1. Simple Example of an Adapted Optimization Problem with Soft Boundary Conditions

We start with the following example:
Max of y
In x<=20;–x+y<=10 ; 3x+2y<=65; 2x+3y<=60, x=>0, y=>0
Figure 1 is showing the search field surrounded by the boundaries given above. Its classical solution would be the point (6, 16)

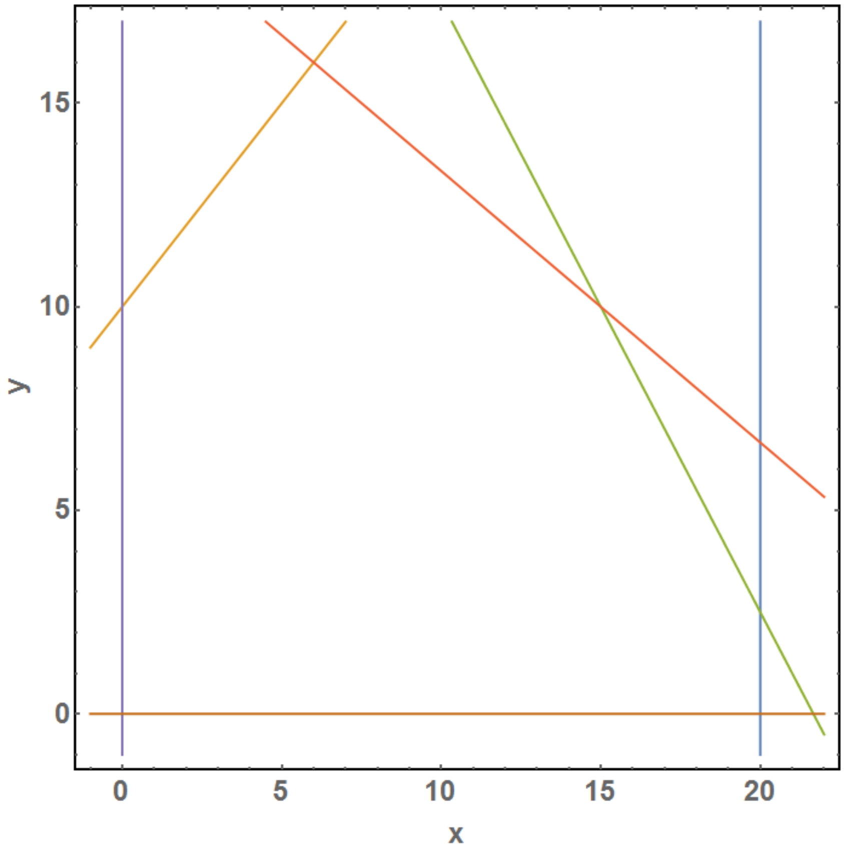

Fig. 1: Search field for the simple linear optimization task given in the text

Now we are adding the additional soft constraint, that x should not be in the region of 10. As an illustrative reason for this one could simply assume the x-axis to be time and that one does not want to plan anything around the typical breakfast-brake around 10 a.m.. Usually, one might be satisfied with a blocking region of about 1 hour but there might be special occasions where we want to have a somewhat bigger constraint. Applying coordinate transforms (e.g. as the ones given in (1) and as demonstrated in figures 2 and 3) we can now reformulate our optimization task using a Gauss-type coordinate transform ending up with the new search field as given in figure 4.

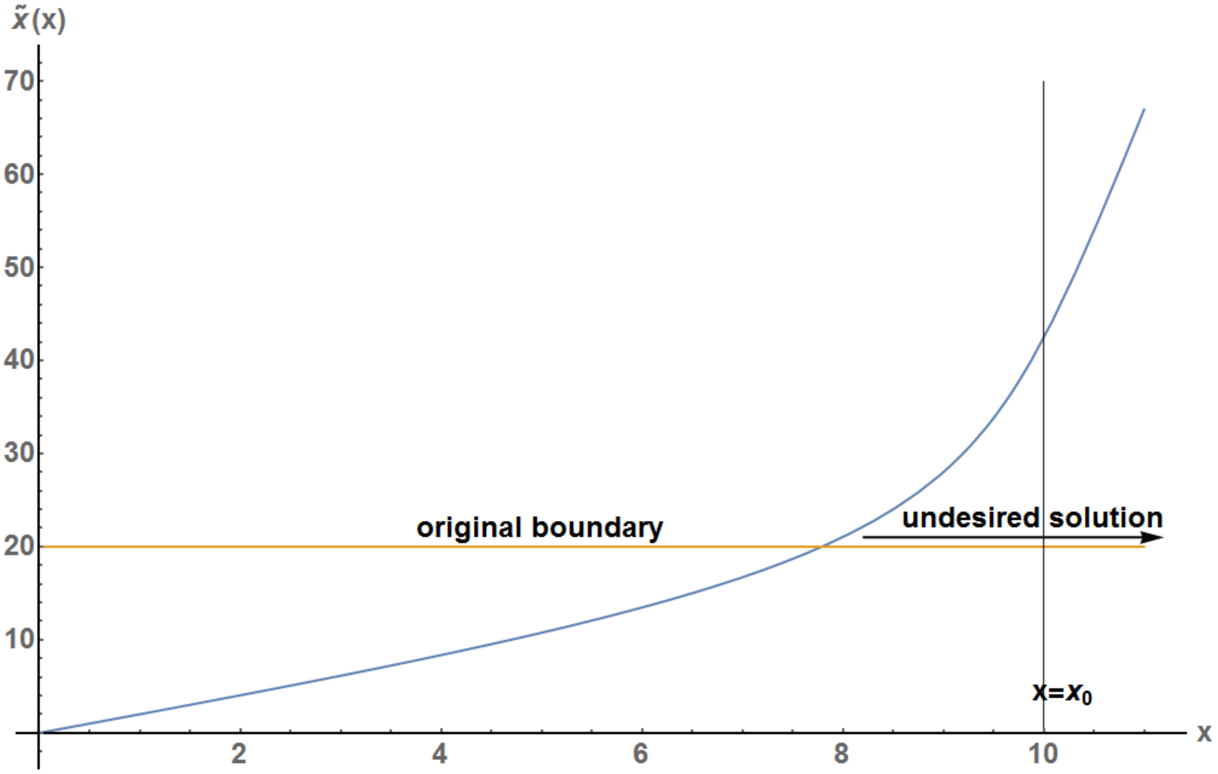

Fig. 2: Lorentz-Type-Coordinate-Transformation as given in equation (1) part one

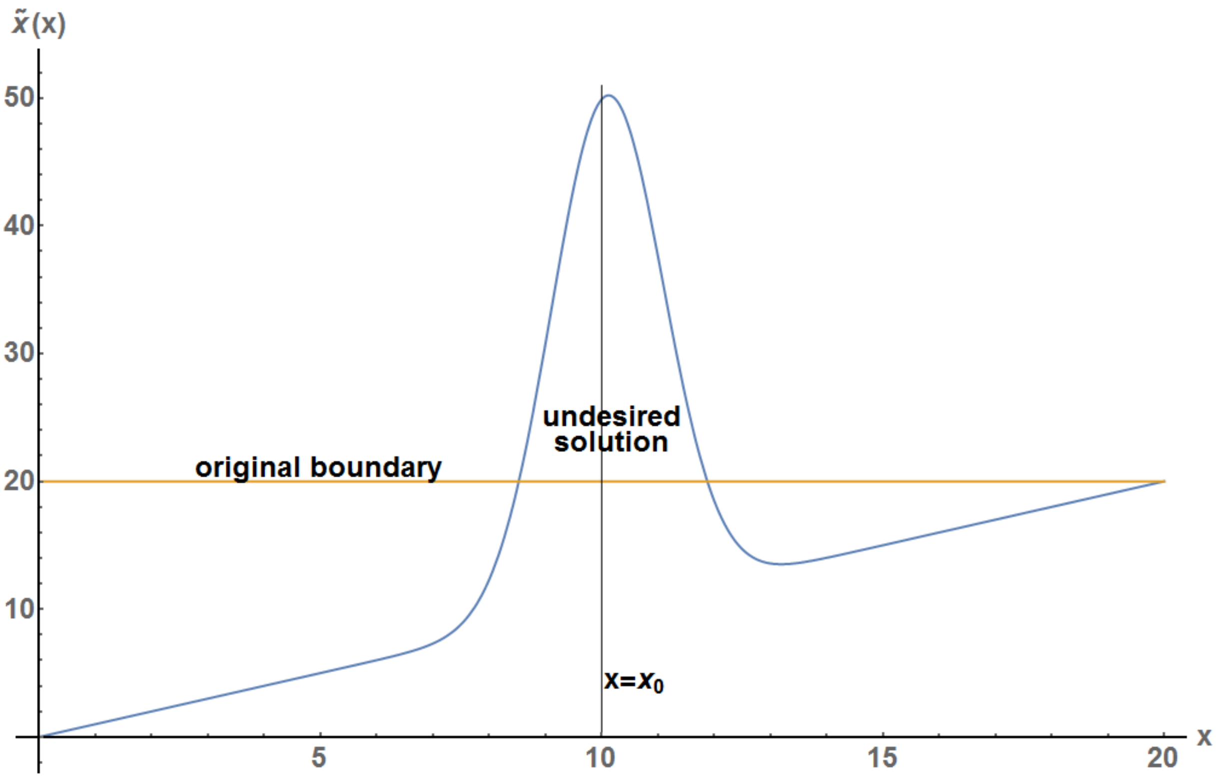

Fig. 3: Gauss-Type-Coordinate-Transformation as given in equation (1) part two

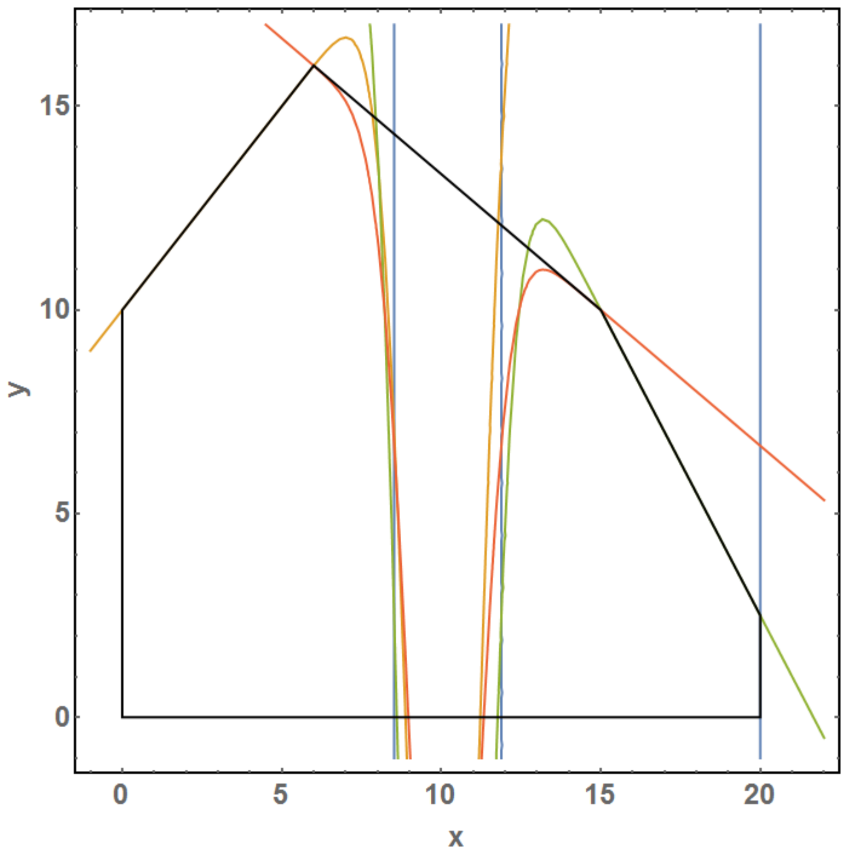

Fig. 4: Search field for the simple linear optimization task given in the text with the additional constraint added via Gauss-Type-Coordinate-Transformation as given in equation (1) part two

The original search field is now separated in two parts being placed around the region to be excluded. One can see that still the point (6, 16) provides the optimum solution. In order to keep the boundaries linear one might chose line approximations instead of the curved boundaries drawn in fig. 4. The "old" optimum becomes a false solution for wider exclusion ranges as shown in fig. 5. So far we had no need for an additional complexity like the Higgs mechanism, but now we are interested in keeping the range around our exclusion (reminder: x=10) more flexible and a function of a certain set of conditions. So, to give an example, it might be advisable after certain events, public holidays or elongated weekends to plan a bit longer for the break on the day after. As every employer would probably be able to name dozens of reasons for such a measure we simple call them "health factors" here and for obvious reasons it is clear, that it might be difficult or impractical to weave them into the original optimization by setting hard constraints. In principle, such a soft boundary condition is well-describable verbally (as we just did here), but it seems more difficult to mathematically formulate these "health factors" within an optimization procedure especially when intending not to perform too many changes on the latter. In addition we want to know the sacrifices the system has to make in order to still accept (or exclude) certain optima. To achieve this, we now resort to the Higgs mechanism.

We add a scalar field Φ dependent only on the parameter x.
We define the Langrangian.

$$L(x) = \frac{1}{2}\left[\left(\frac{\partial\left(\Phi(x)+y(x)\right)}{\partial x}\right)^{2}\right] - U\left(\Phi(x)\right), \tag{10}$$

With a yet undefined potential U. We immediately see that by comparing with the equations given in the previous section we can simply set $\Phi = \Phi_1$ and $y = \Phi_2$ resulting in the following Taylor expansion for U:

$$U\left(\Phi\right)=U\left(\Phi^{(0)}\right)+\frac{1}{2}\frac{\partial^2 U}{\partial\Phi^2}\left(\Phi-\Phi^{(0)}\right)^2+O(\Phi^3)\,. \tag{11}$$

Following the evaluation elaborated above the Lagrangian now reads:

$$L(x)=\frac{1}{2}\left[\dot{\vec{\Phi}}^2-\left|\vec{\nabla}\Phi\right|^2-M^2\Phi^2\right]+O\left(\Phi^3\right)+U\left(\Phi^{(0)}\right), \tag{12}$$

with:

$$M^2=\frac{\partial^2 U}{\partial\Phi^2}\,. \tag{13}$$

Directly giving the "mass" M of the "particle" of the system with the interaction the potential U has created. Regarding our example, the potential should be of the form:

$$U(\Phi=x)=-\frac{q^2}{\sqrt{\left(x-x_0\right)^2}}\quad\rightarrow\quad M=q\sqrt{\frac{2}{\left(\left(x-x_0\right)^2\right)^{3/2}}}\,. \tag{14}$$

Where we have already given the mass M. We immediately realize that the masses of the system are dependent on x assuring higher inertia for values of x close to x_0. Using this parameter within our optimization system and avoiding higher masses, we immediately see that the point x_0 is rigidly being avoided as the mass there is going to infinity. To assure this, we simply add the two boundaries $x-x_0-M<=0$ and $x-x_0+M>=0$.

There also is the option of just using the inertia-information for estimating the costs of a possible optimum solution close to the critical value $x=x_0$ without avoiding it. If these costs (M) are too high still some effort can be put into an adapted optimization system with either more or transformed (curved) boundaries keeping the original constraints in a transformed coordinate system.

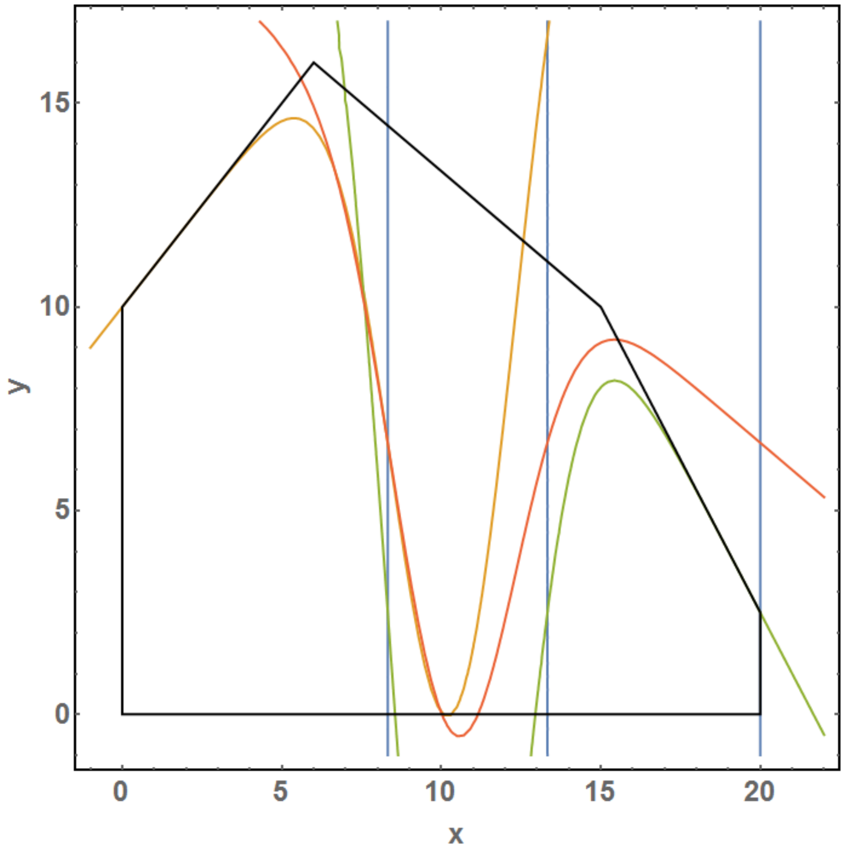

Fig. 5: Search field for the simple linear optimization task given in the text with the additional constraint added via Gauss-Type-Coordinate-Transformation as given in equation (1) part two

Taking yet another characteristic of the "classical" Higgs mechanism, one could also chose potentials of the kind:

$$U(|\Phi(x,y)|^2) = -\mu^2 |\Phi(x,y)|^2 + h|\Phi(x,y)|^4 . \tag{15}$$

The minimum of the potential then is at $|\Phi_0(x,y)| = \sqrt{\frac{\mu^2}{2h}}$ in dependence on both x and y. This way and by choosing a suitable function for $\Phi(x,y)$ one can make sure that the system does not automatically produces mass the moment x comes close to x_0 but only when also y reaches certain values. This is equivalent to the physical Higgs mechanism only locking in place when the total energy of the system is low enough while at higher energies the system can still be treated as being free of masses. Thus, only in situations where the system as a whole reaches a certain state some parameters (x in our case) are likely to produce mass and thus "soft" constraints are becoming active while they do not matter when the total conditions do not support their importance.