2. Find "Shortest" Individual Tour around the World

A somewhat more complex example would be to find a shortest tour around the world where every country has to be visited and individually adapt it to additional "soft" constraints. Here we assume that the traveler does not only seek for the shortest way taking the distances, but also considers things like:

- Likelihood of public transport being on strike in certain countries or suffering from other possible events
- Certain personal preferences regarding customs, languages, political- or law-systems etc. in certain countries
- Likelihood of conflict situations in certain regions possibly affecting the traveler (as an example one might just take the passenger aircraft being shot down from Ukrainian sky)
- Personal preferences with respect to the food and accommodation in certain regions, countries
- In the case of a sales man planning the route also market aspects and customer behaviour might be taken into account
- …

The idea is now to put all these personal constraints into a set of potentials and connecting this with the distances to be taken into account. In order to still keep the system of optimization linear, all the inertia will act on the system such a way, that masses being created due to the proximity of the traveler in certain regions of the world, the mass there will distort the earth surface and thus will effectively increase the distances to be considered there. As a result, such regions now provide effectively longer distances and the system will find different "personalized" routes. Just taking the distances, these routes are longer than the shortest one, of course, but they are "shortest" if taken all constraints, which is to say also those personal or individual "soft" ones. With the help of the Software Mathematica 10.0 and its inbuilt function "FindShortestTour" together with the country-data given in the software, the task of finding the physically shortest way can be solved easily (c.f. fig. 6).
Comparison with an extremely personalized route (fig. 7) the differences are made rather obvious. Of course, in some extreme cases one might suggest to simply skip trips with effective lengths exceeding certain critical values, but as a simple demonstrator of the technology proposed we do not discuss this option here.

Fig. 6: Result for our search of finding the shortest route around the world and visiting every country on the way. The figure was evaluated using the package "Find the Shortest Tour around the World" which is part of the package Mathematica 10.

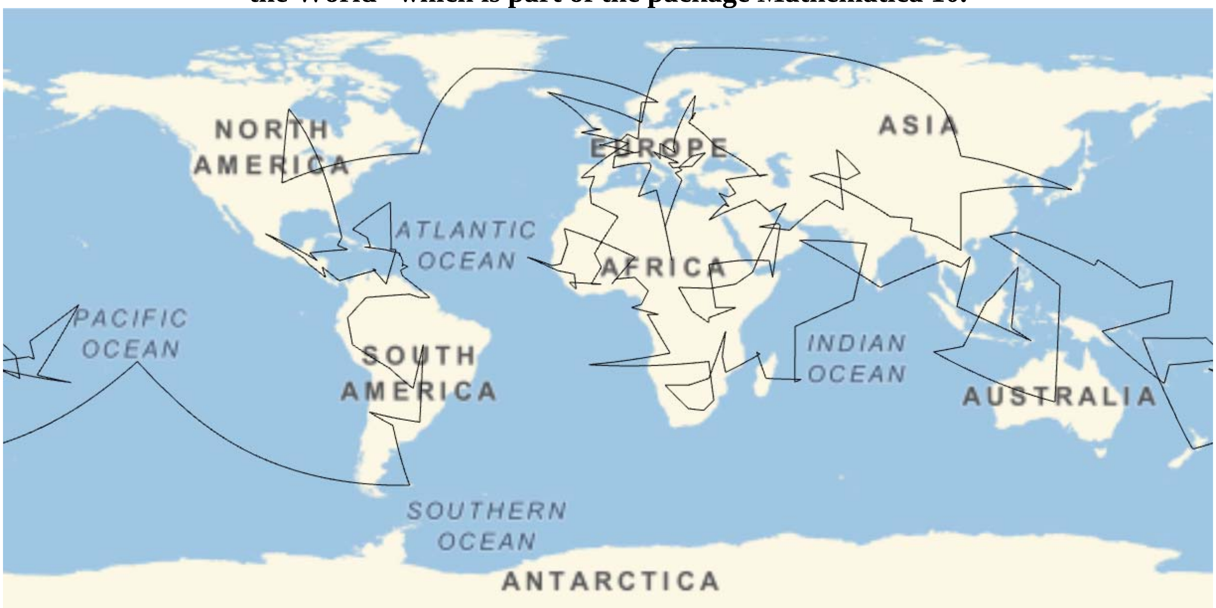

Fig. 7: Result for our search of finding the shortest personalized route around the world and visiting every country on the way using the "soft" weighing conditions as described in the text and evaluating them as "space distorting masses" leading to an effective increase of certain distances in connection with "problematic" countries from the traveler's individual point of view.

In order not to discredit or misleadingly judge on any country we kept the "soft" conditions considered here completely random.

The mass-evaluation and choice of potentials was performed as follows.
At first we introduced a dummy parameter z with $x_i(z)$ being the coordinates of multidimensional space and representing the true geometrical distance between two countries becoming parameter dependent and leading to the Lagrangian:

$$L(z) = \frac{1}{2}\left[\left(\frac{\partial\left(x_i(z)\right)}{\partial z}\right)^2\right] - U\left(x_i(z)\right), \tag{16}$$

with a yet undefined potential U. We immediately see that by comparing with the equations given in the previous section we result in the following Taylor expansion for U:

$$\begin{aligned} U\left(x\right) &= \sum_{k=1}^{Q} U_k\left(x\right) \\ U_k\left(x\right) &= U_k\left(x_i^{(0)}\right) + \frac{1}{2}\left[\sum_{i,j=1}^{N} \frac{\partial^2 U_k}{\partial x_i \partial x_j}\left(x_i - x_i^{(0)}\right)\left(x_j - x_j^{(0)}\right)\right] + O(x^3, y^3) \end{aligned} . \tag{17}$$

With N giving the number of trips and Q the number of "soft" properties being considered with each such trip. The Eigenvalue evaluation has now to be performed on the following matrix:

$$M_{kij}^2 = \left.\frac{\partial^2 U_k}{\partial x_i \partial x_j}\right|_{x_i = x_i^{(0)}} . \tag{18}$$

Leading to:

$$M_{kij}^2 R_{\alpha i} R_{\beta j} = \begin{pmatrix} \frac{\partial^2 U_k}{\partial \tilde{x}_1^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial^2 U_k}{\partial \tilde{x}_2^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial^2 U_k}{\partial \tilde{x}_i^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 U_k}{\partial \tilde{x}_N^2} \end{pmatrix} = M_{k\alpha}^2 \delta_{\alpha\beta} . \tag{19}$$

Giving the "masses" $M_{k\alpha}$ with α=1, 2, …N and k=1, 2, …Q of the N*Q "particles" of the system. Summing up all Q masses for one trip we result in the inertia being connected with this trip and can now decide upon a proper weighting of the effective distance of the trip within our tour optimization. In our example we simply connected higher masses with longer effective distances. The potentials were chosen as follows:

$$U_k = \sum_{i=1}^{N} q_{ki}^2 e^{-\frac{\left(x_i - x_{i0}\right)^2}{\sigma_{ki}}} . \tag{20}$$

Figure 7 shows one completely random individual result. Of course, now the traveler should weigh the distorted routes against his chances of successful business regarding those vertices of his personalized route leading to extra-long distances. He might erase those vertices contributing too much to such increased distances and restart the optimization with a new set of vertices respectively countries to visit.