The Higgs field mechanism

It was shown in [22] that the introduction of a scalar field of the simple form

$$V(|\Phi|^2) = -\mu^2 |\Phi|^2 + h|\Phi|^4 \tag{2}$$

and its coupling to a vector field of none-massive particles can lead to massive particles for this combined scalar and vector field ensemble.
Here we briefly show the "creation" of mass in connection with interacting fields on an example of two scalar fields Φ_1 and Φ_2. With U being a potential of the scalar fields, we can write the Lagrangian:

$$\begin{aligned} L(x) &= \frac{1}{2}\left[\left(\frac{\partial\left(\Phi_1(x,t)+\Phi_2(x,t)\right)}{\partial t}\right)^2 - \left|\vec{\nabla}\left(\Phi_1(x,t)+\Phi_2(x,t)\right)\right|^2\right] - U\left(\Phi_1(x,t),\Phi_2(x,t)\right) \\ &= \frac{1}{2}\left[\dot{\Phi}_1^2+\dot{\Phi}_2^2-\left|\vec{\nabla}\left(\Phi_1+\Phi_2\right)\right|^2\right]-U\left(\Phi_1,\Phi_2\right) \end{aligned}, \tag{3}$$

where the symbol $\vec{\nabla}$ denotes the Nabla operator:

$$\vec{\nabla} f(x \equiv \begin{pmatrix} x \\ y \\ z \end{pmatrix}) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}, \tag{4}$$

and now we expand the potential U near the minimum of the potential part in (3) if considering the whole space:

$$V = \int \left\{ \frac{1}{2}\left[\left|\nabla\left(\Phi_1+\Phi_2\right)\right|^2\right] + U\left(\Phi_1,\Phi_2\right)\right\} dr^3 . \tag{5}$$

We assume a minimum of the expression (5) at the position $\{\Phi_1, \Phi_2\}^{(0)}$ being also a minimum of U (c.f. [23]). Taylor expansion of U around this point now yields:

$$\begin{aligned} U\left(\Phi_1,\Phi_2\right) &= U\left(\Phi_1^{(0)},\Phi_2^{(0)}\right) \\ &+\frac{1}{2}\left[\frac{2\partial^2 U}{\partial\Phi_1\partial\Phi_2}\left(\Phi_1-\Phi_1^{(0)}\right)\left(\Phi_2-\Phi_2^{(0)}\right)+\frac{\partial^2 U}{\partial\Phi_1^2}\left(\Phi_1-\Phi_1^{(0)}\right)^2+\frac{\partial^2 U}{\partial\Phi_2^2}\left(\Phi_2-\Phi_2^{(0)}\right)^2\right] \\ &+\frac{1}{6}\left[\begin{array}{l} \frac{\partial^3 U}{\partial\Phi_1^3}\left(\Phi_1-\Phi_1^{(0)}\right)^3+\frac{2\partial^3 U}{\partial\Phi_1^2\partial\Phi_2}\left(\Phi_1-\Phi_1^{(0)}\right)^2\left(\Phi_2-\Phi_2^{(0)}\right) \\ +\frac{2\partial^3 U}{\partial\Phi_1\partial\Phi_2^2}\left(\Phi_1-\Phi_1^{(0)}\right)\left(\Phi_2-\Phi_2^{(0)}\right)^2+\frac{\partial^3 U}{\partial\Phi_2^3}\left(\Phi_2-\Phi_2^{(0)}\right)^3 \end{array}\right] \\ &+O(\Phi_1^4,\Phi_2^4) \end{aligned} . \tag{6}$$

The linear terms disappear because per definition U was expanded around an extremum leading to zero for the first derivative with respect to the "coordinates" $\{\Phi_1, \Phi_2\}$.

Concentrating now only on the terms of the lowest order, we can construct a symmetric matrix of the form:

$$M_{ij}^2 = \begin{pmatrix} \frac{\partial^2 U}{\partial \Phi_1^2} & \frac{\partial^2 U}{\partial \Phi_1 \partial \Phi_2} \\ \frac{\partial^2 U}{\partial \Phi_1 \partial \Phi_2} & \frac{\partial^2 U}{\partial \Phi_2^2} \end{pmatrix}. \tag{7}$$

Main axis transformation via well-chosen pairs of rotation matrices R_{ij} gives us the Eigenvalues M_α^2 of this matrix:

$$M_{ij}^2 R_{\alpha i} R_{\beta j} = \begin{pmatrix} \frac{\partial^2 U}{\partial \tilde{\Phi}_1^2} & 0 \\ 0 & \frac{\partial^2 U}{\partial \tilde{\Phi}_2^2} \end{pmatrix} = M_\alpha^2 \delta_{\alpha\beta} = M_\alpha^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{8}$$

Written in the field components $\tilde{\Phi}_1, \tilde{\Phi}_2$ and ignoring all terms of order higher than 2 the Lagrangian now reads:

$$L(x) = \frac{1}{2}\left[\dot{\tilde{\Phi}}_1^2 + \dot{\tilde{\Phi}}_2^2 - \left|\vec{\nabla}\left(\tilde{\Phi}_1 + \tilde{\Phi}_2\right)\right|^2 - M_1^2 \tilde{\Phi}_1^2 - M_2^2 \dot{\tilde{\Phi}}_2^2\right] + O\left(\Phi_1^3, \Phi_2^3\right) + U\left(\Phi_1^{(0)}, \Phi_2^{(0)}\right), \tag{9}$$

where the last term only is an unimportant constant. Variation of the resulting action now shows us that the scalar fields have produced two massive particles with masses M_1 and M_2, while all higher order terms, being summed up in the expression $O\left(\Phi_1^3, \Phi_2^3\right)$, just contribute to the interaction between these particles. If necessary these can be treated via perturbation methods.

The procedure can easily be extended to any arbitrary number of scalar fields or to interaction of scalar with vector fields as elaborated in [22] and [23], for instance.