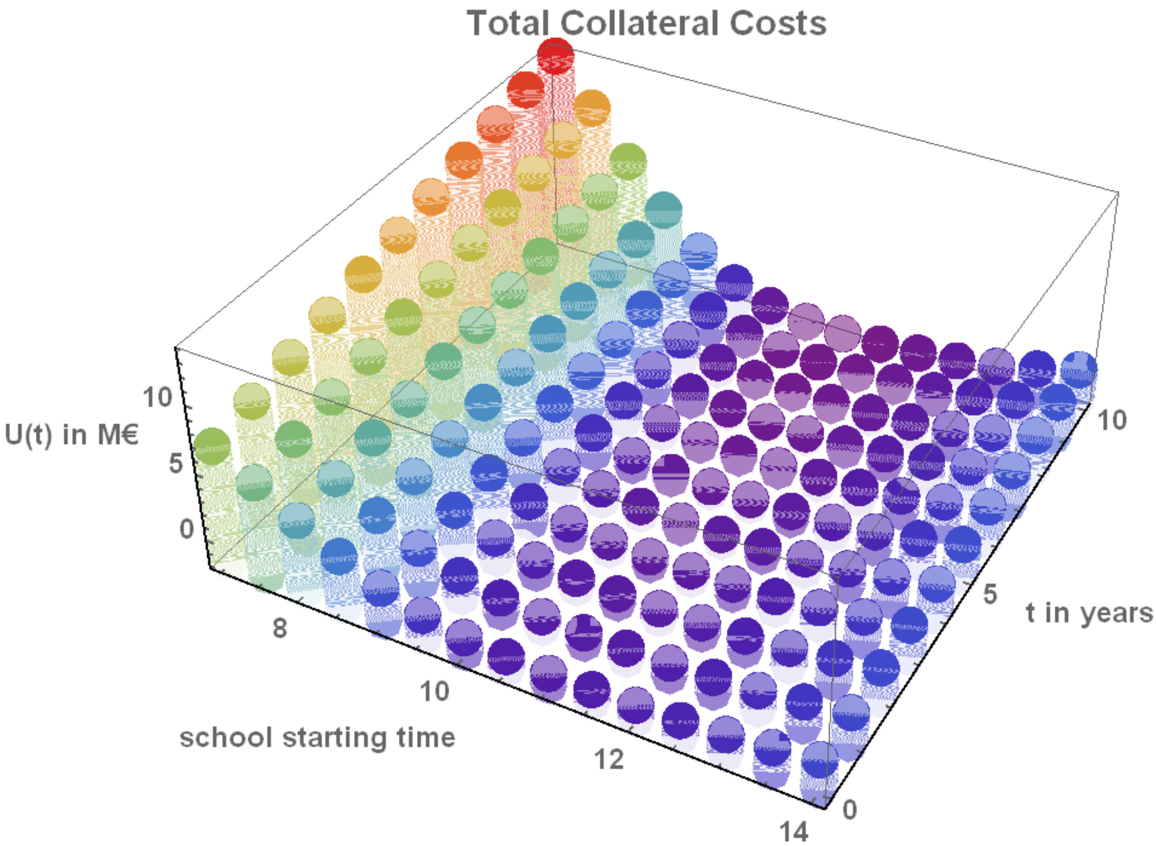

Fig. 38: Total collateral costs per year for pupils averaged between 6 to 18 years for transport duration of 1 hour and a preparation time of 45 minutes. Total chronotype distribution after [34] was taken into account.

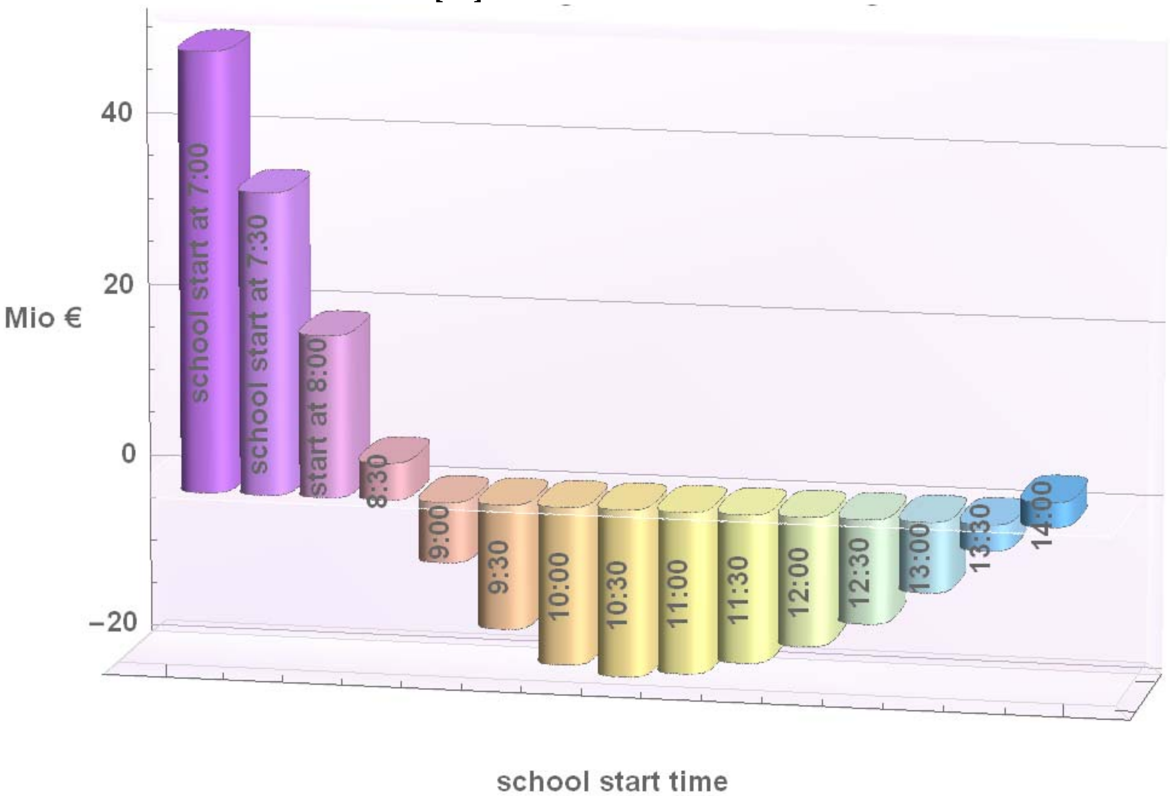

Fig. 39: Total collateral costs after one pupil-generation (10 years) for pupils averaged between 6 to 16 years for transport duration of 1 hour and a preparation time of 45 minutes. Total chronotype distribution after [34] was taken into account.