Combining scalar fields and optimization procedures

As it cannot be the goal for this short contribution to elaborate on the extremely wide and complex field of optimization (nonlinear, linear or integer) we have to refer to the literature with respect to this point. A nice overview about mathematical optimization in general is given in the chapter by Marthaler [1] in the book of Diest. Somewhat more applied contributions especially in connection with transport problems, we are also going to consider here, can be found in [2 – 21].

Now we simply combine field techniques like the Higgs mechanism creating mass and use the information of created mass to steer certain coordinate transformations such that our parameters previously leading to critical conditions will be constrained with further changes on our original optimization system. Here the advantage of the method may be seen. Instead of adding more constraints and thus more complexity to the system, we keep the system but only adapt the shape of convex polyeder as the search field in which we are seeking our solution. Otherwise, the system will not be changed at all, neither its dimension nor the optimization procedure. In essence, the added field couples to the original optimization problem, creates mass in dependence of additional constraints one is interested in and then these masses distort the original system. In addition, as a by-product we obtain information about the strength and character of our "weakly" or "softly" bound critical parameter our original optimization routine "would have liked" to produce. This might be helpful in estimation of mid- and long-term effects not being covered for by the original optimization system but "activated" by the excess of those "weak" boundaries. One might argue that there are more direct, simpler methods to deal with such "weak" boundaries but in highly complex systems or where we already have a working optimization procedure one very often does not want to make principle changes on or in the system itself but prefers to keep all the changes external. Be it due to the complexity of the original system or the functional character of those "weak" boundaries, in some cases it might even be impossible to treat such new constraints internally and thus only considerations separated from the original optimization are of choice. The method proposed here does provide such an option.

In addition to the Higgs mechanism in order to weight the soft constraints one also needs coordinate transformation techniques in order to restrict the search field in adaptation to the needs arising together with the new weak or soft boundary condition. The coordinate transforms we propose here are mainly of the Lorentz type as being applied in special relativity (e.g. [24, 25]) or of Gauss-Type (c.f. fig. 2 and 3).

$$\tilde{x} = x\left(1 \pm \frac{M}{\sqrt{1-\left(\frac{x}{x_0}\right)^2}}\right); \quad \tilde{x} = x\left(1 \pm \frac{M \cdot e^{-\frac{1}{2}\left(\frac{x-x_0}{\sigma}\right)^2}}{\sqrt{2\pi\sigma^2}}\right) \tag{1}$$